Evaluating Investments, irr, mirr, npv. AM, TR, and HM

Floyd Vest, October, 2011 (Preliminary Version)

Consider an investment of $5000 as Initial Equity (IE) which yields cash flows indicated on the following time-line. What was the rate of return on the investment?

<table>
<thead>
<tr>
<th></th>
<th>$-5000</th>
<th>$3500</th>
<th>$-2500</th>
<th>$3800</th>
<th>$-3300</th>
<th>$14,500</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
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<td>2</td>
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<td>3</td>
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<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Calculating mirr. We will approach this evaluation using the Modified Financial Management Rate of Return abbreviated “mirr, modified irr”. Assume that the cost of money is 5.5% and money is saved to pay the negative cash flows. This gives

\[ P_1 (1 + .055)^2 = 2500 \text{ and } P_1 (1 + .055)^4 = 3300, \text{ with } P_1 = 2500(1 + .055)^{-2}, \text{ and} \]

\[ P_2 = 3300(1 + .055)^{-4} \text{ so that the amount invested was } 5000 + P_1 + P_2 = 9909.95. \]

We will separate the $9909.95 into \( P_3, P_4, P_5 \) so that $9909.95 = P_3 + P_4 + P_5, and \( P_3(1+i) = 3500, P_4(1+i)^3 = 3800, \text{ and } P_5(1+i)^5 = 14,500 \text{ where } i = \text{mirr}. \) These equations give \( P_3 = 3500(1+i)^{-1}, P_4 = 3800(1+i)^{-3}, \text{ and } P_5 = 14,500(1+i)^{-5} \) so that

\[
(1) \quad 9909.95 = 3500(1+i)^{-1} + 3800(1+i)^{-3} + 14,500(1+i)^{-5}.
\]

We are discounting the positive cash flows at the rate \( i = \text{mirr} \). Solving for \( i \) gives \( i = \text{mirr} = 23.458\%. \) (See the Exercises and Side Bar Notes for TI84 code for this calculation.) If a person uses the mirr function in a spreadsheet, they will be asked for the “cost of money rate” to be applied to negative cash flows.

Calculating irr. Another well known approach to evaluating an investment is to calculate the Internal Rate of Return (irr). The irr is the rate that discounts the cash flows to the Initial Equity (IE). This procedure would be similar to the above discounting of positive cash flows for calculating mirr. Calculating \( \text{irr} = i \) requires solving

\[ 5000 = 3500(1+i)^{-1} - 2500(1+i)^{-2} + 3800(1+i)^{-3} - 3300(1+i)^{-4} + 14,500(1+i)^{-5}. \]

Calculating gives \( i = \text{irr} = 35.0446\%, \text{ compared to mirr} = 23.458\%. \) (See the Exercises and Side Bar Notes for using the TI83/84 for the calculation.)

Why is it that irr is so high? Consider saving money at the rate \( i = .350446 \) to acquire funds to pay the negative cash flows. This gives

\[ P_1(1+i)^2 = 2500 \text{ and } P_2(1+i)^4 = 3300, \text{ with } P_1 = 2500(1+i)^{-2}, \text{ and } P_2 = 3300(1+i)^{-4}. \]

So that the amount invested is $5000 + 2500(1+i)^{-2} + 3300(1+i)^{-4}. This amount invested is equal to the positive cash flows discounted at the rate \( i \). This gives

\[ 5000 + 2500(1+i)^{-2} + 3300(1+i)^{-4} = 3500(1+i)^{-1} + 3800(1+i)^{-3} + 14,500(1+i)^{-5}. \]

Solving for $5000 gives

\[
(2) \quad 5000 = 3500(1+i)^{-1} - 2500(1+i)^{-2} + 3800(1+i)^{-3} - 3300(1+i)^{-4} + 14,500(1+i)^{-5}.
\]

Notice that this is the equation (definition) of \( i = \text{irr} \). The \( i = 35.0446\% \) is assuming that the investor invested money at this high rate to accumulate funds to pay the negative cash
flows, while mirr assumes those saving earned 5.5%. By using the irr, the investment is only $7763.28 while using the 5.5% the investment is $9909.95 and thus yields a lower rate of return. While irr is commonly used, one needs to examine the investment to see which rate of return is appropriate.

