Scott’s Example: “Honey I hocked the car but it’s worth it.” Denton Record Chronicle, Aug. 5, 2013. Scott says that the credit union offered loans on used cars for 1.99%. Just for fun he borrowed $33,000 on his 2010 Lexus RX 350. At 3% inflation, he said he got paid $839 for his trouble. He took out a five year loan paying back a total of $34,674 in principle and interest. His monthly payments would be $34,674 ÷ 60 = $577.90. To figure the purchasing power of the 60 monthly payments at the time of taking out the loan, he used his Texas Instruments Financial Calculator and got $32,161. The profit = 33,000 – 32,161 = $839. He said “it’s not so big a deal,” but it would buy a lot of hamburgers.

We will follow Scott’s line of thought but do the calculations our way:

Example 1: the monthly payments. First, we will recalculate the monthly payments: TI 83/84 Code for the TVM Solver: N = 60, I% = 1.99, PV = -33,000, P/Y = 12, PMT End and get PMT = $578.27 per month. Total principle and interest = 60 × 578.27 = $34,696.20. Scott said the total of the payments was $34,674. We are off. See the Exercises for the time lines, formulas, calculations, and attempting to get Scott’s numbers.

Example 2: the purchasing value of the monthly payments. We will ask what rate \( r \) compounded monthly gives an annual rate of 3% inflation. Calculating gives

\[
(1 + r)^{12} = 1 + .03. \quad \text{Solving gives} \quad r = \frac{1.03^{1/12} - 1}{1 - 1.03^{1/12}} = \frac{.24602}{1 + r}.
\]

To figure the purchasing power of the monthly payments at the time of the loan, we use the TI 83/84 TVM Solver: N = 60, P/Y = 1, PMT:BEGIN, PMT = 578.27, I% = -.24602 and solve for FV = $32,214.33. We get the profit = 33,000 – 32,214.33 = $785.67. The credit union gave Scott $33,000 but the purchasing power of the payments was only $32,214.33. (See “Living and Investing with Inflation”, in this course, for the \( d = \frac{-r}{1 + r} \).) Scott said he made $839 in purchasing power. No big deal, his point is well made. (Again, see the Exercises for the derivations of formulas for the above calculations and attempting to get Scott’s numbers.) When budgeting, think inflation.

Example 3: “There’s plenty of value in penny-pinching.” Scott Burns, Denton Record Chronicle, July 14, 2013. Scott says that at today’s top income tax rate is 39.6%. A taxpayer would have to earn $165 before taxes to have $100 to spend. (We are not counting social security employment tax and medicare tax.) Today, a five year Treasury note yields 1.51%. You’d need to invest $6,622 to get before tax annual interest of $100. Scott will keep his $100. (See the Exercises.) Scott says that parsimony, pinching pennies, has never been more valuable. (See the Exercises for other examples.) When budgeting, think before taxes, after taxes, and opportunity costs.

If interest rates rise, the value of parsimony declines. So the value fluctuates – but unlike investing stocks, you will never lose money by spending carefully.
Opportunity costs. Scott says that 25 years ago, the five year Treasury note paid 8.47% per year. At this rate, what is the value of his $100 invested from age 20 to age 70? (See the Exercises and Side Bar Notes.)

Example 4: Expense ratios for stock funds. Expense ratios of mutual funds affect the fund’s return and higher expense ratios are harder to overcome. Consider the following example from Scott Burn’s “What Vegas can teach you about mutual fund investing” Denton Record Chronicle, June 10, 2012, in which he calculates the effect of higher expense ratios. For large blend managed stock funds, in the last fifteen years, the average return was 6.82%, and average expense ratio was 1.17%. For the Vanguard 500 Index stock fund, the average return was 5.57%, and average expense ratio was .17% . Only forty percent of the managed funds outperformed the index fund. In the large blend managed fund, the investor paid one percentage point more expense ratio than in the index fund. They paid 1% of the average value of the managed fund for a 60% chance of not doing any better than the index fund, and a 40% chance of doing better. On average, they paid one percentage point for a 40% chance at the average advantage of 6.82% - 5.57% = 1.25%. This one percentage point in additional costs can be thought of as 80% of the average advantage of 1.25%, and 14.7% of the average return of 6.82%. Should they pay on average 80% of a dollar for a 40% chance of average winnings of a dollar? When budgeting, think expense ratio and commissions.

