A Master Time Value of Money Formula

Floyd Vest

For Financial Functions on a calculator or computer, Master Time Value of Money (TVM) Formulas are usually used for the Compound Interest Formula and for Annuities. (See Formula 7 below. See the Appendix in the TI83 (p. A-55) or TI84 manuals. The manuals are almost identical for finance. You can download parts of a manual at www.ti.com/calc.) The following is a derivation of the TVM Formula for Future Value (FV) with examples and exercises.

You Try It #1
Check the Appendix, Financial Functions in the TI83/84 manual and find the Time Value of Money Formula for FV. Compare it to Formula 7 in this article.

Compound Interest Formula. First we need to derive the Compound Interest Formula, which is

(1) \( FV = PV(1 + i)^N \), where \( FV \) is the future value, \( PV \) is the present value, \( i \) is the interest rate per compounding period, and \( N \) is the number of compounding periods. We are using the terminology in the TI83 and TI84 manuals.

Example 1
For an example we calculate the \( FV \) for \( PV = \$100 \) invested for ten years at the annual nominal rate of 4%, compounded quarterly. The rate per compounding period is

\[ i = \frac{0.04}{4} = \frac{\text{annual nominal rate}}{\text{number of compounding periods per year}}. \]

\( N \) is the number of compounding periods, which is \( 10 \times 4 = 40 \) quarterly periods. So

\[ FV = 100 \left(1 + \frac{0.04}{4}\right)^{40} = \$148.89. \] The investment of \$100 grew to \$148.89 in ten years.

You Try It #2
For Example 1 above, what is the Future Value if 4% is compounded daily? Money Market Funds usually compound daily.
For the derivation, consider that compound interest earns interest on interest as recorded in the following table:

<table>
<thead>
<tr>
<th>Period</th>
<th>Beginning PV</th>
<th>Interest</th>
<th>FV at end of period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period 1</td>
<td>PV</td>
<td>$i(PV)$</td>
<td>$PV(1 + i)$</td>
</tr>
<tr>
<td>Period 2</td>
<td>$PV(1 + i)$</td>
<td>$i[PV(1 + i)]$</td>
<td>$PV(1 + i) \times PV(1 + i) = PV(1 + i)^2$</td>
</tr>
<tr>
<td>Period 3</td>
<td>$PV(1 + i)^2$</td>
<td>$i[PV(1 + i)^2]$</td>
<td>$PV(1 + i)^3$</td>
</tr>
<tr>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>Period N</td>
<td>$PV(1 + i)^{N-1}$</td>
<td>$i[PV(1 + i)^{N-1}]$</td>
<td>$PV(1 + i)^N$</td>
</tr>
</tbody>
</table>

We conclude, from the last line of the table, the Compound Interest Formula for $FV$ to be $FV = PV(1 + i)^N$ as described above.

Future Value of an Ordinary Annuity. To continue the derivation of the TVM Formula, we need the formula for the Future Value of an Ordinary Annuity of $N$ payments ($PMT$), each drawing compound interest at the rate $i$ per period as illustrated on the following timeline.

<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>. . .</th>
<th>$N - 1$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PMT</td>
<td>PMT</td>
<td>PMT</td>
<td>PMT</td>
<td>PMT</td>
<td>PMT</td>
<td></td>
</tr>
</tbody>
</table>

By the Compound Interest Formula, the $FV$ of the Ordinary Annuity is the accumulation of $PMT$s and the interest on each:

$FV = PMT(1 + i)^{N-1} + PMT(1 + i)^{N-2} + PMT(1 + i)^{N-3} + \ldots + PMT(1 + i)^{1} + PMT$.

Multiplying through by $(1 + i)$ we get

$(1 + i)FV = PMT(1 + i)^N + PMT(1 + i)^{N-1} + PMT(1 + i)^{N-2} + \ldots + PMT(1 + i)^{2} + PMT(1 + i)$.