**Graphing a polynomial.** Equation 2 makes a nice polynomial

\[ y(x) = -5000(1 + x)^5 + 3500(1 + x)^4 - 2500(1 + x)^3 + 3800(1 + x)^2 - 3300(1 + x)^1 + 14,500 \]

where \( x = 1 + \text{irr} \) which should have a zero at \( x = 1.340446 \). A person might be surprised at the \( \text{irr} = 35.0446\% \) and wonder if there is a better zero and do a graph of the polynomial for \( 0 \leq x \leq 2 \). This is left as an exercise. Then one could look for zeros other than \( 0 \leq x \leq 2 \). To do this, Descarte’s Rule of Signs would be useful:

**Descarte’s Rule of Signs**

The number of positive zeros of a polynomial is less than or equal to the number of sign changes in \( y(x) \). The number of negative zeros is less than or equal to the number of sign changes in \( y(-x) \). In both cases, the number of zeros can be less by an even integer. Counting the sign changes in \( y(x) \), we get five. Counting the sign changes in \( y(-x) \), we get zero, to build the following table of possible zeros.

<table>
<thead>
<tr>
<th>positive</th>
<th>negative</th>
<th>complex</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>4</td>
</tr>
</tbody>
</table>

We will not look for negative zeros.

Next, we could use the Upper Bound Theorem to look for zeros greater than two.

**Upper Bound Theorem**

For a positive number \( c \), if \( y(x) \) is divided by \( x - c \) and the resulting quotient and remainder have no sign changes, then \( y(x) \) has no real zeros greater than \( c \). A lower bound can be found by determining an upper bound for \( y(-x) \). This requires dividing \( y(x) \) by \( x - 2 \). This is left as an exercise for the student. Since the resultant quotient and remainder have no sign changes, then \( y(x) \) has no real zeros greater than 2. Thus \( y(x) \) and Equation 2 have the one real zero which is approximately \( x = 1 + \text{irr} = 1.350446 \), with \( i = \text{irr} = 0.350446 = 35.0446\% \).

**Side Bar Notes:**

**Solving for mirr.** We will use in the TI84 the irr( function to solve Equation 1 for \( i = \text{mirr} \). On page 14-8 of the TI83 manual, you find irr(CF0,CFList). To do this, put the sequence of positive and zero cash flows: 3500, 0, 3800, 0, 14500 in a List. On the home screen,

Code and commentary: 2nd \{ 3500, 0, 3800, 0, 14500 2nd \} STO> Alpha L 7 Enter. This puts the numbers in the List named L7. Then 2nd Finance 8
Solving for net present value, npv(. Net present value is used to evaluate investments. Solving uses npv(interest rate, CF0, CFList). For the above example, code and commentary: 2nd Finance 7 Your see npv( Write 23.458, (-)9909.95, Alpha L 7 ) Enter and read npv = 0 as it should be. If npv is negative, what does that mean? If npv is positive, what does that mean?

Arithmetic Mean (AM) versus Total Return (TR). It is important to know the difference between arithmetic mean (AM) and total return (TR) for reporting returns on financial investments. The generally correct method is total return (TR). (There are other terms for Total Return such as effective rate, geometric mean, yield, and return.)

Example of Total Return (TR). Consider the annual rates of return -5% for the first year, and 15% the second year. The total return TR = i such that 

\[(1 - .05)(1 + .15) = (1 + i)^2, \text{ so } i = TR = \sqrt{(1 - .05)(1+.15)} - 1 = .0452272 = 4.52272%,\]
as the average total return (TR). For the arithmetic mean, \(AM = \frac{- .05 + .15}{2} = .05 = 5\%\).

It is a mathematical fact that AM \(\geq\) TR. The arithmetic mean, although useful in some applications, tends to overestimate the actual return on investments. Financial professional know this and may take advantage of it. So, it is important to know the difference, and ask questions.