Example 5: Expense ratio for bond funds. In the last fifteen years, a Vanguard Index Bond fund had a return of 7.08% and expense ratio of .22%. The average managed bond fund had a return of 5.79% and an expense ratio of .89% . Almost no managed bond funds beat the index fund. Investors in the managed bond fund paid .67 percentage point (.89 percentage point - .22 percentage point = .67 percentage point) for finding the needle in the haystack. On average in the managed fund, they earned 7.08% - 5.79% = 1.29 percentage points less (Scott Burns, Denton Record Chronicle, June 10, 2012).

Example 6: The median-price of houses versus the price of the Wilshire 5000. The Wilshire 5000 Total Market Index of stocks represents the collective price of most of the stocks traded in the United States. In “How to sort out expensive paycheck, portfolio houses,” Denton Record Chronicle, June 27, 2010, Scott Burns presented most of the data in the following

Table 1. \( P_w \) gives the price of the Wilshire 500, \( P_H \) gives the median price of a house, \( C_{HW} \) gives the cost of a median priced house in terms of the number of shares in the Wilshire 5000.

<table>
<thead>
<tr>
<th>Year</th>
<th>( C_{HW} )</th>
<th>( P_w )</th>
<th>( P_H )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980</td>
<td>50.9 shares</td>
<td>$1221</td>
<td>$62,200</td>
</tr>
<tr>
<td>2000</td>
<td>11.8 shares</td>
<td>$12,488</td>
<td>$147,300</td>
</tr>
<tr>
<td>2010</td>
<td>14.3 shares</td>
<td>$12,454</td>
<td>$177,900</td>
</tr>
</tbody>
</table>

Table 1 – Stock prices versus housing prices.

Examining the table we can see that for 1980 housing and stock index prices, \( 62,200 \div 1221 = 50.9 \) shares of the Wilshire 5000 required to buy the median priced house. In terms of shares in the Wilshire 5000, by the peak of the internet bubble in the stock market, the relative cost of shelter had dropped by 77%.
See the Exercises for calculating for the above time periods, the annual rate of increase in prices of the Wilshire 5000, the annual rate of increase in housing prices, the annual rate of inflation, and the design and history of the Wilshire 5000 index of stocks.

**Example 7: A negative tax bill.** If you have twelve stocks which you like and four of them go down in price. Call them A, B, C, and D. Sell A and B and put the money in C and D. After 31 days, sell the older tranches of C and D and buy back into A and B. (Forbes, March 4, 2013, p. 58) Consider the following example:

You own 100 shares each of A, B, C and D purchased at $40 per share, and they all go down to $30 per share. Sell 100 shares each of A and B for $30 per share. The proceeds is $(30+30)(100) = $6000. Originally you paid $(40+40)(100) = $8000. You have a capital loss of $8000 – $6000 = $2000. Put $3000 in C and $3000 in D at $30 per share. After 31 days, sell the shares of C and D for which you paid $40 per share. Your proceeds are $6000. You have another capital loss of $2000. You have a total capital loss of $4000. At a 15% capital gains tax rate, this reduces your tax bill by $.15(4000) = $600. When budgeting, learn about taxes.

What should you do with the last proceeds of $6000 and the tax savings of $600? There may be three charges for the sale and purchase costing 3($9) = $18. There may be a limit on capital losses for which you don’t have capital gains to post against. See the Exercises for owning 1000 shares.