Next we subtract to get

$(1 + i)FV - FV = PMT(1 + i)^N - PMT$, with intermediate terms subtracting out. Solving for $FV$ we have

$$FV = PMT \left[ \frac{(1+i)^N - 1}{i} \right]$$

as the formula for the Future Value of an Ordinary Annuity where $PMT$ is the payments over $N$ periods, each drawing compound interest at the rate $i$ per period. For an Ordinary Annuity, the $PMT$s occur at the end of periods.
Example 2
Consider payments of $500 = PMT invested at the end of each year for \( N = 30 \) years at the rate \( i = 6\% = 0.06 \) per year. The Future Value of the savings is

\[
FV = 500 \left[ \frac{(1 + 0.06)^{30} - 1}{0.06} \right] = 39,529.09.
\]

The amount invested was \( 30 \times 500 = 15,000 \) in \( PMTs \) to accumulate to $39,529.09.

You Try It #3
For Example 2 above, what is the Future Value of $500 per month for thirty years in a retirement investment program?

If there is also a \( PV \) invested at time 0 on the time line, then

\[
(3) \quad FV = PMT \left[ \frac{(1 + i)^N - 1}{i} \right] + PV(1 + i)^N. \quad \text{See the Exercises for an example.}
\]

Future Value of an Annuity Due. Consider \( N \) payments \( PMT \) occurring at the beginning of each period as illustrated on the following timeline:

\[
\begin{array}{cccccccc}
0 & 1 & 2 & 3 & \ldots & N - 1 & N \\
PMT & PMT & PMT & PMT & \ldots & PMT & PMT \\
\end{array}
\]

In this case each \( PMT \) draws compound interest for an additional period and accumulates to the \( FV \) so

\[
(4) \quad FV = (1 + i)PMT \left[ \frac{(1 + i)^N - 1}{i} \right] + PV(1 + i)^N \quad \text{for an Annuity Due.}
\]

For the Master TVM Formula, the following formula is used for two cases:

\( k = 0 \) if end of period payments,
\( k = 1 \) if beginning of period payments,

with \( G_i = 1 + i(k) \), where \( i \) is the interest rate per period, to give

\[
(5) \quad FV = G_i \times PMT \left[ \frac{(1 + i)^N - 1}{i} \right] + PV(1 + i)^N
\]

Note that for end of period payments, \( G_i = 1 + i(0) = 1 \) giving Formula 3; and for beginning of period payments \( G_i = 1 + 1(i) = 1 + i \) giving Formula 4.
Algebraic manipulation is conducted on Formula 5 to get the TVM Formula for FV to be

\[
FV = \frac{-PMT \times G_i}{i} + (1 + i)^N \left[ PV + \frac{PMT \times G_i}{i} \right].
\]

The algebra is left as an exercise.

**Cash Flow Sign Conventions.** One property of the final Master TVM Formula is that sign changes are made to accommodate Cash Flow Sign Conventions. For example consider a Cash Flow timeline with PV and PMTs negative and FV positive.

\[
\begin{array}{cccccccc}
0 & 1 & 2 & 3 & \ldots & N-1 & N \\
\downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \uparrow \text{FV} \\
PV & PMT & PMT & PMT & PMT & PMT & PMT \\
\end{array}
\]

In an investment of PV and PMTs, money going out is considered negative, and indicated under the timeline, and FV coming in at the end is positive, and indicated with an upward arrow on the timeline.

**Master TVM Formula.** To accommodate this sign convention, we will in Formula 6 change -PMT to PMT and PV to -PV to get Formula 7.

\[
(7) \quad FV = \frac{PMT \times G_i}{i} - (1 + i)^N \left[ PV + \frac{PMT \times G_i}{i} \right], \text{ where we have an annuity with payments PMT, interest rate per period of } i, N \text{ periods, and a } PV \text{ at time zero. } G_i \text{ is described above.}
\]

This Master TVM Formula for FV is in the manuals for the TI83 and TI84. Financial functions on other calculators may have different sign conventions. (See the TIBA35, TIBAII, and HP19B.)