Proof of a simple case. Consider the following proof for consecutive annual returns \(a\) and \(b\). To make the point of the proof clear, we will work the proof backwards and leave it as an exercise to reverse the steps.

Theorem: If \(a > -1, \) and \(b > -1, \) then \(\frac{a+b}{2} \geq \sqrt{(1+a)(1+b)} -1.\)

“Proof”:\[
\left[\frac{a+b}{2} + 1\right]^2 \geq \left[\frac{1+1}{2}\right]^2
\]
\[
\frac{a^2}{4} + 2\left(\frac{a}{2}\right)\left(\frac{b}{2}\right) + \frac{b^2}{4} + a + b + 1 \geq 1 + a + b + ab
\]
\[
\frac{a^2}{4} + \frac{ab}{2} + \frac{b^2}{4} - ab \geq 0
\]
\[
\left(\frac{a}{2} - \frac{b}{2}\right)^2 \geq 0
\]

Wall Street Journal Example (Oct. 8, 2003, p D2) They considered a return of 100% the first year and -50% the second year. \(AM = \frac{1+(-.5)}{2} = .25 = 25\%.\)

\(TR = \sqrt{(1+1)(1- .5)} - 1 = 0.\) So in this dramatic example, the AM exceeds the TR and may give misinformation.
Long term returns on stocks. Consider the following table giving average annual rates of returns for stocks.

Annual Rates of Return on Stocks (End of 1926 through 1988), Roden p. 287

<table>
<thead>
<tr>
<th></th>
<th>Total return</th>
<th>Arithmetic mean</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small Company Stocks</td>
<td>12.3%</td>
<td>17.8%</td>
<td>35.6</td>
</tr>
<tr>
<td>Common Stocks</td>
<td>10.0%</td>
<td>12.1%</td>
<td>20.9</td>
</tr>
</tbody>
</table>

If a person gets an advertisement of a 17.8% average return on small company stocks, they should investigate what kind of average is being quoted.

Harmonic Mean and Dollar cost averaging. Dollar cost averaging takes advantage of the stock market’s only certainty, stock prices fluctuate. All one does is invest equal amounts in a given investment at regular intervals. As a result, the average share cost will always be less than or equal to the average share price. Share price could apply to stock prices or to Share prices of a mutual fund. Consider the charts below:

<table>
<thead>
<tr>
<th>Rising Market</th>
<th>Investment</th>
<th>Share Price</th>
<th>Shares Acquired</th>
<th>Investment</th>
<th>Share Price</th>
<th>Shares Acquired</th>
</tr>
</thead>
<tbody>
<tr>
<td>$400</td>
<td>$5</td>
<td>$49</td>
<td>235</td>
<td>$400</td>
<td>$16</td>
<td>25</td>
</tr>
<tr>
<td>400</td>
<td>8</td>
<td></td>
<td>50</td>
<td>400</td>
<td>10</td>
<td>40</td>
</tr>
<tr>
<td>400</td>
<td>10</td>
<td></td>
<td>40</td>
<td>400</td>
<td>8</td>
<td>50</td>
</tr>
<tr>
<td>400</td>
<td>10</td>
<td></td>
<td>40</td>
<td>400</td>
<td>8</td>
<td>50</td>
</tr>
<tr>
<td>400</td>
<td>16</td>
<td></td>
<td>25</td>
<td>400</td>
<td>5</td>
<td>80</td>
</tr>
<tr>
<td>Total</td>
<td>$2000</td>
<td>$49</td>
<td>235</td>
<td>$2000</td>
<td>$47</td>
<td>245</td>
</tr>
<tr>
<td></td>
<td>Average share cost $8.51 ($2000/235)</td>
<td>Average share cost $8.16 ($2000/245)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Average share price $9.80 ($49/5)</td>
<td>Average share price $9.40 ($47/5)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notice that in each chart, the Average share costs is less than the Average share price. To generalize, let $P_i$ denote each of five share prices, and $I$ denote the dollar amount of the regular investment. The Average share price $= \frac{\sum_{i=1}^{5} P_i}{5}$.