**Example 8:** McDonald’s stock (MCD) has had a seven fold increase in dividends in the last ten years (Kiplinger’s Retirement Report, Jan. 2013). We went to finance.yahoo.com and put in MCD on July 19, 2013 and the Price was $100.27, Dividend Yield was 3.00%, and yearly sum of Dividends was $3.08. We traced the price back to Dec. 29, 2008 to be $63.75. The approximate annual rate of price increase $r$ is given by $63.75(1 + r)^{4.5} = 100.27$, to get $r = .106 = 10.6\%$ per year. In the last ten years the dividend $D$ has increased seven fold. This gives $D(1 + i)^{10} = 7D$ which gives $i = 21.5\%$ per year for the average rate of increase in dividends.

For $D$, the dividend ten years ago, $D(1 + .215)^{10} = 3.08$. This gives $D = $.44. Check: $7(.44) = 3.08$. See the Exercises, Side Bar Notes, and References for more about stock dividends. When budgeting, think about dividends.

**Side Bar Notes:**

- **Doing research on mutual funds, indexes, and stocks.** Try for example morningstar.com/Fidelity Puritan, or /VTSAX for Vanguard Total Stock Market Index Fund, or /VZ for Verizon stock.

- **Opportunity costs.** “The best piece of money advice I’ve ever been given is to look at every single one of my expenditures and to not think about it in today’s dollars but in future dollars. So, for example, if I’m buying something that costs $300, what’s the opportunity cost of not having that $300 in my retirement account? Maybe it $20,000,” says Alexa von Tobel, the 29-year-old founder of LearnVest, a personal finance advice site that recently became a registered investment advisor with the SEC (Forbes, June 24, 2013, p. 109). How could $300 grow to $20,000 from age 30 to age 70? What would be the real value of the $20,000 in terms of
age 30 dollars? Was the $300 pretax or after tax money? What was her marginal income tax bracket? What was the pretax value of the $300? See the Exercises.

Upward mobility. “Historically, our nation has enjoyed remarkable economic mobility. About 60 percent of the households that were in the lowest income quintile in 1999 were in a higher quintile ten years later. During the same decade, almost 40 percent of the richest households fell to a lower quintile.” (Imprimis, May/June 2013) When budgeting, think upward mobility.

Did actively managed mutual funds beat their index? According to the Standard and Poor’s Indices Versus Active Funds Scorecard, “Over the last five years ending in 2011, 60 to 80 percent of all actively managed domestic funds failed to beat their index” (Scott Burns, Denton Record Chronicle, May 30, 2012).

Diversify broadly. “With a small list of stocks, it’s easy to miss out entirely on the top 10% performers. From 1980 to 2008, for a portfolio containing most of the U S stocks, the return was 10.4%. Minus the top 10%, the return was 6.6%. Minus the top 25%, the return was -2.1% The table said these are total returns, which should indicate reinvestment of dividends. (Money, June 2009, p. 54). See the articles in this course on stocks.

For the Wilshire 5000 Total Market Index of stocks (^W5000), see Wikipedia.org and finance.yahoo.com .

Value investing. In contrast to the Wilshire 5000, well known successful investors use value investing. See “Value investing” in Wikipedia.org and indexes based on a discipline of value investing, where the total stock capitalization of the company compares favorably to the “value” of the company.

Making loans with loan sharks. According to Texas state reports, 70 percent of borrowers are unable to repay a short-term loan by the original terms. On average it costs about $840 for the average customer to repay a $300 loan (Denton Record Chronicle, March 21, 2013). When budgeting, think shop for interest rates.

Family budgeting. In our discussions of various kinds of Penny Pinching in this article, we will not address such topics as family budgeting. For used books on family finance, see barnsandnoble.com or amazon.com. Such books can be bought for less than the cost of shipping. Do a search under books, family budgeting, priced from low to high. Try Crown Ministries.

The unpredictable stock market. People sometimes dump stocks when the market goes down. From 1991 to 2010, the S&P 500 returned an average 9.1% per year. If they had been on the sidelines for just the 20 best days, they would have made 3% (Money, June 2011).

Continuing care retirement community in Sarasota, FL requires a six-figure up front fee, an ongoing monthly fee, and a minimum income. This provides lifetime care, a smooth on-site transition from independent living to assisted living to nursing care. Check the terms in your area, and ask the social worker what kinds of assistance some of their clients get. Do the math for people your age present a written report to your class.
**Exercises:** Show your work. Provide calculations with formulas and time lines. Label numbers, variables, and answers.