**Example 3**

We will do an example for Formula 7. Considering the above timeline, we invest PV = -$100,000, PMT of -$10,000 at the end of each year for N = 25 years in a retirement program averaging 6% per year. Notice the sign conventions. What is the FV of the savings program? Substituting we have \( G_i = 1 \) since we have end of period PMTs.

\[
FV = \frac{-10,000}{0.06} - (1 + 0.06)^{25} \left[ -100,000 + \frac{-10,000}{0.06} \right]. \quad FV = $977,832.19.
\]

**You Try It #4**

For Example 3 above, investments earn 6% compounded monthly and the family has $200,000 in investments and are saving $500 at the end of each month for the next 25 years. How much is in their retirement fund?
To get the TVM Formulas for $PV$ and $PMT$ from Formula 7, you simply use algebra to solve. To solve for $N$, you can use logarithms. These solutions are left as exercises. To solve for $i$ for an Annuity requires an iterative program. See the references for some iterative calculator programs. To get the Compound Interest Formula from Formula 7, simply let $PMT = 0$, and remember the sign changes. For the Compound Interest Formula, you can solve algebraically for $i$.

The concept of discounting and PVOA. From the Compound Interest Formula, $FV = PV(1 + i)^N$ we get $PV = FV(1 + i)^{-N}$. This is referred to as discounting $FV$ to get $PV$. For an Ordinary Annuity, we can discount each of the $PMT$s to get a $PV$ of an Ordinary Annuity (PVOA) as the sum, $PVOA = PMT(1+i)^{-1} + PMT(1+i)^{-2} + PMT(1+i)^{-3} + \ldots + PMT(1+i)^{-N}$.

It left as an exercise to show that the PVOA of an Ordinary Annuity is

$$PVOA = PMT \left[ \frac{1-(1+i)^{-N}}{i} \right],$$

and to derive a formula for the Present Value of an Annuity Due (PVAD).

The concept of discounting is important because it comes up in loans, mortgages, long term financial planning, inflation, bond pricing, discounting Cash Flows to Initial Equity or Net Present Value in business planning, Yield to Maturity (YTM), Internal Rate of Return (IRR), other applications. Note here that the term $PV$ has taken on two meanings.

Examples for Exercises

The following is an example of the format of some of the application problems in the Exercises requesting formulas, answers, commentary, timelines, knowns, unknowns, variables, formulas, Financial Functions, code, and summary.

Instructions: Solve the following problems with formulas, a scientific calculator pad, list and label knowns, unknowns, draw a time line. Use any financial formulas that are convenient.

Then, solve with Financial Functions on a calculator or computer. See the calculator manual for the required code. The problems are taken from Chapter 14 of the TI83 and TI84 manuals. Write code and commentary. Identify the output, and put on units. Summarize the answer to the problem.

Problem: Consider a problem on p. 14-3 of the TI83 manual. A car costs $9000. You can afford monthly payments of $250 a month for four years. What (APR) annual percentage rate will make this possible?
Answer: From the problem, $N = 4 \times 12 = 48$ months. $PV = \$9000$. $PMT = \$250$ a month. $FV = 0$. $P/Y = 12$. Pmt:End since we have an ordinary annuity. The question is, what is $I\%$? Using Formula 8 for the $PV$ of an Ordinary Annuity, we have

$$9000 = 250 \left[ \frac{1 - (1 + i)^{-48}}{i} \right],$$

where $i$ is the monthly rate as a decimal and $APR = 12i$.

There is no closed form formula for solving for $i$ for an annuity.

TI83 code and commentary for the problem:

2\textsuperscript{nd} Finance Enter (To select TVM Solver.) (-)250 Enter 0 Enter 12 Enter \checkmark Enter (To select Pmt:End for payments.) Press \checkmark \checkmark \ldots (To highlight the I\% entry.) Press Alpha Solve (To solve for I\%). (You see a cursor on I\%.) You read I\% = 14.90\%.

Summary: With an APR of $I\% \leq 14.90\%$, the purchaser can afford the car.

($I\% = 100(i) \times$ (the number of compounding periods per year).)
Exercises

For some of the following exercises, solve with mathematics of finance formulas and with financial functions on a calculator. Use any mathematics of finance formula that is convenient. For all problems, show all your work, label all inputs, show formulas, label all answers, and summarize. For the basic mathematics of finance formulas and their derivations, see Luttman or Kasting in Unit 1 of this course.


\[ FV = PV \left(1 + \frac{r}{k}\right)^N, \text{ where } r \text{ is the annual nominal rate compounded } k \text{ times per year.} \]

\[ N \text{ is the number of compounding periods. I\% is a percent. } r = \frac{I\%}{100}. \]

2. For a mortgage of $100,000 at 18% per year for 30 years, what is the monthly payment? See p. 14-4 in the TI 83 Manual.

