Calculating the Value of Social Security Benefits for Different Retirement Ages

Floyd Vest

Many people calculate the value of Social Security Benefits for different ages of retirement. There are dozens of different methods of calculation, dozens and dozens of different personal conditions, assumptions, and needs. People often claim that it is best to begin the benefits at age 62. The following is a series of calculations displaying this conclusion. These calculations are followed by more serious approaches.

Our Person A used the Social Security Quick Calculator (ssa.gov/OACT/quickcalc/index.html). They entered a date of birth of 6/15/1949 and retirement at 9/2011. They selected estimates in today’s dollars, and entered $40,000 as 2011 income, then clicked Calculate. The Calculator response was that you are retiring at age 62 and 3 months at $874 per month.

To evaluate the age 62 choice, Person A calculated $874 \times 12 = $10,488 per year. They had read that the average life expectancy for a white male at age 66 is 16.4 years. To calculate the value of the future benefits, they calculated the Future Value of an annuity of 10,488 per year for 20.4 years at 6% to get $FV = 399,027$. (For derivations of the basic formulas for Compound Interest and Annuities, see the first unit of this course. For the $FV$ formula for an ordinary annuity, see the Side Bar Notes.)

Person A considered retirement at age 66, which is normal Social Security retirement age for their age group. They entered 9/2015 and got the response that you are retiring at age 66 and 3 months at $1235 per month. The Calculator is assuming that Person A works until 9/2015.

To evaluate the age 66 choice, they calculate $1235 \times 12 = $14,820 per year. Calculating the value of future benefits, the Future Value of an annuity of $14,820 per year for 16.4 years at 6% gives $FV = 395,263$.

Comparing $395,263$ for age 66 with $399,027$ for age 62, they conclude that age 62 is the best choice

**Summary 1: With Average Life Expectancy**

<table>
<thead>
<tr>
<th>Age</th>
<th>S. S. Benefit</th>
<th>Future Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>62</td>
<td>$10,488</td>
<td>$399,027</td>
</tr>
<tr>
<td>66</td>
<td>$14,820</td>
<td>$395,263</td>
</tr>
</tbody>
</table>

Person A had second thoughts and decides to consider the Calculator choice of estimates in inflated (future) dollars. For age 62, they got the same $874 per month. For age 66, they entered the same 9/2015 for retirement date and $40,000 for 2011 income. The Calculator responded that you are retiring at age 66 plus 3 months with a benefit of $1,321 per month ($15,852 per year). A check of the option of listing Estimated
Earnings reveals that earnings from age 62 to age 65 are increased at inflation rates. Again calculating the Future Value of the retirement benefits, using 6%, 16.4 years, and $15,852 gives $FV = 422,788$. But a friend comes along and says that for comparison, this should be discounted to age 62 which gives $422,787.77(1 + 0.06)^{-4} = 334,888$. This calculation again “proves” the case for age 62. (See Exercise 1 for other common ways to value Social Security. See Exercise 11 for doing these $FV$ calculations on the TI83/84 TVM Solver. See Exercise 9 for doing these $FV$ calculations with 2%. You get different results. Which long-term interest rate makes more sense?)

Summary 2: With Average Life Expectancy, Age 66 with inflation

<table>
<thead>
<tr>
<th>S. S. Benefit</th>
<th>Future Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age 62</td>
<td>$10,488</td>
</tr>
<tr>
<td>Age 66</td>
<td>$15,852</td>
</tr>
<tr>
<td>Age 66 discounted</td>
<td></td>
</tr>
</tbody>
</table>

The reader probably knows that Social Security Benefits are indexed to inflation and that the above calculations didn’t consider it. This suggests that we should make a more serious calculation resulting in a comparison. We will continue with the 6% and the average life span, although these are questionable. All calculations will be converted to age 62. The formula to be used is called the Present Value of an Annuity with Rents Increasing at a Constant Rates with $r = 6\%$ and $I = 3.24\% = 0.0324$ (COLA) which is the average inflation rate for the last 98 years. (See usinflationcalculator.com.) The formula is

\[ P_1 = R \left( \frac{1 - (1 + y)^{-x}}{y} \right), \quad \text{where} \quad y = \frac{r - I}{1 + I}. \]

Think of $P_1$ as the amount in a fund that earns interest at the rate $r$ and provides for yearly withdrawals starting with $R(1 + I)$ at the end of the first year, and increasing each year at the inflation rate $I$ for $x$ years. (See “Taking the Long View of Life,” (2002) in this course for a derivation of this formula.)