#1. Calculate with a formula and a scientific calculator pad the monthly payments in Example 1.

#2. Do a time line for Example 2 and derive a formula for FV. Then use a calculator to calculate FV from the formula.

#3. Scott said that at the end of June 2012, inflation had reduced the purchasing power of a dollar by 9.9% over the previous five years. Go to usinflationcalculator.com estimate this percent decline. See the article in this course “Using USInflationcalculator.com.”

#4. Scott said 2009 was a rare year of price deflation. Go to usinflationcalculator.com and check this. What was the increase in the purchasing value of the dollar? What was the percentage increase? See “Living and Investing with Inflation,” in this course.

#5. Copy Scott’s numbers and their meanings on one page. Do calculations to see how close you can get to Scott’s numbers.

#6. Consider reducing your cable bill by $50 per month. One measure of the value of this reduction is owning Time Warner Stock (TWC). TWC sell for about $111 per share and pays 2.3% dividends per year. (a) Estimate how much you would have to invest in TWC to get $600 a year. (b) How many shares would you have to buy? (c) With dividends taxed at 20%, how many shares would you need? (c) You may be on Verizon. Go to finance/yahoo.com and look up Verizon stock. Calculate. How often per year are periodic dividends paid? (See the article in this course “High Dividend Yields on Stocks and Low Interest Rates on CDs and Bonds.”)

#7. Opportunity costs: (a) Estimate the value of $1200 ($100 per month) accumulating at 8.47% per year from age 20 to age 70? (b) At 3.2% inflation, what is the real value in terms of age 20 dollars?

#8. For the average ETF (Exchange Traded Fund), the expense ratio was .05%. For the average mutual fund it was 1.3% (Forbes, June 24, 2013, p. 131). (a) At a return of 5% before expense ratios, calculate the after expense value of $100,000 in fifty years for each type of fund. (b) At what rate of return does the expense ratio of .05% give exactly twice that of 1.3%? Considering the answers to part (a), it appears that this is a reasonable question but the answer is negative.

#9. Malkiel in his famous book gave the following examples for an expense ratio of .46%: AT a rate of 9% before expense ratio, with expense ratio, $10,000 grows to $15,064 in five years, $22,693 in ten years, and $34,186 in fifteen years. Check his figures. What formula did he use? (Malkiel, A Random Walk Down Wall Street)

#10. Expenses for 529 savings plans for educational costs offered by the States range from .65% to 2.8% (Kiplinger’s, May 2005). (a) Calculate the after expense future value of $5000 a year for 15 years at 7% before expense ratio for these two examples. (b) For four years of anticipated tuition and fees beginning in 15 years, how much a year should be saved? Make your own figures including inflation. (c) What is the advantage of a 529 program? See “Some 529 Plans and Other Options for Saving for College” in this course. Future and current college
students can use 529 plans. Twelve percent work their way through college. See “The Financial Story of Mr. and Mrs. Dick and Jane” in this course. Read about their neighbor, John the engineer.

#11. The expense ratio for a fidelity fund was below average. But if you invest $10,000 over the next ten years, you will pay $1444 in expenses which is 10.6% of your profit after expenses according to Fund Analyzer provided by the brokerage industry’s self-regulatory body (Kiplinger’s Personal Finance, 8/2013, p. 18). (a) Calculate the profit after expenses for the investment. (b) Calculate the future value after expenses of the investment. (c) What was the average annual rate of return after expenses? (d) What was the future value before expenses? (e) What was the rate of return before expenses? (f) Estimate the expense ratio.

#12. Target retirement funds automatically turn more conservative as you age. Forbes June 24, 2013 gives the example from BlackRock, Life Path 2040, for a saver age 40, where when $10,000 is invested for ten years, with a return of 5% after expenses, there is a total expense of $2478. (a) What was the future value at 5%? (b) If the expenses hadn’t been charged, what would be the sum? (c) From this estimate the annual expense ratio. (d) What expense would be charged if the investment was $300,000? (e) The Vanguard Target Retirement 2040 uses index funds and has a ten year cost of $230 on $10,000. Estimate the expense ratio? How does it compare to BlackRock? (f) You could build your own age 40 ten year program, but you would need to change the allocation each year. Start with 62.7% in Vanguard Total Stock Market Index Fund (VTSAX), 27.2% in Vanguard Total International Stock Index Fund (VTIAX), and 10.1% in Vanguard Total Bond Market Index Fund (VBTLX). Your ten year cost would be about $109. Look up the expense ratios for each of these funds.

#13. Slot machines have an expense ratio of 13%. (a) In how many plays is your $100 down to $1? What would you do with it? (b) The expense ratio for the lottery is 50%. In how many plays is your $100 down to $1?

The following problems are for Table 1 above.

#14. (a) If it takes 11.8 shares of the Wilshire 5000 (W5000) to buy the $147,300 house, what is the price of a share of W5000? How does this compare to Table 1? Check this pattern with 2010 houses.

#15. Check Table 1 for year 2010, for $P_w$, and $C_{hw}$ by going to finance.yahoo.com and put in $^W5000$ to estimate $P_w$ for 2010. We got an estimate for $P_w$ by averaging W5000 prices for the beginning of the year and end of the year. Where is $^W5000$ on the date of your reading this exercise?

#16. Complete the following table. The $r_H$ gives the annual rate of increase in the median price of houses. The $r_w$ gives the annual rate of increase in W5000 prices. $I =$ the annual inflation rate which is calculated from usinflationcalculator.com.

$$
\begin{array}{ccc}
       & r_H & P_w & I \\
\end{array}
$$

From 1980 to 2000
From 2000 to 2010
From 1980 to 2010

#17. Discuss the table: (a) How has the rate of increase \( r_H \) compared to \( P_W \) for the different time periods. Why was \( P_W \) flat from 2000 to 2010 and \( r_H \) was small? See Wikipedia.org. How did \( r_H \) compare to I? (b) How have new house in these periods compared in features to older houses? How do these prices compare to prices in your area? (c) Could you make money by investing in rent houses? Discuss. See the references. (d) In your area could you easily get by with a house priced below the median? What price would be acceptable? What is the amount of a 20% down payment? What is the advantage of a 20% down payment? (See several articles in the references on housing, inflation, and investment in stocks.) (e) Describe five ways you could get better than average appreciation on a house?

#18. (a) From Wikipedia.org, when did the Wilshire 5000 Index start? How many stocks and what stocks did it represent then? How many stocks are in the Wilshire 5000 in recent times? (b) How is the Index calculated? (c) What is the history of the ups and downs of the Wilshire 5000? By how much and when did major losses in the index occur? (d) How many dollars are represented for each point in the index? Which version of the Wilshire 5000 is represented by \(^W5000\)?

#19. Investigate and describe a mutual fund that is a Total Stock Market Index fund. (a) Does the return of the fund outperform the index in rate of increase of dollars? Why? (b) How does the fund select stocks from the thousands of stocks in the index? When was the fund started? (c) What is the expense ratio? How does this expense ratio compare to that for other types of funds? Do you pay a commission to buy the fund? (d) What are the advantages of investing in this index fund? Does this index fund outperform managed stock funds?

#20. Do Example 7 for 1000 shares of each of stocks A, B, C and D. What would you do with the tax savings and the last sales proceeds?

#21. Assume a stock price increases at 3.2% per year. Beginning price is $30 per share. The dividend rate starts at 4%. Dividends increase at 1% per year. For \( x = 0 \) to \( n \) years, give an equation for price \( P \). Given an equation for dividend \( D \). Give an equation for dividend rate \( R \). For years 0 to 15, graph \( P, D, \) and \( R \). For more interesting mathematics about dividends, see “High Dividend Yields on Stocks and Low Interest Rates on CDs and Bonds” in this course.

#22. (a) Look up a high dividend stock fund and study its philosophy, performance, expense ratio, and dividend rate. Did it hold stocks that have a history of increasing their dividends? How did it compare to its benchmark and to other funds? Did it sell for a commission? What were some of its major holdings and their dividend rates? (b) What does the term “total return” mean? See measuringworth.com for a long term simulation total return table for the S&P500 Index. Compare the return of the Index with the Total Return of the Index over a period of years. What was the dividend rate for some of the years? What would be a disadvantage of the Wilshire 5000 Index?

#23. (a) In Example 3, Scott used a top income tax rate of 39.6%. He could have added in Social Security tax of 6.2% up to earnings of $113,700 (Your employer also pays 6.2%), and
Medicare tax of 1.45% on all income. Do Example 3 by adding in these numbers. (It is likely that the individual has income over $113,700 on which there is no Social Security tax. This upper limit could change.) (b) Do an example for yourself when you are fully employed in your chosen profession. For a self employed person the Social Security tax is twice 6.2%. If your employer hadn’t paid the Social Security tax, he could have paid you more. This could be added in.

#24. The frequently recommended 4% rule for retirement withdrawals says withdraw 4% the first year and then adjust annually for inflation. Money magazine said that if you started with $100,000 in 1990, you would have $264,657 in 2011. Do the research and math to check their figures.

**Answers:**

#24 We assume that their time line starts Jan. 1, 1990 and goes through 2011. This was 22 years. We went to measuringworth.com and estimated for the period, the total return of the S&P
500 to be \( r = 8.5\% \). On usinflationcalculator.com, we got inflation \( I = 2.74\% \). The following formula tells how much is required to fund these 22 years:

\[
P = 4000 \left[ \frac{1 - (1 + y)^{-22}}{y} \right] \quad \text{where} \quad y = \frac{.085 - .0274}{1 + .0274} = .056.
\]

\( P = \$49,859 \). So there would be \( $100,000 - 49,859 = \$50,141 \) which is left to grow at 8.5\% and gives \( 50,141(1+.085)^{22} = \$301,750 \) left in the fund. Money said \$264,657 would be left in the fund. Given the uncertainty of their time line and the rate of return on the investment, their figure seems reasonable. If we back \( r \) down to 8\% and \( I = 2.74\% \) we get \( 47,918(1+.08)^{22} = \$260,508 \) left in the fund. See “The Mathematics of Financial and Social Responsibility” in this course.

References

For a free course in financial mathematics with emphasis on personal finance, see COMAP.com. Click on the box for the free financial mathematics course, register, and COMAP will e-mail you a password. Simply click on an article in the annotated bibliography, download it, and teach it.

Unit 1: The Basics of Mathematics of Finance  Unit 2: Managing Your Money  Unit 3: Long-Term Financial Planning  Unit 4: Investing in Bonds and Stocks  Unit 5: Investing in Real Estate  Unit 6: Solving Financial Formulas for i, iteration programs  Unit 7: Related Topics  Unit 8: More Advanced or Technical Topics  The course has around sixty articles at this time all related to this article. Some of them are:


“High Dividend Yields on Stocks and Low Interest Rates on CDs and Bonds,” Jan. 2013.

“‘Examining a Lifetime of Automobile Purchase Expenses’ A spreadsheet demonstration. By careful management, Family B accumulated \$1,678,860 .

“An Alternative Investment for Low CD Rates: A Fuel Efficient Car.”

“The Financial Story of Mr. and Mrs. Dick and Jane.” Family finances.

“Living and Investing with Inflation, Fisher’s Effect.”

“Buying a House Versus Renting.”

“The Mathematics of the Return on Home Ownership.”

“Some Mathematics of Investing In Rental Property.”


“Some 529 Plans and Other Options for Saving for College.”

We will write an article on “Using measuringworth.com, a Total Return Table for a S&P 500 Stock Portfolio.”
Teachers’ notes: Have your students prepare written reports, with mathematics, for the class. Send some of them to COMAP.