3. For a mortgage of $100,000 at 8.5% per year for 30 years, what is the monthly payment? See p. 14-6 of the TI 83 Manual.

4. For a 30-year, 11% mortgage with $1000 a month payments, what is the amount of the mortgage? See p. 14-7 of the TI 83 Manual.

5. On p. 14-3 of the TI 83 Manual, one problem is I\% = 6\%, PV = 9000, PMT = 350, FV = 0, P/Y = 3, what is N? Do this problem.

6. Set up the Cash Flow equation for the second timeline on p. 14-8 and verify the answers. The npv (Net Present Value) at 6\% = $2920.65 means the sum of the cash flows discounted at 6\% gives $2920.65. The (Internal Rate of Return) irr = 27.88\% makes the npv equal to zero. The Internal Rate of Return is the rate that discounts the remaining cash flows to the initial equity CF0. To do this calculation with a scientific calculator pad, store the 1 + i in STO and use RCL.

7. By algebra, derive Formula 6 from Formula 5.

8. Derive from Formula 3, the TVM Formula in the Appendix of the TI 83 Manual for \( N = \frac{\ln \left( \frac{PMT - FV \times i}{PMT + PV \times i} \right)}{\ln(1 + i)} \) for \( G_i = 1. \)
9. Derive from Formula 3, the TVM Formula in the Appendix of the TI 83 Manual for $PV$ with $G_i = 1$, which is 
$$PV = \left[ \frac{PMT}{i} - FV \right] \times \frac{1}{(1+i)^N} - \frac{PMT}{i}.$$ 
Indicate the required sign changes to satisfy the TVM sign conventions for cash flows.

10. Derive from Formula 3, the TVM Formula in the Appendix of the TI 83 Manual for $PMT$ with $G_i = 1$, which is 
$$PMT = -i \left[ PV + \frac{FV}{(1+i)^N-1} \right]$$ 
and indicate the required sign changes to satisfy the TVM sign conventions for cash flows. You may want to use the identity 
$$\frac{PV(1+i)^N}{(1+i)^N-1} = PV + \frac{PV}{(1+i)^N-1}.$$

11. What do you change in Formula 7 to get the Compound Interest Formula? Make changes and show the formula.

12. Derive the Formula for the Sum of an Ordinary Annuity from Formula 7. What strange results do you get and what sign change is required?

13. Derive a formula for the Present Value of an Annuity Due.

14. Write a paragraph with cash flow timelines discussing some of the entries and sign changes required in the TVM Solver and required by the TVM formulas.

15. Consider a fund of $PV$ dollars that provides annual withdrawals of $PMTs$ for $N$ years. Draw a cash flow timeline with sign conventions and give the signs for $PV$ and $PMTs$. Give a formula for this fund and withdrawals.

16. (a) Consider a loan of $PV$ dollars that is repaid with $PMTs$. Draw a cash flow timeline with sign conventions and give the sign changes for the variables. Give a formula for this loan and payments.
(b) Notice that financial variables have multiple meanings. It is this multivalency and abstractness that gives the power and generality to mathematics. In some publications, even more generic financial terms such as Rent ($R$), Principle ($P$), and Sum ($S$) are used. Sometimes applied problems in finance require formulas that are not among the common ones. Then you need to derive your own formula. For an example of such a derivation, derive a formula for the Sum $S_n$ of the first $n$ terms of a geometric sequence with first term $a$, common ratio $r$, second term $ar$, and $n$th term $ar^{n-1}$. Use the technique used to derive Formula 2 above.

17. A simplification of a formula for $i$ in the Appendix to the TI83 Manual is 
$$i = e^{\ln(1+i)} - 1.$$ 
Prove this formula. Use the definition of $\ln(1+i)$.

18. Some problems require two timelines. A person makes a debt of $2000$ that must be paid back in ten years at 5% interest compounded semiannually. To accumulate money
to pay the debt, after one year they start saving each year with the last deposit made when the debt is due. The investment pays 4% compounded annually. How much should they save each year?