The Average share cost $= \frac{5I}{\sum_{i=1}^{5} \frac{I}{P_i}} = \frac{5}{\sum_{i=1}^{5} \frac{1}{P_i}}$. (see the Exercises for explaining this formula.)

We are observing that $\frac{5}{\sum_{i=1}^{5} \frac{1}{P_i}} \leq \frac{\sum_{i=1}^{5} P_i}{5}$. By definition, the left side is the Harmonic Mean (HM) and the right side is the Arithmetic Mean (AM). We have observed a well known inequality, $HM \leq AM$. 
Consider a proof of the simplest possible case for $P_1$ and $P_2$ with $P_1 \geq P_2$, and both positive. We note that 
\[
\frac{2}{\frac{1}{P_1} + \frac{1}{P_2}} = \frac{2P_1P_2}{P_1 + P_2} = \text{HM}(P_1, P_2).
\]
\[
(P_1 - P_2)^2 \geq 0
\]
\[
P_1^2 - 2P_1P_2 + P_2^2 \geq 0
\]
By adding $4P_1P_2$ to both side, we get
\[
P_1^2 + 2P_1P_2 + P_2^2 \geq 4P_1P_2 \quad \text{or} \quad (P_1 + P_2)(P_1 + P_2) \geq 4P_1P_2.
\]
Dividing both sides by $2(P_1 + P_2)$, which is greater than zero, we get 
\[
\frac{(P_1 + P_2)}{2} \geq \frac{2P_1P_2}{P_1 + P_2}.
\]
Thus $\text{AM}(P_1, P_2) \geq \text{HM}(P_1, P_2)$. This is why dollar cost averaging works.

In finance, the harmonic mean is the preferred method for averaging multiples such as the price/earnings ratio. A common error is to average them with the arithmetic mean.

Yield to Maturity (YTM). The term Yield to Maturity has several different meanings and applications. We may discuss this in a later article.
Exercises: Show all your work. Label numbers and answers. Summarize. Substitute numbers into formulas.

1. (a) Use 20% in Equation 1 and calculate and interpret npv. (b) Write the mathematical equation for npv. (c) Use a scientific calculator pad to check the npv.

2. (a) Calculate npv for Equation 2 at 22.40%. Interpret. (b) What should npv be at 35.044%?

3. For the IE = -$2000 and end of year cash flows of $1000, -$2500, $0, $5000, and $3000: (a) Calculate and interpret npv at 6%. (b) Calculate and interpret irr. (c) Calculate and explain mirr, with the cost of money at 6%.

4. Do the calculations to get the $9909.95 in Equation 1.

5. (a) Use the idea of separating the IE = $5000 into $P_1, P_2, ..., P_5$ (used in the above discussion of mirr) to derive the definition of $i = irr$. State the definition. (b) Then do a derivation in general terms for $CF_1, ..., CF_n$. Do you think this derivation reveals the problem with irr discussed above?

6. Calculate the future value of the positive cash flows in the mirr example at 20% and then discount the FV to 9909.95 at a rate $i$. What is $i$? Interpret.

7. Divide the above polynomial $y(x)$ by $x - 2$ and interpret the results.

8. (a) Make a polynomial from Equation 1 and apply Descarte’s Rule of Signs and the Upper Bound Theorem. Summarize and explain. (b) Graph the polynomial to estimate the zero. Use Trace to estimate the zero. (c) To get a better look at the polynomial, use Window: $x_{min} = -10, x_{max} = 10, x_{scl} = 1, y_{min} = -10,000, y_{max} = 40,000, y_{scl} = 1000$. Draw a hand graph. What is the $y$-intercept? Are there any local maximum or minimum points?

9. (a) For Equation 2, show that the amount invested in IE and for funds to pay the negative CFs is $7363.28. Explain. (b) Use a calculator to solve Equation 2 for irr.

10. For an interesting polynomial, graph, use UBLB Theorem and Descarte’s Rule for $P(x) = x^3 - 4x^2 - 5x + 14$. Are there any upper bounds and lower bounds of zeros? Are there any rational zeros. If so what are the other zeros. Did you find any relative maximum, minimum points, any inflection points, and $y$ and $x$ intercepts?