Using $I = 0.0324$ and $r = 0.06$ and $y = \frac{0.06 - 0.0324}{1 + 0.0324}$, and applying this formula for age 62 retirement with $R = 10,488$ and $x = 20.4$ years, gives $P_1$ at age 62 of $163,283$. (See Exercise 12 for using the TI83/84 TVM Solver for this $PV$ calculation.)

For age 66, using the above inflated dollar yearly benefit of $15,852$ and 16.4 years gives $P_1$ at age 66 = $208,264.02$. Discounting this at 6% for four years back to age 62 gives $164,965$. Comparing this present value with the above $163,283$ for age 62 suggests: Why work four more years for the difference?
Summary 3: With Average Life Expectancy, Calculating with SS COLA

<table>
<thead>
<tr>
<th>S. S. Benefit</th>
<th>Present Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age 62</td>
<td>$10,488</td>
</tr>
<tr>
<td>Age 66</td>
<td>$15,852</td>
</tr>
<tr>
<td>Age 66 discounted</td>
<td></td>
</tr>
</tbody>
</table>

But to some people, the point is to not be average. Who wants to plan to die at the average age? The objective is to have adequate retirement income for as long as you live. We will redo the above calculations with this objective in mind. For the last twenty years, it has been stated that for an average 65 year old couple, chances are 20% that one or both will live to age 95. Recently, it has been announced that in the last fifteen years, average longevity for 65 year olds has increased by five years. We will use age 95 and continue with the questionable 6%.

For age 62, we have 33 years, $R = $10,488, and calculate $P_1$ at age 62 = $228,056.
For age 66, we have 29 years and $R = $15,852 to get $P_1$ at age 66 to be $317,061.10.
Discounting this value back to age 62 at 6% gives $317,061.10 (1 + 0.06)⁻⁴ = $251,142. Age 66 beats age 62 by $23,000.

Summary 4: With Age 95 Target, Calculating with SS COLA

<table>
<thead>
<tr>
<th>S. S. Benefit</th>
<th>Present Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age 62</td>
<td>$10,488</td>
</tr>
<tr>
<td>Age 66</td>
<td>$15,852</td>
</tr>
<tr>
<td>Age 66 discounted</td>
<td></td>
</tr>
</tbody>
</table>

Making another comparison, for age 62, the Social Security benefit at age 95 is $10,488(1 + 0.0324)³³ = $30,038. For age 66, at age 95 the benefit is $39,965. Some people would be willing to work another four years to accumulate higher yearly Social Security benefits as well as other retirement investments.

Social Security offers an extra benefit for delayed retirement up to age 70. For this date of birth of 6/15/1949 and retirement age of 70, they use a benefit formula of 1.32 times the normal age 66 retirement benefit (See ssa.gov/OACT/quickcalc/early_late.html. “...we will tell you the effect of delayed retirement as a percentage of your primary [age 66] amount.”) We can estimate the age 70 benefit to be $1.32(15,852)(1 + 0.0324)⁴ = $23,771.14. Receiving this benefit increasing each year at the rate of inflation for 25 years has a present value $P_1$ at age 70 of $429,406.79. But discounting it at 6% back to age 62 for eight years gives $269,415. However, the benefit at age 95 = $52,753. Again some people would like this additional security in their old age. (It is not clear if the 1.32 factor includes inflation, or includes work, from age 66 to 69. Scott Burns says it does not.) (See scottburns.com.)
Summary 5: With Age 95 Target, SS COLA, and Age 70

<table>
<thead>
<tr>
<th></th>
<th>S. S. Benefit</th>
<th>Present Value</th>
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<tbody>
<tr>
<td>Age 62</td>
<td>$10,488</td>
<td>$228,056</td>
</tr>
<tr>
<td>Age 66 discounted</td>
<td>$15,852</td>
<td>$251,142</td>
</tr>
<tr>
<td>Age 70</td>
<td>$23,771</td>
<td>$429,407</td>
</tr>
<tr>
<td>Age 70 discounted</td>
<td></td>
<td>$269,415</td>
</tr>
</tbody>
</table>

It now looks like age 70 is the winner over age 62 by $41,359.

**You Try It #1**
Which method for valuing Social Security benefits makes the most sense, a Future Value method or a Present Value method? Why? Are there any assumptions under which the Future Value makes sense?

**You Try It #2**
Is Social Security enough money for retirement? Why would a person prefer to calculate the Present Values at age 66? What conclusions would you or someone else make about the numbers in these tables? Give any discussion you want. Do your own calculations if you wish. See the Side Bar Notes for ideas.
Exercises

1. (a) How would the comparison of age 62 and age 66 come out if the person simply added the annual benefits for the years of life expectancy, without considering inflation? (b) How would it come out if they used simple interest?

2. Use the formula for the Future Value of an Ordinary Annuity to do the calculation for age 62, which gave $399,027. What consideration was missing in this calculation?

3. If a loaf of bread costs about two dollars today, what did it cost 98 years ago, using the long-term average inflation rate of 3.24%?

4. Draw a timeline for the formula for the Present Value of an Ordinary Annuity with Rents Increasing at a Constant Rate (Formula 1). Put expressions involving \( R \) at various points. Do the calculations for age 66, for 29 years of drawing social security, resulting in \( P1 = \$251,142 \) in Summary 4. Explain the calculations and the meaning of this amount of money.

5. There are many assumptions, needs, attitudes, personal problems, and calculations involved in considering Social Security retirement. Discuss at least twenty of them. If you wish, do your own calculations. Ask people you know how they would do the calculations. Do you think some people would choose the calculations that tell them what they want to hear?

6. Derive the formula for the Future Value of an Ordinary Annuity.

7. Apply the formula for the Present Value of an Ordinary Annuity with Rents Increasing at a Constant Rate to an example of long-term financial retirement planning.

8. Help someone you know with their Social Security decision. Use one of the more accurate Calculators at ssa.gov. You may need to get their Social Security Statement in order to do a good job. Present your work to your class. Discuss the person’s individual problems, knowledge, and other conditions. Consider spousal benefits. Don’t give out their Social Security number.

9. If the author had used 2% instead of 6% as the long-term value of money (as some insurance companies would), how would age 62 come out in the Future Value comparison? Summarize for 2% and 6% with a table. Guess why the author chose 6%.
10. The following formula can be used in the Y= functions on a TI graphing calculator. It is a version of the formula for \( P1 \) in this lesson (Formula 1). It can be used to calculate the Present Value of Social Security benefits, or for long term financial planning, or for other uses.

Mathematically, the formula is

\[
Y1 = (1 + K)^B \left( \frac{E \left(1 + \frac{R - A}{1 + A}\right)^{-M}}{1 + \frac{R - A}{1 + A}} \right).
\]

When applied to Social Security benefits as was with \( P1 \) in this article:

- \( K \) is the annual discount rate.
- Put \( B \) in as negative and you discount for \( B \) years.
- \( E(1 + A) \) is the Social Security benefit at the end of the first year.
- \( R \) is the annual rate of return on money.
- \( A \) is the average inflation rate or COLA rate for Social Security.
- \( M \) is the number of years of drawing Social Security benefits.

The calculator formula for \( Y1= \) is

\[
((1 + K) \times B) \times E(1 - (1 + (R - A)/(1 + A)) \times (-M)) + ((R - A) + (1 + A)).
\]

Use ALPHA to put in letters. To check your calculator code, apply it to the age 70 example with \( E = 23771.14 \) and discounting to age 62 with a negative \( B \) to get \( Y1 = 269415 \) (As in Summary 5).

11. (a) Use the Financial Functions on a TI83 or TI84 for the age 62 example in this article with $10,488 per year for 20.4 years at 6% to get FV = $399,027.

TI83 code and commentary for the problem.

2nd Finance Enter (To select TVM Solver.) 20.4 Enter 0 Enter (In PV.) (-)10488 Enter 0 Enter (For FV.) 1 Enter (For 1 payment per year.) (Select Pmt:End.) \( \wedge \wedge \) (To highlight FV.) Alpha Solve And read: 399027.13

(b) Apply the TVM Solver to the age 66 example in this article which gets FV = $395,263.
12. Use the TVM Solver to calculate the age 62 example which gets $P1 = 163,283. To store in $I\%$, on home screen: $(.06 - .032) \div 1.0324 \times 100 \sto 2^{nd}$ Finance $\triangleright$ (To Vars.) $\checkmark$ (To highlight $I\%$.) Enter Enter $2^{nd}$ Finance Enter Enter (And you see 2.67 in $I\%$.) (Then finish the problem.)

(For derivations of the TVM formulas used in the TVM Solver, see “A Master Time Value of Money Formula” (2011) in this course.)

Using Financial Functions saves a lot of calculator strokes in doing the above examples in this article. You can now use them to help people with their Social Security calculations, or for long-term financial planning. (See “Taking the Long View of Life” (2002) in this course.)

13. In an article about Social Security on www.kiplinger.com entitled “Make the Right Moves to Boost Benefits,” the authors give an example of a white male with age 62 benefits of $1125 a month and retirement age 70 benefits of $1980 a month, a difference of $855 a month. Then for comparison, they consider the cost at age 70 of making up the extra $855 a month by buying an immediate lifetime income annuity of $855 per month. He would have to pay the insurance company $116,660 and that’s without an inflation adjustment or a survivor benefit. For a white male age 70, average life expectancy is 13.7 years. What is the interest rate paid by the insurance company?

Based on $855(12) = 10,260$ per year and averaging rates on an ordinary annuity and an annuity due, we get the interest rate paid by the insurance company is approximately 2.87%.

Problem: Using a calculator or computer, see if you can get this answer. Use any mathematical approach you prefer. Write in general form the mathematical formulas involved and then enter the numbers and indicate the unknown. Draw timelines. (For more information on Social Security, see kiplinger.com.)

14. It is recommended that people acquire retirement investments in addition to Social Security. In a recent study by Morningstar, the mutual fund rating company, Morningstar reported that low-cost no-load mutual funds out performed high-cost load funds on average.

For example, with domestic large-cap blend funds, in the least expensive ten percent, the expense ratio was 0.30 percent or lower, in the highest ten percent, expense ratios were at 1.54 percent. Over the three years ending April 30, the lower cost funds had a return advantage of 3.93 percent per year (Scott Burns, Denton Record Chronicle, June 19, 2011).

For a retirement saver investing $R$ dollars per year from age 20 to age 70, what percent difference does this make in the Future Value? Use $5\% = 0.05$ as a base rate with $5\% + 3.93\% = 8.93\%$ for the higher and $5\%$ for the lower. $FV$ for the lower = $R(209.348)$; for the higher paying fund, $FV = R(795.15204)$. 

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(a) Verify the above numbers.
(b) The higher rate accumulates at age 70 what percent of the lower rate? The difference is what percent of the lower?
(c) For a more concrete example, assume $5000 per year is invested and make the comparison.
(d) To consider what the 20 year old will need at age 70 to fund retirement, we calculate

\[ P = 50,000(1 + .0324)^{30} \left[ \frac{1 - (1 + y)^{-30}}{y} \right], \text{ where } y = \frac{0.08 - 0.0324}{1 + 0.0324} \]

to get \( P = 3,959,396.50 \).

Do you recognize this formula from this article? Draw a timeline and explain the numbers that are used.

15. To get enough money for a comfortable life and retirement, some people plan to win the Lottery. Some people even spend the milk money on the Lottery. When Maria Longonia won the $3.6 million Texas Jack Pot, she received $190,300 at the beginning of the first year and $180,000 a year for the next nineteen years. This totals $3,610,300. If the insurance company paid 2.5% per year, what did the Texas Lottery pay for the annuity? If the insurance company paid 3.5% what would the Texas Lottery pay? How much would they save by shopping rates?

But, if you don’t win the first time you play the Lottery, keep in mind, the only sure thing about luck, is it will change. If you hit the big one, quit playing and give someone else an even chance. If you really want to win, get a rabbit’s foot, but keep in mind, it didn’t bring the rabbit much luck. Of course, you probably know that you are more likely to be struck by lightning than to win the Jack Pot.
Side Bar Notes

You probably know this. Between early retirement and before full retirement age, one can earn extra money, but if they make too much then for every two dollars, their benefit is reduced by one dollar. (See ssa.gov.)

Once a person has reached full retirement age, there is no penalty for extra earnings. Social Security additionally provides benefits to a nonworking spouse. Social Security provides survivor and disability benefits.

A paper from the Boston College for Retirement Research indicates the optimal age for maximizing benefits is for a younger and lower-earning spouse to take Social Security benefits at age 62. The higher-earning and older spouse should take Social Security benefits at age 68 or 69. (scottburns.com). (See kiplinger.com.)

www.ssa.gov has several Social Security Calculators: Retirement Estimator (accesses a person’s earnings records), Online Calculator, Quick Calculator, and Detailed Calculator. It has a life expectancy calculator. One can request a Social Security Statement giving the history of their earnings.

Income taxes on Social Security. If a person’s modified adjusted gross income is high enough, then up to 85% of their Social Security benefit is taxed at their marginal income tax rate. See IRS 1040 Instructions, 2010.

Some average life expectancy tables. (See www.ssa.gov.)

<table>
<thead>
<tr>
<th>Age</th>
<th>White men</th>
<th>White women</th>
<th>Black women</th>
<th>Black men</th>
</tr>
</thead>
<tbody>
<tr>
<td>65</td>
<td>17.2</td>
<td>20.0</td>
<td>18.6</td>
<td>15.2</td>
</tr>
<tr>
<td>66</td>
<td>16.4</td>
<td>19.2</td>
<td>17.9</td>
<td>14.6</td>
</tr>
</tbody>
</table>

Recent annual inflation. As of April 2011 for the year, the Consumer Price Index is up 3.2%. Groceries are up 3.9%. MONEY, July 2011.

The $FV$ formula. The formula for the Future Value $FV$ of an ordinary annuity with interest rate $i$ per year, for $n$ years, with payments $PMT$ at the end of each year, is

$$FV = PMT \left[ \frac{(1+i)^n - 1}{i} \right].$$
How long will you live? Recently it has been published that for a couple age 60, chances are 40% that one or both will live to age 94. (MONEY, Jan./Feb. 2011) What are the chances that their money will last as long as they do? This statistic applies to people who are age 60 in about 2010. To what age will younger people live?

How much a year does it cost to live during retirement? The Federal Bureau of Labor Statistics reported that in 2009 the average person in the 65 to 74 age range spends about $43,000 a year, while the 55 to 64 range spends more than $52,000. (Smart Money, April 2011) How much will it cost when you are this age? But, who wants to plan to be average? Have your students study long-term financial planning. See “Taking the Long View of Life” (2002) in this course.

Will Social Security run out of money? According to the Congressional Budget Office, Social Security payments in 2011 will exceed program revenue by $130 billion. This shortfall is about equal to the benefits received by 9.2 million retirees. (Scott Burns, Denton Record Chronicle, April 17, 2011)

Can current retirees count on a four or five percent withdrawal rate from their retirement savings? A financial economist Moshe Milevsky says that they would be pushing their luck. With current investments paying less than 2%, he doesn’t expect interest, dividends, and stock appreciation to support withdrawals at this rate. The “probability of ruin” – going broke in retirement, is high. So what is the solution? Spend less. Work longer. (Scott Burns, Denton Record Chronicle, July 17, 2011) For the young, save adequate funds for retirement while you are young. Financial Mathematics problem: How much in retirement funds is needed to provide for inflation, longevity, standard of living, and to supplement Social Security, if your retirement fund earns only 2%?

People who planned for retirement. A recent study by HSBC found that people who planned for retirement has five times the assets of those who didn’t (MONEY magazine, August 2011).

Retirement Planning. For retirement planning software, see guidedchoicenow.com/quickadvice. By the Nobel laureate Harry Markowitz. But you get only part of it. There are hundreds of retirement planning softwares on the internet. Check brokers, mutual fund families, finance magazines, etc.

Center for Retirement Research, Boston College. See crr.bc.edu/ and for “The Social Security Claiming Guide” PDF 28 pages, see crr.bc.edu/special_projects/social_security_claiming_guide.html.

Report of the National Commission on Fiscal Responsibility and Reform, 2010 (fiscalreformcommission.org) The Commission has recommended that by 2050, the Social Security National Retirement Age (NRA Age) be 68 and by 2050, and it be Age 69 by 2075. Problem: Write an interpolation equation that gives NRA Age as function of Date in terms of year. Answer the question, when will a person with a certain current age and Date of Birth (DOB) reach the Age where their Age equals the NRA Age so they can retire with full Social Security Benefits. For a person Age 20 in 2010, DOB = 1990, they will reach full retirement Age at Age 68.33 years, in the year 2058.33.

Standard and Poor’s has downgraded the credit rating of the United States Federal Debt. On August 6, 2001, the U.S. credit rating was downgraded from AAA (the top rating) to AA+. The U.S. has been AAA since it was initiated in 1917. It was downgraded for the first time.
References

For the basics of mathematics of finance, see Kasting and Luttman below:


For the mathematics of long term financial planning for retirement and family security see the following:

Vest, Floyd, “Taking the Long View of Life,” HiMAP Pull-Out, Consortium 83. Derives Annuities with Rents Increasing at a Constant Rate and uses them to calculate the amount of money required to fund retirement, considering inflation, cost of living, rate of return on investments, longevity. Calculates saving required. Reviews basic Annuity Formulas and Compound Interest.

Answers to Exercises

1. (b) For an end of period simple interest annuity with simple interest rate \( r \), and annual benefits \( R \), and for \( n \) years, \( FV = nR \left[ \frac{2 + (n-1)r}{2} \right] \) and \( PV = \left( \frac{nR}{1 + nr} \right) \left[ \frac{2 + (n-1)r}{2} \right] \).

These formulas should help in part (b). Draw a timeline and derive these formulas. By simple interest at the rate \( r = 0.06 \) and \( n = 20.4 \) years, and \( R = \$10,488, \ PV = \$152,193 \). Do you think that someone who used simple interest on this comparison could derive these formulas?

3. Let \( x \) = the cost of a loaf of bread 98 years ago. \( x(1 + 0.0324)^{98} = \$2 \). \( x = \$0.09 \) a loaf.

4. Use Formula 1, with \( r = 6\% = 0.06, I = 3.245\% \) per year, and 29 years from age 66 to age 95. \( P1 = \$317,061 \). This is the amount of money which would fund Social Security benefits from age 66 to age 95 with an inflation COLA of 3.24\% and \( r = 6\% \) is the interest rate earned by the fund with the age 66 benefit of \$15,852. The fund is exhausted at age 95. To get the discounted value of \$317,061 back to age 62, calculate \( 317,061(1 + 0.06)^{-4} = \$251,142 \). (Do a graph approximating the amount of money in the fund from age 66 to age 95. You will see how the fund works. What do you think the graph will look like? Do a graph approximating the social security benefits from age 66 to age 95. Why do we say your graphs are an approximation? Derive a formula for the amount required in a fund to last forever. Can the answer be a finite number?)

14. (b) The higher earnings rate product accumulates at age 70 a balance which is 379.82\% of the balance at the lower earnings rate product. The higher rate has a balance at age 70 that is 279.82\% more than the balance for the lower rate.

(c) The lower rate accumulates \$1,046,740. The higher rate accumulates \$3,975,760.

PS: Low-cost index funds out perform 70\% of managed funds. There is no need for an index fund to have an expense ratio higher than 0.20\% (Scott Burns, Denton Record Chronicle, June 19, 2011).

15. At 2.5\%, the annuity cost the Texas Lottery \$2,886,500. At 3.5\%, it cost \$2,658,070. By shopping rates, the Texas Lottery might save \$228,430.
Teachers’ Notes

Have your students study inflation. In 1933: Milk cost 42 cents per gallon, Bacon cost 25 cents per pound, Eggs cost 16 cents per dozen, Hamburger cost 11 cents per pound, and Bread cost 7 cents a loaf (Seek Publishing, Remember When). What were the rates of inflation? For instructional materials, see “Living and Investing with Inflation” in this course.

A lifetime file on personal finance. Have you students set up a lifetime file on personal finance and include the materials from this course on that file. A financial calculator will last them a lifetime. Although Social Security will change, they now know how to use ssa.gov. They also know to monitor the financial world and periodically recalculate. They know about financial responsibility and the need to learn to make money and invest money, and that average is not good enough.

Code for the TI83/84 TVM Solver is in problems 11 and 12.

For the basics of Mathematics of Finance, see Luttman and Kasting, in this course.