19. From *Money* magazine, June 2011, p. 65: You don’t start saving until age 45, but then dutifully invest $10,000 a year until 65, assuming annualized 8% returns. At age 65 you’ll have $460,000. Your brother starts at 35, saves $10,000 a year but quits at age 45. At age 65 he has $675,000. Make the mistake of starting to save late, and you’ll have to work hard to make up the error. Problem: Do the calculations to get their numbers. Keep trying and you’ll get them. Explain *Money* magazine’s calculations.

20. As this course progresses, accumulate at least twenty reasons for knowing the financial mathematics and calculator skills to complement the Financial Functions on a calculator. Items such as The Financial Functions on a calculator are referred to as, “black boxes.”

21. From *Forbes*, May 9, 2011: In comparing ETFs (Exchange Traded Funds), it is important to calculate the effect on earnings of expense ratios charged for managing the investment. Vanguard MSCI Emerging Market (VWO, 49) has an expense ratio of 22 basis points. BlackRock’s iShares charges 69 basis points.

If your $10,000 earns 9% before expenses, the difference in expense ratios will balloon into a difference in final amount of $16,000 in your pocket in 2041. Over 30 years that difference of half a percentage point compounds to 13%. Problem: See if you can get these numbers. If you keep trying, you will figure it out. What lesson did you learn about investing? Did you compound with \((1 + 0.09 - 0.0022)\) or \((1 + 0.09)(1 - 0.0022)\)? Which is correct? How much difference does it make?
Side Bar Notes

Load Funds and No-load Funds. From USA Today, May 27, 2011: “Let’s look at the American Funds Growth Fund of America. The A shares … have gained 4.17% a year for the last decade. … To put this in perspective: A $10,000 investment in the fund would have gained $5,046 in 10 years.”

“If you paid the fund’s maximum 5.75% sales charge, however, your total return was 3.55%, … . Your gain has now shrunk to $4,174. You owe taxes on the distributions. … your after tax return would be 3.21%.”

“And if you had sold the fund after 10 years and paid taxes on you gains, you’d be left with a 2.96% average annual gain.”

“So you invested $10,000 in a taxable account, paid the sales charge, paid taxes on distributions and gains. Your $10,000 is now $13,387. At that rate, you’d double your money in about 24 years.”

“But you can reduce your cost. One easy way of course, is to buy a no-load fund. You pay no commission.” If you had the money invested in a Roth IRA or 401k, it would be after-tax money and there would be no income taxes. See IRS Publication 590.

For a no-load fund, see www.vanguard.com for the Vanguard Wellington Fund, which is the nation’s oldest balanced fund. 6.20% in the last 10 years. Expense ratio 0.34%. Some salesmen of load funds will tell you that you will pay indirectly a commission on a no-load fund. How is that with a 0.34% expense ratio on the no-load fund, and a 0.69% expense ratio on the load fund.

Problem: See if your can reproduce the above calculations. You can do most of them.
References

Most of the following references are cited in the annotated bibliography for this course, and provide derivations, exercises, and calculator programs for practice with the financial formulas, calculator Financial Functions, and graphs.


Answers to You Try Its

You Try It #2 \( FV = 100 \left( 1 + \frac{0.04}{365} \right)^{3650} \) $= 149.18.

You Try It #3 \( FV = 500 \left[ \left( \frac{1 + \frac{0.06}{12}}{\frac{0.06}{12}} \right)^{360} - 1 \right] \) $= 502,257.49

You Try It #4 \( FV = 200,000 \left( 1 + \frac{0.06}{12} \right)^{300} + 500 \left[ \frac{1 + \frac{0.06}{12}}{\frac{0.06}{12}} \right]^3 - 1 \) $= 892,993.93 + 346,496.97 = 1,239,490.90
Answers to Exercises

1. To solve the problem with a formula, use the Compound Interest Formula 9 given above: Substituting in Formula 9, we have \(2000 = 1250 \left(1 + \frac{r}{12}\right)^{84}\) since there are \(7 \times 12 = 84\) compounding periods. Using algebra and a scientific calculator pad to solve for \(r\), we have \(r = 0.06733 = 6.733\%\) as the annual nominal rate. For the financial calculator code and explanation for the TVM Solver, see page 14-3 of the TI 83 Manual, or see the TI 84 Manual.