11. (a) Given that anticipated dividends of a stock at the ends of years 1 through 4 are $2.68, $3.23, $3.89, and $4.69, and the anticipated price of the stock at end of year 4 is $179.21. At 20%, what is the net present value? What is the meaning of npv in this case? (b) If the stock brought $10 more, what is the irr? What is the accumulated value at the end of 4 years, if the dividends are reinvested at the irr? (c) Does the irr assume reinvestment of dividends at the irr rate? (d) If the stock brought $179.21, and they
reinvested dividends at 6% and wanted a 20% return on their investment, what could they pay for the stock?

12. Prove the Upper Bound Theorem.

13. A person could play with the graph of the rational function made from Equation 2. (a) Investigate the graph of \( y(x) \) for \( x = 1+i > 0 \). Use Window: \( x_{\text{min}}=0, x_{\text{max}}=10, x_{\text{scl}}=1, y_{\text{min}}=-10,000, y_{\text{max}}=10,000, y_{\text{scl}}=100 \). (b) Graph for \( x = 1+i < 0 \). Use Window: \( x_{\text{min}}=-1000, x_{\text{max}}=0, x_{\text{scl}}=100, y_{\text{min}}=-10,000, y_{\text{max}}=10,000, y_{\text{scl}}=100 \). (c) Draw a hand graph including asymptotes, intercepts, and zero. Describe asymptotic behavior of \( y(x) \).

14. For consecutive returns of \( a \) and \( b \), prove \( TR = \text{antilog} \left[ \frac{\log(1+a) + \log(1+b)}{2} \right] - 1 \).

Generalize this theorem.

15. For the table giving Annual Rates of Return for Stocks, (a) Use \( TR \) to calculate the accumulation from $100 invested for 63 years in small company stocks. (b) Apply the same rate to simple interest calculations. (c) Use AM and simple interest. Use AM and compound interest. (d) Discuss.

16. Write in correct order the steps in the backwards “Proof” for the theorem \( \text{AM} \geq TR \). State conditions which justify steps where needed. Write a proof that: if \( a > -1 \) and \( b > -1 \), then \( \frac{a+b}{2} + 1 > 0 \), and explain to which step this applies.

17. Write out carefully, in words, a derivation of why the

\[
\text{Average share cost} = \frac{5I}{\sum_{i=1}^{5} P_i} = \frac{5}{\sum_{i=1}^{5} \frac{1}{P_i}}.
\]

18. (a) Calculate the Average share cost and the Average share price for the following chart:

<table>
<thead>
<tr>
<th>Investment</th>
<th>Share price</th>
<th>Shares acquired</th>
</tr>
</thead>
<tbody>
<tr>
<td>$400</td>
<td>$10</td>
<td>40</td>
</tr>
<tr>
<td>400</td>
<td>8</td>
<td>50</td>
</tr>
<tr>
<td>400</td>
<td>5</td>
<td>80</td>
</tr>
<tr>
<td>400</td>
<td>8</td>
<td>50</td>
</tr>
<tr>
<td>400</td>
<td>10</td>
<td>40</td>
</tr>
</tbody>
</table>

(b) Verify numerically the formulas in problem 17. (c) Use the formulas to calculate

HM of \( P_i \) and AM of \( P_i \).
19. There is a famous inequality that $AM \geq GM \geq HM$ where $GM(a_1, a_2, \ldots, a_n) = \sqrt[n]{a_1 \times a_2 \times \cdots \times a_n}$. (a) Write this inequality in terms of the general formulas for AM, GM, and HM. (b) Explain how GM is involved in calculating TR.
Answers to Exercises:

1. (a) npv = $1033.02 This means that the investment made a better rate of return than 20%. If npv is positive, then the business meets its goals.
(b) npv = 1033.02 = -9909.95 + 3500(1+i)^{-1} + 3800(1+i)^{-3} + 14500(1+i)^{-5}
where i = 20%.

2. (a) npv(22.4,-5000,L6) = $2070.67. The business meets expectations and made a higher rate of return than 22.4%. (b) npv(35.0446, -5000, L6) = -.0034 as expected.