2. We will use Formula 8 above. Substituting we get

\[
100,000 = PMT \left[ 1 - \left(1 + \frac{0.18}{12}\right)^{-360} \right] \left( \frac{0.18}{12} \right).
\]

Solving and using a scientific calculator pad gives \(PMT = 1507.09\) as the monthly payment. We could have used Formula 3 above with \(FV = 0\) and \(PMT\) would have come out negative. For a financial calculator code, see p. 14-4 of the TI 83 Manual, or see the TI 84 Manual, for the TVM Solver.

3. For this problem, the TI 83 Manual presents the \texttt{tvm_Pmt} financial function, which is menu item 2 under \texttt{2^nd Finance}.

Code and comments: Put the knowns in the TVM Solver. Put 0 in PMT.
Code: \texttt{2^nd Finance} \(\triangleright\) (to \texttt{tvm_Pmt}) \ Enter \ Enter \ Enter \ Enter \ Enter \ Enter \ Enter

(On the home screen you read \(-768.91\). The monthly payment is \$768.91.)

4. Substituting into Formula 6 and calculating with a scientific calculator pad gives \(PV = 105,006.35\), as the amount of the mortgage.

The TI 83 Manual presents the \texttt{tvm_PV ([N,I\%,PMT,FV,P/Y,C/Y])} function for this problem. On the home screen, you store with the \texttt{STO>} key, 360 in \(N\), 11 in \(I\%\), \((-)1000\) in PMT, 0 in FV, 12 in P/Y and calculate PV.

Code and comments: (On the home screen.) \(360 \rightarrow 2^\text{nd} \text{Finace} \rightarrow \text{Enter}
(To Sto 360 in N.) \Alpha : 11 \rightarrow 2^\text{nd} \text{Finace} \rightarrow \text{Enter}
(To I\%) \text{Enter}
Alpha : \((-)1000 \rightarrow 2^\text{nd} \text{Finace} \rightarrow \text{Enter}
(To PMT.) \text{Enter} \ Alpha : 12
→ 2^\text{nd} \text{Finace} \rightarrow \text{Enter}
(To P/Y.) \text{Enter} \ Enter \ 2^\text{nd} \text{Finace} \rightarrow
(To tvm_PV.) \text{Enter} \ Enter \ Enter \ Enter \ Enter (You read 105006.35.) The amount of the mortgage is \$105,006.36. (It is more convenient to enter values directly into the TVM Solver when possible.) The above code didn’t store values to the TVM Solver.

5. Substituting into Formula 8 above and using logs to solve, you get \(N = 36.47\) payments. The TVM Solver gives the same answer.
11. To get the Compound Interest Formula \( FV = PV(1 + i)^N \) from Formula 7, set \( PMT = 0, \ G = 1 \) and make a sign change.

12. To get the Formula for the Sum of an Ordinary Annuity from Formula 7, set \( PV = 0 \) and \( G = 1 \), and make a sign change to get Formula 2.

13. To get a Formula for the Present Value of an Annuity Due, use Formula 8 and since \( PMTs \) are at the beginnings of periods, use
\[
PVAD = PMT \left[ \frac{1-(1+i)^{-n}}{i} \right] (1+i).
\]

15. For the fund, \( PV \) is negative and \( PMT \) is money coming in and is positive.

16. For the loan, \( PV \) is money coming in and is positive, \( PMTs \) are money going out and are negative.

18. They should save $272.96 at the end of each year for ten years.
**Teachers’ Notes**

**The TVM Formulas.** This article presents derivations of the basic mathematics of finance formulas found in Luttman and Kasting in Unit 1 of this course, but in the form of the TVM Formulas in the appendix to the TI83/84. These formulas utilized time line sign conventions for variables. For example for a loan, \( PV \) is positive since it is thought of as money coming in, and \( PMT \) is negative since it is thought of as money going out. All of this is explained in the article.

TI 83/84 Financial Functions are indicated in some the exercises. Students can use other calculators if they wish. Instructions for use of the TI83/84 are include.

**A lifetime file on financial mathematics.** Have students set up and maintain a lifetime file on personal finance and financial mathematics. In the future, this article will help students use their TI83/84 in personal finance. Students should be encouraged to buy their own calculators. They will last them a lifetime.

**Additional examples and exercises.** The articles in the references provide additional examples and exercises which students can do with their graphing calculators and financial calculators. Some give programs for solving for \( i \) in annuity formulas, as well as associated graphs.