3. (a) npv(6,-1200,CFLis)t = $2920.65  (b) irr(-2000, CFList) = 27.88%.

6. At 20%, FV = 427,227.60. Solving 9909.95 = 27,229.60(1 + i)^{-5} gives i = 22.4%. If cash flows earn 20%, the investment made 22.4% on the cost.

9. 2500(1 + i)^{-2} + 3300(1+i)^{-4} + 5000 = $7763.28 at i = .350446.

10. There is a rational zero at -2 and P(x) = (x + 2)(x^2 - 6x + 7). The other two zero are 3-√2 and 3 + √2. By the UBLB Theorem, 5 is an UB and -3 is a LB.

11. (a) npv = 2.68(1+.20)^{-1} + 3.23(1+.20)^{-2} + 3.89(1+.20)^{-3} + 183.90(1 + .20)^{-4} = $95.41. Or use npv(20,0,CFList) = 95.41. If they paid $95.41 for the stock, they made 20% on the investment. (b) irr = 21.54% if the stock brought $189.28. (c) With reinvestment of dividends, FV= 2.68(1+i)^3 +3.23(1+i)^2 + 3.89(1+i)^1 + 193.90, for i = irr. So 95.41 = FV(1 + i)^{-4}. This gives 95.41 = [2.68(1+i)^3 + 3.23(1+i)^2 + 3.89(1+i)^1 + 193.90](1 + i)^{-4} which simplifies to 95.41 = 2.68(1+i)^{-1} + 3.23(1+i)^{-2} + 3.89(1+i)^{-3} + 193.90(1 + i)^{-4}. This last equation is the equation (definition) for irr. So i = irr assumes reinvestment of dividends at the irr rate. (See the articles in this course on stock pricing models.) Does npv( assume reinvestment of dividends? (d) $93.96 or less.

A follow-up discussion of the answer to problem 11: Opportunity cost and risk premium. They could have made 5% risk free without buying in the stock. This difference between 20% and 5% is called the “risk premium” (Alexander, p. 488). The 5% will be called the “opportunity cost” or “risk free rate.” If they paid $93.96 for the stock, then the money was tied up in the stock and didn’t earn the 5%. At 5% it would have grown to $114.20 in four years. The “net gain” is $194.84 – $114.20 = $80.64. If there is no a positive net gain, then the investment doesn’t meet the “opportunity cost test”. At 5%, and additional $66.34 would earn the $80.64. This number is another version of “risk premium”.

19. (b) \( TR = \sqrt[n]{(1+r_1)(1+r_2)\cdots(1+r_n)} - 1. \) The \( \sqrt[n]{(1+r_1)(1+r_2)\cdots(1+r_n)} \) is \( \text{GM}[1+r_1, 1+r_2, \ldots, 1+r_n]. \)
Teachers’ Notes

For a copy of the TI 84 manual, see http://www.ti.com/calc.

For a free course in financial mathematics, with emphasis on personal finance, for upper high school and college, see COMAP.com. Register and they will e-mail you a password. Simply click on an article in the annotated bibliography, download it, and teach it.

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Unit 3: Long-Term Financial Planning  Unit 4: Investing in Bonds and Stocks
Unit 5: Investing in Real Estate  Unit 6: Solving Financial Formulas for i.

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Unable to get jobs, and wouldn’t know what to do with the money, if they had it. There are many jobs available but college graduates find themselves unprepared. A study by the Association of American Colleges and Universities has found that 87 percent of employers believe that education in U. S. colleges is not competitive in the global market. Sixty-three percent say recent college graduates don’t have the skills to be employed. A recent Roper Organization study found that nearly half of recent college graduates don’t think they got their money’s worth.  (Denton Record Chronicle, Oct. 7, 2011)

For the geometry of AM, GM, and HM, see http://en.wikipedia.org/wiki/Harmonic_mean.

More financial mathematics. The Teacher Retirement System of Texas is authorized to use derivatives in its investment portfolio and to use external managers to invest up to 30% of TRS funds. TRS is authorized to increase its allocation to hedge funds from 5% to 10%  (TRS Newsletter, July 2011).

References:


