TechMAP Sampler

TechMAP is a new NSF-funded project that has developed high school modules in technical applications of mathematics. The mathematics required by state tests is taught in algebra and geometry courses, but for many students this is not enough. TechMAP modules demonstrate these skills in vocational fields; they help reinforce existing mathematics courses through actual career activities. These modules are self-contained with exercises, teacher notes, handouts, etc., and can be used individually or together. Each module takes one to three weeks of classroom time.

Enclosed in this sampler you will find:

- Matrix of TechMAP modules, including module title, mathematics course, and mathematics topic.
- Lesson 1 of *Data, Data, Everywhere* including teacher notes as well as the answers to this lesson.
- Lesson 1 of *The Right Package* including teacher notes as well as the answers to this lesson.
- TechMAP order form.
## TechMAP Modules

<table>
<thead>
<tr>
<th>Module Title</th>
<th>Software</th>
<th>Mathematics Course</th>
<th>Mathematics Topic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bracing Grids</td>
<td>Geometry</td>
<td>Discrete Math</td>
<td>Pythagorean Theorem Graph Theory</td>
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<tr>
<td>Plan the Work, Then Work the Plan</td>
<td>Discrete Math</td>
<td></td>
<td>Scheduling Bin Packing</td>
</tr>
<tr>
<td>On the Botpath*</td>
<td>X</td>
<td>Geometry Algebra I &amp; II</td>
<td>Coordinates, Slope Systems of Equations</td>
</tr>
<tr>
<td>Where Shall I Sell My Wheat?</td>
<td>X</td>
<td>Algebra II Precalculus</td>
<td>Conics (hyperbola)</td>
</tr>
<tr>
<td>GPS</td>
<td>Geometry</td>
<td></td>
<td>Sphere Latitude &amp; Longitude</td>
</tr>
<tr>
<td>Sound</td>
<td>Algebra II</td>
<td></td>
<td>Exponents Logarithms</td>
</tr>
<tr>
<td>The Right Package*</td>
<td>X</td>
<td>Geometry</td>
<td>Surface Area Volume, Measurement</td>
</tr>
<tr>
<td>Tim and Tom’s Financial Adventure</td>
<td>Algebra II</td>
<td></td>
<td>Exponential Functions Growth &amp; Decay</td>
</tr>
<tr>
<td>Circle Relationships in Carpentry</td>
<td>X</td>
<td>Geometry</td>
<td>Circles (central &amp; inscribed angles)</td>
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<tr>
<td>Insulation</td>
<td>Algebra II</td>
<td></td>
<td>Logarithms, Equations, Linear Regression</td>
</tr>
<tr>
<td>Clocks</td>
<td>Algebra II</td>
<td></td>
<td>Direct Variation Indirect Variation</td>
</tr>
<tr>
<td>Strength of Beams</td>
<td>Algebra I</td>
<td></td>
<td>Direct Variation Indirect Variation</td>
</tr>
<tr>
<td>Formulas for Success*</td>
<td>Algebra I</td>
<td></td>
<td>Expressions Equations</td>
</tr>
<tr>
<td>The Right Size*</td>
<td>Geometry</td>
<td></td>
<td>Measurement</td>
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<td>Module</td>
<td>Subject(s)</td>
<td>Topic(s)</td>
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</tr>
<tr>
<td>---------------------------------------</td>
<td>------------------</td>
<td>-----------------------------------------------</td>
<td></td>
</tr>
<tr>
<td>What Are the Chances?*</td>
<td>Probability &amp; Statistics</td>
<td>Probability: Basic, Joint, Conditional</td>
<td></td>
</tr>
<tr>
<td>Line Up*</td>
<td>Algebra I Statistics</td>
<td>Linear Relationships</td>
<td></td>
</tr>
<tr>
<td>Similarity*</td>
<td>Geometry</td>
<td>Similar Figures (sides, areas, volumes)</td>
<td></td>
</tr>
<tr>
<td>What's Next?*</td>
<td>Algebra I &amp; II</td>
<td>Sequences, Exponential Functions</td>
<td></td>
</tr>
<tr>
<td>Data, Data, Everywhere*</td>
<td>Algebra I, Statistics</td>
<td>Statistical Graphs Averages</td>
<td></td>
</tr>
<tr>
<td>Over the Limit*</td>
<td>Algebra I &amp; II</td>
<td>Inequalities</td>
<td></td>
</tr>
<tr>
<td>Circular Motion</td>
<td>Algebra II Precalculus</td>
<td>Radians, Parametrics, Trig Functions</td>
<td></td>
</tr>
<tr>
<td>Finding the Right Triangle</td>
<td>Geometry Algebra I &amp; II</td>
<td>Right Triangle Trigonometry</td>
<td></td>
</tr>
</tbody>
</table>

*Denotes modules targeted at high-stakes tests. Each of these modules includes a supplemental section of problems taken from the MCAS or other state tests. These supplemental problems are related to the mathematical content of the module.*
TechMAP
DATA, DATA, EVERYWHERE

TECHNOLOGY & SCIENCE

TEACHER EDITION

COYAP
Data, Data, Everywhere

Marsha Davis
Data are collected every day. Whether you know it or not, you contribute to the amount of data. Every time you make a phone call, the number, location, date, and length of the call are recorded. When you rent movies at your local video store, the titles, dates rented, dates returned, and fines are saved in the store’s computer. When you use a store card to get discounts, the date, products you buy, and amount you spend are stored in a data bank. At the end of each school term, your school records and files your grades for future reference.

The situation with data is much like the situation in Samuel Taylor Coleridge’s poem *The Rime of the Ancient Mariner*. The ancient mariner (old sailor), stuck in the middle of an ocean and very thirsty, exclaims:

Water, water, everywhere,
Nor any drop to drink.

Sometimes, we are stuck in a sea of data and still thirsty for information. For example, look at the data in Figure 1. These numbers are neatly organized in columns and rows. However, they don’t tell you much.

![Figure 1. Data consisting of 165 numbers.](image)

Before you can make sense of these data, you need to know what they represent. Then you need to process the numbers in order to extract useful information.

The data in Figure 1 are 165 responses from 12th-grade students to the question:

“On the average over the school year, how many hours per week do you work in a paid or unpaid job?”
Each student picked one of the following choices:

(1) None
(2) 5 or fewer hours
(3) 6 to 10 hours
(4) 11 to 15 hours
(5) 16 to 20 hours
(6) 21 to 25 hours
(7) 26 to 30 hours
(8) More than 30 hours

Now you know what the numbers in Figure 1 represent. However, what do they tell you about student work patterns? If you have trouble answering this question, think about this: the complete data set contains responses from 13,543 students. This question and the data in Figure 1 are part of the ongoing study Monitoring the Future: A Continuing Study of American Youth.

In this module you will learn techniques for organizing, summarizing, and displaying data. These techniques will help you extract useful information from data.
Lesson 1

Frequency Tables
Graphical Displays
Activity 1
Exercise Set 1

Category Data

Data come in two basic types—**category data** and **numeric data**. The techniques you use to analyze data depend on the type. In this lesson you will learn how to analyze category data.

**Category data** describe a characteristic of the items in the sample. For example, if you describe cars by color (red, white, silver, black, other), the result is category data. Other examples of category data are pizza toppings (cheese, pepperoni, veggie), hair color (red, blonde, brunette), and types of wood (oak, cedar, pine).

**Numeric data** are numbers that have meaning in terms of size. For example, if you measure the heights of the students in your class, the data are numeric. Just because data are numbers does not make them numeric data. For example, the data in Figure 1 of the module opener are numbers. However, each of the numbers represents a category (1 = “None,” 2 = “5 or fewer hours,” 3 = “6 to 10 hours,” and so forth). Hence, the data in Figure 1 are category data.

**FREQUENCY TABLES**

Look again at Figure 1, which gives student responses to the question:

“On the average over the school year, how many hours per week do you work in a paid or unpaid job?”

It is difficult to learn anything from the 165 numbers. However, you can learn a lot when the data are organized into the frequency table in **Figure 2**.

<table>
<thead>
<tr>
<th>Category</th>
<th>Frequency (number of data in category)</th>
<th>Percent (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) None</td>
<td>38</td>
<td>23.0</td>
</tr>
<tr>
<td>(2) 5 or fewer hours</td>
<td>14</td>
<td>8.5</td>
</tr>
<tr>
<td>(3) 6 to 10 hours</td>
<td>17</td>
<td>10.3</td>
</tr>
<tr>
<td>(4) 11 to 15 hours</td>
<td>12</td>
<td>7.3</td>
</tr>
<tr>
<td>(5) 16 to 20 hours</td>
<td>34</td>
<td>20.6</td>
</tr>
<tr>
<td>(6) 21 to 25 hours</td>
<td>18</td>
<td>10.9</td>
</tr>
<tr>
<td>(7) 26 to 30 hours</td>
<td>16</td>
<td>9.7</td>
</tr>
<tr>
<td>(8) More than 30 hours</td>
<td>16</td>
<td>9.7</td>
</tr>
<tr>
<td>Total</td>
<td>165</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Figure 2. A frequency table.
Here’s how to make this table.

- The first column contains a list of the possible data values (categories).
- The second column shows the number of times (frequency) each data value occurs. For example, you find thirty-eight 1s, fourteen 2s, etc. in Figure 1.
- The third column gives the percentage (rounded to the nearest tenth of a percent). For example, the percentage for category 1 is calculated as follows: \[ \frac{38}{165} \times 100\% = 23.0\% . \]

To calculate the percentage, divide the frequency by the total number of data and multiply the result by 100%:

\[
\text{percentage} = \frac{\text{frequency}}{\text{total number of data}} \times 100\%
\]

- The last row of the table gives the totals. The total for the second column is the sum of the numbers in column 2. This tells you the number of data. The total for the third column is the sum of the percents in column 3. This total should be 100%. (Sometimes this total is slightly less or more than 100% due to roundoff.)

Finally, the data are in a form that gives you information about students’ work patterns. You now know that 23.0% of the students do not work at any job during the school year (category 1). That is the highest percent for any of the categories. However, category 5 (students who worked between 16 and 20 hours per week) has the next highest frequency. The 5s account for 20.6% of the responses.

**GRAPHICAL DISPLAYS**

You’ve heard the saying “A picture is worth a thousand words.” Sometimes it’s easier to understand information if it is presented in the form of a picture or chart. **Bar charts** and **pie charts** are the most common displays for category data.

**IN THE NEWS ...**

**Why are music sales falling? DOWNLOADING**

LOS ANGELES, June 16, 2003 — While there are a variety of reasons contributing to the downturn in music sales, the two problems of Downloading and Burning are clearly the most potent ones, according to a recent national survey of 12 to 44 year olds conducted by Edison Media Research for the trade publication *Radio & Records*.

In 2003, roughly 36% of those asked said that they had downloaded music from the Internet for playback at another time. The graph below shows the breakdown from light downloaders (20 or fewer files) to heavy downloaders (100 or more files).
The bar chart in Figure 3 shows something about downloading patterns. For example, the tallest bar in the chart is above the category “100 or more.” That bar represents around 45% of those who download music files.

The information on students’ work patterns in Figure 2 could also be displayed in a bar chart. Here are the directions on how to make a bar chart:

**Creating a Bar Chart**

**Step 1:** Draw a horizontal axis and a vertical axis.

**Step 2:** Count the number of categories. Mark evenly-spaced tick marks on the horizontal axis—one tick mark for each category. Label them with the category names.

**Step 3:** Scale the vertical axis for frequency (or percent). Your tick marks on this scale should be equally-spaced. The numbers next to these tick marks should jump by the same amount as you move from one tick mark to the next.

**Step 4:** Centered over each category on the horizontal axis, draw a rectangle (bar). The rectangle’s height should equal the frequency (or percent) of the category. The bars should have the same width. They should not touch each other.

**Step 5:** Label the axes and give the graph a title.

**Figures 4-8** show you how to apply these five steps to make a bar chart for the students’ work data from Figure 2.
Notice from Figure 8 that roughly half as many students work 6 to 10 hours as 16 to 20 hours. That's because the bar over the category “6 to 10 hours” is about half the height of the bar over the category “16 to 20 hours.”

In the examples shown, the horizontal axis is used for the category and the vertical axis for the frequency or percent. Sometimes people get creative in their bar charts. They might switch the axes and put the categories on the vertical axis. Also, they might give the bars characteristics of the topic. The newspaper clipping in Figure 9 is a great example.
A pie chart is another way to display category data. A pie chart is a circle divided into slices. Each slice represents a category. The pie chart in Figure 10 is based on data from the music survey conducted by Edison Media Research. (See the In the News segment near Figure 3.) The survey asked:

“Have you ever gone on to buy an artist’s CD after first downloading a track for free from the Internet?”

The responses are represented by the pie chart in Figure 10.

Drawing a pie chart by hand requires a protractor to measure angles. Figure 11 shows the angles of the slices. These angles are computed by multiplying the angle of a full pie, 360°, by the percentage for the slice:

angle for the Yes slice is 60% of 360°: \(0.60 \times 360° = 216°\).

angle for the No slice is 40% of 360°: \(0.40 \times 360° = 144°\).
Computer software such as Excel is often used to make the pie charts. What is important is to be able to look at a pie chart and estimate the percentage in each category from the size of the slice. Figure 12 displays three percents that can guide your estimates.

50% is $\frac{50}{100} = \frac{1}{2}$ of the pie (180°).

25% is $\frac{25}{100} = \frac{1}{4}$ of the pie (90°).

12.5% is $\frac{1}{8}$ of the pie (45°).

Now that you have learned how to make frequency tables, bar charts, and pie charts, you are ready to analyze data in Activity 1.
In this activity, you will investigate more data from Monitoring the Future: A Continuing Study of American Youth. You will focus on the four questions in Figure 13. Data from 50 students who said their present high school program was “vocational, technical, or commercial” are in Figure 14.

### Activity 1

<table>
<thead>
<tr>
<th>Question</th>
<th>Response</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Gender</td>
<td>What is your gender? Male, Female</td>
</tr>
</tbody>
</table>
| 2. Work | On the average over the school year, how many hours per week do you work in a paid or unpaid job?  
1 = “None”, 2 = “5 or fewer hours”, 3 = “6 to 10 hours”, 4 = “11 to 15 hours”, 5 = “16 to 20 hours”, 6 = “21 to 25 hours”, 7 = “26 to 30 hours”, 8 = “More than 30 hours” |
| 3. Drive | During the average week, how much do you usually drive a car, truck, or motorcycle?  
1 = “Not at all”, 2 = “1–10 miles”, 3 = “11–50 miles”, 4 = “51–100 miles”, 5 = “100 to 200 miles” and 6 = “More than 200 miles” |
| 4. Ticket | Within the LAST 12 MONTHS, have you received a ticket (OR been stopped and warned) for moving violations, such as speeding, running a stop light, or improper passing?  
Yes, No |

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<table>
<thead>
<tr>
<th>Gender</th>
<th>Work</th>
<th>Drive</th>
<th>Ticket</th>
</tr>
</thead>
<tbody>
<tr>
<td>Female</td>
<td>5</td>
<td>4</td>
<td>No</td>
</tr>
<tr>
<td>Female</td>
<td>1</td>
<td>4</td>
<td>No</td>
</tr>
<tr>
<td>Female</td>
<td>5</td>
<td>1</td>
<td>No</td>
</tr>
<tr>
<td>Female</td>
<td>5</td>
<td>5</td>
<td>Yes</td>
</tr>
<tr>
<td>Male</td>
<td>8</td>
<td>3</td>
<td>No</td>
</tr>
<tr>
<td>Male</td>
<td>3</td>
<td>5</td>
<td>Yes</td>
</tr>
<tr>
<td>Male</td>
<td>6</td>
<td>5</td>
<td>Yes</td>
</tr>
<tr>
<td>Male</td>
<td>8</td>
<td>1</td>
<td>No</td>
</tr>
<tr>
<td>Female</td>
<td>7</td>
<td>5</td>
<td>No</td>
</tr>
<tr>
<td>Female</td>
<td>7</td>
<td>5</td>
<td>No</td>
</tr>
<tr>
<td>Male</td>
<td>1</td>
<td>1</td>
<td>No</td>
</tr>
<tr>
<td>Male</td>
<td>6</td>
<td>6</td>
<td>No</td>
</tr>
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<td>Male</td>
<td>3</td>
<td>4</td>
<td>No</td>
</tr>
<tr>
<td>Male</td>
<td>5</td>
<td>1</td>
<td>No</td>
</tr>
<tr>
<td>Male</td>
<td>8</td>
<td>5</td>
<td>No</td>
</tr>
<tr>
<td>Female</td>
<td>7</td>
<td>3</td>
<td>No</td>
</tr>
<tr>
<td>Male</td>
<td>1</td>
<td>6</td>
<td>Yes</td>
</tr>
</tbody>
</table>

### Figure 13.
Questions and possible responses.

### Figure 14.
Data from 50 students in vocational, technical, or commercial programs.
1. First, consider the data from the question on gender.
   a) What percentage of the students are male? Explain your calculations.
   b) What percentage of the students are female? Explain how you got your answer.
   c) Which of these pie charts represents your results from (a) and (b)? Explain how you chose your answer.

2. Next, consider the data on the number of hours that the students work.
   a) Make a frequency table for the hours students work. Make it similar to Figure 2.
   Your frequency table in (a) is based on a sample of 50 students attending vocational, technical, or commercial programs. The frequency table in Figure 2 is based on a sample of 165 high school students (all types of programs are included).
   b) Do the work patterns of students in vocational, technical, or commercial programs appear to be different from the work patterns of high school students in general? Explain. Use numbers from both tables (your table from (a) and the table in Figure 2) in your explanation.
   c) Draw a bar chart that represents the information in your frequency table in (a).
The next two questions focus on driving habits.

3. a) What percentage of students do not drive cars, trucks, or motorcycles during an average week?
   b) What percentage of students do drive cars, trucks, or motorcycles during an average week?
   c) Make a pie chart that represents the information from (a) and (b). Explain how you determined the angles of the slices.

4. Focus only on the students who drive during an average week (categories 2–6).
   a) How many of the students in this sample drive during an average week?
   b) Of the students who drive during an average week, what percentage gets a ticket or warning? Explain how you got your answer. Show your calculations.
   c) Based on your answer to (b), do you think that police target high school students? What other data would be helpful in answering this question?
1. The bar chart in Figure 15 is based on data from the 2003 National Record Buyers Study.

Number of Music CDs Purchased
By Teens Ages 12 to 17

- 1 to 5
- 6 to 10
- 11 to 15
- 16 or more

a) Estimate the percentage of teens that bought 16 or more CDs.
b) Estimate the percentage of teens that bought 1 to 5 CDs.
c) Which of the following is closest to the percentage of teens that bought 6 or more CDs?
   - A. 25%
   - B. 23%
   - C. 47%
   - D. 68%
d) Suppose the bar chart was based on data from 1500 teens. About how many teens do you think bought 6 or more CDs in the 12-month period? Show how you got your answer.

2. Participants in the 2003 National Record Buyers Study were asked to respond to the following statement: “You are less passionate about music these days than you used to be.” The results are displayed in Figure 16.
Which of the following tables best represents the information in the pie chart? Explain how you arrived at your answer.

A.

<table>
<thead>
<tr>
<th>Response</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agree</td>
<td>30</td>
</tr>
<tr>
<td>Disagree</td>
<td>71</td>
</tr>
<tr>
<td>Neither</td>
<td>2</td>
</tr>
</tbody>
</table>

B.

<table>
<thead>
<tr>
<th>Response</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agree</td>
<td>40</td>
</tr>
<tr>
<td>Disagree</td>
<td>50</td>
</tr>
<tr>
<td>Neither</td>
<td>2</td>
</tr>
</tbody>
</table>

C.

<table>
<thead>
<tr>
<th>Response</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agree</td>
<td>25</td>
</tr>
<tr>
<td>Disagree</td>
<td>73</td>
</tr>
<tr>
<td>Neither</td>
<td>2</td>
</tr>
</tbody>
</table>

D.

<table>
<thead>
<tr>
<th>Response</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agree</td>
<td>41</td>
</tr>
<tr>
<td>Disagree</td>
<td>58</td>
</tr>
<tr>
<td>Neither</td>
<td>1</td>
</tr>
</tbody>
</table>

3. As part of the 2002 survey from Monitoring the Future: A Continuing Study of American Youth, 12th-grade students were asked: How intelligent do you think you are compared to others your age? The frequency table in Figure 17 shows the results.

<table>
<thead>
<tr>
<th>Response</th>
<th>Frequency</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Far below average</td>
<td>121</td>
<td></td>
</tr>
<tr>
<td>Below average</td>
<td>745</td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>4102</td>
<td></td>
</tr>
<tr>
<td>Above average</td>
<td>6679</td>
<td></td>
</tr>
<tr>
<td>Far above average</td>
<td>1048</td>
<td></td>
</tr>
</tbody>
</table>

Figure 17. Results on the intelligence question.
a) How many students answered this question?

b) Copy the table in Figure 17. Fill in the percent column with the percentages for each category. Round your answers to the nearest tenth of a percent.

c) Represent the categories and percents from (b) in a bar chart.

4. Refer to the bar chart on air conditioning in Figure 9. The caption reads that two thirds of homes have some type of air conditioning. Do you agree with this statement? Explain.

5. The bar chart in Figure 18 shows the amounts that consumers spent on artificial nails/extensions and manicures in 1993, 1994, and 1995.

   a) In 1995 consumers spent an estimated $5.2 billion on nail services. Estimate the percent that was spent on artificial nails/extensions. (Round your answer to the nearest tenth of a percent.) Explain how you got your answer.
b) Which of the pie charts below best represents consumer spending in 1995?

6. The bar chart in Figure 18 shows the amounts that consumers spent on artificial nails/extensions and manicures in 1993, 1994, and 1995.

   a) Measure the length of the bar (finger) that represents the amount spent for artificial nails/extensions in 1993. Then measure the length of the bar that represents the amount spent for manicures in 1993.

   b) Based on your measurements, approximately how many times as much was spent on artificial nails/extensions as on manicures?

7. The pie chart in Figure 19 represents grades given in a math course.

   a) Which of the following is closest to the percent of students who got a B in the course?

      A. 120%
      B. 45%
      C. 33%
      D. 22%
b) Approximately what percentage of the class got As?
c) Approximately what percentage of the class did not get As?

8. **Figure 20** shows the recommended percentages of protein, fat, and carbohydrates for one diet plan.

![Pie chart of protein, fat, and carbohydrates.](image)

- **Carbohydrates** 55%
- **Protein** 30%
- **Fat** 20%

a) According to these recommendations, what percentage of your diet should be protein?
b) Represent the same information in a bar chart.
A Note on Context

This module investigates the statistical techniques for extracting information from data. We live in a world of data. There are databases with information about the food we buy, videos we rent, phone calls we make, and tollbooths we drive through. When data are collected and used properly, displays and statistics based on the data can provide useful information, which can guide decisions.

Topics of the data contained in this module are varied. They include work patterns of 12th-grade students, impact of music downloading on music sales, number of career home runs for players with all-time top batting averages, daily pulse rates and blood pressure readings for nursing home patients, and treadwear ratings for tires.

The module introduces students to techniques for analyzing both category and numeric data. Frequency tables, bar charts, and pie charts are used to analyze category data. Numeric data are displayed with stem-and-leaf plots, line plots, histograms, and box plots. In addition, students learn how to calculate basic statistics such as mean, median, mode, and range. These techniques help unlock the information contained in the data so the data can tell their story.

Technology Note

Graphing calculators are recommended for use throughout this module. In Lesson 3 in particular, students should be encouraged to use graphing calculators to calculate statistics and display data. Since graphing calculators are allowed on many standardized exams, student scores may improve if students know how to use them.

Module Opener

The purpose of this opening reading is to convince students that a lot of data by itself provides little information. For example, the numbers in Figure 1 look more like something out of *The Matrix* movies rather than information on the work patterns of 12th-grade students.

The data from Figure 1 are a small subset of the data collected for the study *Monitoring the Future: A Continuing Study of American Youth (12th-Grade Survey), 2002*. The 2002 study is part of an annual series. The series is designed to document changes in important values, behaviors, and lifestyles of American youth. More information can be found at the National Archive of Criminal Justice Data, where it is cataloged as study number 3753. The URL is

http://webapp.icpsr.umich.edu/cocoon/NACJD-STUDY/03753.xml
Lesson 1

Materials List
Protractors
Rulers (Exercise 6)

Preliminary Reading
The purpose of this reading is to give students background in analyzing category data. The techniques introduced include frequency tables, bar charts, and pie charts. It may be helpful if students check for themselves the frequencies in Figure 2. Have different groups of students count the 1s, 2s, 3s, 4s, 5s, 6s, 7s and 8s in Figure 1.

This would also be a good time to review how to compute percentages. Students should use calculators for the division. However, you may want to remind students that when you multiply a number by 100, you need only move the decimal point two places to the right. Stress some equivalences between some common fractions and percentages. For example, 1/2 is equivalent to 50%, 1/4 to 25%, 3/4 to 75%, and 1/3 to 33.3%. Unless students are told to round to the nearest percent, encourage students to retain one decimal when writing percents.

The In the News segment is from the National Record Buyers Study III, conducted by Edison Media Research. The complete report (filled with a variety of bar charts and pie charts) can be found at the following URL:


In this lesson students learn how to make frequency tables, bar charts, and pie charts for category data. If you want students to draw a pie chart by hand, you will need to provide protractors. Some state exams provide protractors and rulers during the exam. However, for most questions students need only approximate the correct size of a pie slice. Hence they should be familiar with basic percentages: 50% is half of the pie, 33.3% is one-third of the pie; 25% is one-fourth of the pie; 12.5% is one-eighth of the pie.

Activity 1
The purpose of this activity is to give students an opportunity to practice what they have learned on a set of real data from the study Monitoring the Future: A Continuing Study of American Youth. These data are from students who identified their high school program as vocational, technical, or commercial. (The data from Figure 1 included high school students enrolled in all types of programs.) The activity includes data from four questions on gender, work, and driving.

Students could use protractors to answer Question 1, which involves pie charts. However, giving students an opportunity to reason what the correct size of the slice should look like is probably more beneficial. The correct angle for the slice representing the Males should be 62% of 360° or 223.2°. However, 62% is between
50% and 75%. Hence, the slice representing 62% should be between one-half and three-quarters of a pie.

Make sure in Question 2(b) that students compare percents and not frequencies. This is an open response question. Many state exit exams contain open response questions and students need practice answering them. Set aside some time to discuss student answers to this question.

Question 3 would be a good place for students to use protractors.

Exercise Set 1

The purpose of this set of exercises is to provide additional practice with frequency tables, bar charts, and pie charts. Some of the questions are patterned after questions that appear on state exit exams.

Exercise 1 is a multi-part question that focuses on estimating percents from a bar chart. Part (d) asks students to calculate a percentage of a particular quantity.

Exercise 2 asks students to estimate percentages in each category given a pie chart.

Exercise 3 gives the frequencies and sample size and asks students to calculate the percents. Then they draw a bar graph, which has a percent scale on the vertical axis.

Exercise 4 asks students to reconcile a discrepancy between 68% and the fraction 2/3.

Exercises 5 and 6 are based on a rather elaborate bar graph, in which the bars are drawn as fingers. In Exercise 5, students estimate a percentage from the bar graph and then select a pie chart that represents that percentage. In Exercise 6, students estimate how many more dollars are in one category than another by comparing the height of the bars (fingers). They will need rulers for this exercise.

Exercises 7 and 8 are based on pie charts. In Exercise 7, students estimate percentages from a pie chart. In Exercise 8, they draw a bar chart that represents the same information contained in a pie chart.
Activity 1

1. a) There were 31 males out of 50 students.
   
   \[
   \text{Percent of males} = \left(\frac{31}{50}\right) \times 100\% = 62.0\%.
   \]

   b) Percentage of females = 100% – 62% = 38%; or, \( \left(\frac{19}{50}\right) \times 100\% = 38.0\% \).

   c) C ; Sample explanation: Pie chart A shows more females than males. Pie chart B shows equal numbers of males and females. Since there are more males than females in the sample, both A and B should be rejected. In D, the slice representing the females appears to be roughly 1/4 th or 25% of the pie. This percentage is too low. Hence, by a process of elimination, the correct answer is C. Furthermore, the slice of pie C that represents the males is larger than the slice in B and smaller than the slice in D; hence, the slice for males in pie C represents between 50% and 75%. This fits in with the actual percentage of 62%.

2. a)

<table>
<thead>
<tr>
<th>Category</th>
<th>Frequency</th>
<th>Percent (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) None</td>
<td>8</td>
<td>16</td>
</tr>
<tr>
<td>(2) 5 or fewer hours</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>(3) 6 to 10 hours</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>(4) 11 to 15 hours</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>(5) 16 to 20 hours</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>(6) 21 to 25 hours</td>
<td>7</td>
<td>14</td>
</tr>
<tr>
<td>(7) 26 to 30 hours</td>
<td>7</td>
<td>14</td>
</tr>
<tr>
<td>(8) More than 30 hours</td>
<td>9</td>
<td>18</td>
</tr>
<tr>
<td>Total</td>
<td>50</td>
<td>100</td>
</tr>
</tbody>
</table>

b) It does not make sense to compare frequencies since the sample size for the frequency table in Figure 2 is 165 students and the sample size for the frequency table from (a) is 50 students. In order to make a comparison, the percentages must be used. This is an open response question and hence there are many good answers.

Sample answer: In the frequency table from Figure 2, the highest percentage was in the category of none: 23% of the high school students (all programs) did not work during the school year. This percentage is lower for students enrolled in a vocational, technical, or commercial (votechcom) program; only 16% of those students did not work during the school year. The highest
percentage for the votechcom students fell in the category of working 16 to 20 hours. This was the second highest percentage for the mixed high school students (all programs). The two percentages, 20% for votechcom students and 20.6% for the mixed high school students, are about the same. The largest difference is in the percentages for students who work more than 20 hours per week: 46% for the votechcom students and only 30.3% for the mixed high school students. In conclusion, a higher percentage of the mixed students than the votechcom students did not work during the school year. A considerably higher percentage of votechcom students than mixed students worked 21 or more hours.

c)

Note: Students may choose a shorter title. They may also replace the frequency scale with a percentage scale, in which case the numbers on the vertical axis would double. However, the relative sizes of the bars to each other should remain the same for all histograms, regardless of the scale chosen for the vertical axis.

3. a) Ten students reported they did not drive at all. That is $\frac{10}{50} \times 100\% = 20\%$ of the students.

b) If 20% of the students did not drive at all, then $100\% - 20\% = 80\%$ did drive during an average week.
c) The angle for the slice representing the non-drivers is 20% of 360°: 
\[(0.20)(360°) = 72°\]. Thus, the angle for the slice representing the drivers is 
\[360° – 72° = 288°\]. The pie chart below is drawn using a protractor.

![Pie Chart Showing Driving Status of Students in Vocational, Technical, or Commercial Programs](image)

If students do not have protractors, they can approximate this pie chart by 
first drawing a quarter slice (for 25%) and then making it a little smaller. 
More precisely, they could divide the one-quarter slice into 5 equal slices and 
remove one of these smaller slices as shown below:

![Pie Chart with Approximation](image)

The four small slices remaining will represent 20%.

4. a) 40

b) Sample answer: First, we crossed out all the students who answered 1 to 
the Drive question. That left 40 students who answered 2–6, meaning they 
drove more than one mile during an average week. From these remaining 
students, we counted the number of Yes responses to the Ticket question. 
There were 14 Yesses. The percentage of Yesses from the students who 
drive is \[(14/40) \times 100\% = 35\%\].

c) This is a very open question and there are many good answers. 
Sample answer: On the surface, it appears that police do target high school 
students when they give out tickets or warnings because the percentage of 
35% is very high. One possibility is that a high percentage of high school 
students exceed the speed limit when driving, run stop signs or pass 
improperly. Perhaps the percentage of high school students who engage in 
these practices is higher than for drivers in general. Thus, high school 
students are more likely to get stopped. Another possibility is that the 
percentage of all drivers that get tickets or warnings in a single year is also
about 35%. Hence, data from another survey is needed. There should be three types of questions on this survey: (1) a question on high school status (high school student, out of high school), (2) a question asking if the person frequently exceeds the speed limit, runs stop signs, or passes improperly, and (3) the ticket question from Figure 13.

Exercise Set 1

1. a) Around 15%
   b) Around 53%
   c) C
   d) Using the answer to (c) or 47%, 47% of 1500 is \(0.47)(1500) = 705\).

2. D; Sample explanation: A and B can be eliminated because the percentages do not add up to 100. In C, the percentage for Agree is one-quarter of the pie. However, the actual slice for Agree is more than a quarter of a pie. Hence C can be eliminated. The remaining answer is D.

3. a) 12,695
   b) Here are the calculations:

\[
(121/12,695) \times 100\% \approx .953\%; 
(745/12,695) \times 100\% \approx 5.868\%; 
(4,102/12,695) \times 100\% \approx 32.312\%; 
(6,679/12,695) \times 100\% \approx 52.611\%; 
(1,048/12,695) \times 100\% \approx 8.255\%.
\]

<table>
<thead>
<tr>
<th>Response</th>
<th>Frequency</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Far below average</td>
<td>121</td>
<td>1.0</td>
</tr>
<tr>
<td>(2) Below average</td>
<td>745</td>
<td>5.9</td>
</tr>
<tr>
<td>(3) Average</td>
<td>4,102</td>
<td>32.3</td>
</tr>
<tr>
<td>(4) Above average</td>
<td>6,679</td>
<td>52.6</td>
</tr>
<tr>
<td>(5) Far above average</td>
<td>1,048</td>
<td>8.2</td>
</tr>
</tbody>
</table>

c)
4. Sample answer #1: According to the bar chart in Figure 9, 32% of homes have no air conditioning. That means that $100\% - 32\% = 68\%$ of the homes have some sort of air conditioning. Two thirds is equivalent to $\left(\frac{2}{3}\right) \times 100\% = 66.7\%$. If you round up to the nearest percent, you only get 67%, a little short of 68%.

Sample answer #2: For their caption, *USA Today* wanted a fraction with a small denominator that was close to 67%—$\frac{2}{3}$ would work for that purpose.

5. a) Sample (answers may differ slightly): The longest finger represents the amount spent on artificial nails//extensions. It represents approximately $4.2$ billion. This is $\left(\frac{4.2 \text{ billion}}{5.2 \text{ billion}}\right) \times 100\% \approx 80.8\%$ of the total amount that consumers spent on nail services in 1995.

b) B

6. b) $\left(\frac{\text{length of 1993 finger for artificial nails}}{\text{length of 1993 finger for manicures}}\right) = 3$. Hence, approximately three times as much was spent on artificial nails//extensions as on manicures.

7. a) C

b) $(\frac{60}{360}) \times 100\% \approx 17\%$. (Answers may vary slightly.)

c) $100\% - 17\% = 83\%$. (Answers may vary slightly.)

8. a) 15%

b)
CAREERS & FINANCE

TEACHER EDITION
The Right Package

Gary Froelich
Most people do not get very far in a day without using a package. The tube that holds your toothpaste is a package. So is the bottle that holds your shampoo. And many people have a breakfast of cereal that comes in a box and milk that comes in a carton.

Many jobs involve packaging. If you work in a supermarket, nearly every product you handle is packaged. Sometimes a product is packaged more than once. For example, soda is often sold in cans, which are packages. Twelve cans are packaged in a paperboard container. And the customer might carry the 12-pack home in a bag, which is another package.

Here are a few facts about packaging.

- The packaging business contributes over a trillion dollars to the American economy each year.
- In the United States, the packaging industry employs over a million people.
- Beverage packaging alone is a huge business. In the United States, over 150 billion beverage containers are sold annually.
- About 10 cents of every dollar spent in a grocery store pays for packaging.
- About a third of America's garbage is packaging.
- Most companies that use packaging are trying to use less. For example, Sunoco says it has reduced the amount of paperboard in its packages by 14%. This amounts to a savings of about 2000 tons a year.

Since packaging is so common, knowing a little about it can be helpful in many careers. Perhaps one day you will work for a company such as Royal Paper Box, which designs and makes cartons. Or perhaps you will work for a company such as United Parcel Service that transports cartons around the world.

This module focuses on paperboard containers, one of the most common types of packaging. Of course, the most common shape of such containers is the rectangular box like the ones that contain cereal. But there are many other shapes out there—the one in Figure 2, for example. As you work through this module, keep your eyes open for cartons with unusual shapes.

There are many things that make a package the right package. For example, a package that sits on the shelf of a store must be attractive and convenient. But all packages have to hold the right amount of product and use packaging material efficiently.
In this module, you consider two of the most important questions about any package:

- How much material is needed to make the package?
- How much does the package hold?

**IN THE NEWS ...**

**Women In Packaging Founder Honored Among 50 Most Influential Leaders**


JoAnn R. Hines, founding executive director of Women in Packaging, Inc., has been honored as one of the packaging industry’s 50 Most Influential Leaders of the 20th Century. Hines earned the designation from *Boxboard Containers International (BCI)*, a trade publication covering news in the boxboard, carton and paperboard-container industries.

BCI compiled the list to recognize the important individuals who made packaging the third-largest industry in the United States.

“Ms. Hines’ foresight continues to bring the packaging industry some of its best leaders,” wrote Michelle Nordlinger, who nominated Hines for the award. “Her talents and experience make her a visionary, a true business leader, a mentor and an innovator.”
Area

PRISMS

Most cartons are shaped like a prism. A prism has two congruent parallel surfaces that are polygons. (Congruent means they have the same size and shape.) These two surfaces are sometimes called the bases of the prism. All the other surfaces are rectangles.

If you’re not familiar with the term polygon, it means a figure with several sides. The simplest polygon is a triangle. Other examples are squares, rectangles, and hexagons.

A prism is usually named for its bases. For example, the Toblerone package in Figure 3 is a triangular prism. The Droste package in Figure 4 is a hexagonal prism.

Cartons start life flat. Then they are folded into a prism shape. For example, the FedEx Tube is shown in its original state in Figure 5. It is folded into the prism shape shown in Figure 6.

To be able to find the amount of paperboard in a carton, you need to know some basic area geometry. No doubt you have worked with area formulas in the past. Here’s a brief review of some key facts.
MEASURING AREA

Area is measured in square units. The size of the square used to measure area varies. For example, area might be measured in square centimeters. That is, in squares 1 centimeter on a side. **Figure 7** is 1 square centimeter.

Another common area measure is square inches. **Figure 8** is 1 square inch.

Of course, there are larger units of area measure such as a square mile and a square kilometer.

RECTANGLES

To find the area of an object, you could divide it into squares and count them. But it’s much easier to calculate the area. The simplest area to calculate is a rectangle.

**Figure 9** shows a rectangle. The sides measure 5 cm and 3 cm.

As you can see, the rectangle in Figure 9 contains $5 \times 3 = 15$ square centimeters.

The area of a rectangle is calculated by multiplying the length by the width: $\text{Area} = \text{length} \times \text{width}$.

TRIANGLES

**Figure 10** shows a triangle with one 90° angle. You may recall that an angle that measures 90° is called a right angle.

**Figure 11** shows that this triangle is half of a rectangle with the same length and width.

So the area of the triangle is half the area of the rectangle.

The area of a triangle is calculated by multiplying 0.5 by the base by the height: $\text{Area} = 0.5 \times \text{base} \times \text{height}$.

The base can be any of the triangle’s sides. The height is perpendicular to the base and goes through the opposite corner or vertex. **Figures 12 and 13** show two ways to calculate the area of the same triangle.
Rectangles and triangles are both polygons. The triangle's area formula is especially handy for calculating areas of other polygons. That's because any polygon can be divided into triangles. So, you can just find the area of each triangle and add them up. Figure 14 shows a hexagon divided into triangles.

To find the area of any polygon, divide it into triangles. Use the formula for the area of a triangle on each of them. Then add these areas.
In this activity, you calculate the number of square inches of paperboard in a carton shaped like a prism. You'll also consider how the flat version of the prism folds into the real thing!

**Figure 15** is a reduced version of Handout 1. It folds into a triangular prism.

A flattened version of a solid figure is sometimes called a net.

1. Notice that some of the edges are numbered. Two edges are numbered 1. When the net is cut out and folded into a triangular prism, the two edges numbered 1 meet. Find and label the edge that meets each of edges 2, 3, 4, and 5.

2. Cut out Handout 1 and fold it into a prism.
3. Use area formulas to find the total number of square centimeters of paperboard in the prism’s surface. Be sure to do the following:
   ■ Draw the height you use to find a triangle’s area.
   ■ Use a ruler to measure edges and heights to the nearest tenth of a centimeter.
   ■ Write all measurements on the prism.
   ■ Write the area of each rectangle or triangle somewhere on it.

4. How well do you think your answer in Question 3 gives the amount of paperboard needed to make a carton? (For example, a candy carton similar to the Toblerone or a shipping carton similar to the FedEx.) Explain.

---

**How precise is your area calculation?**

You may have seen rules for rounding area calculations. To be as precise as possible about area, you should calculate a range based on the precision of your measurements.

Suppose you measure one side of a rectangle as 2.3 cm. If you measure carefully, the length is closer to 2.3 than to 2.2 or 2.4. It could be as low as 2.25 cm or as high as 2.35 cm.

If you measure the other side as 4.6 cm, then it’s between 4.55 cm and 4.65 cm.

So, the least the area can be is $2.25 \times 4.55 = 10.2375 \text{ cm}^2$.

The most the area can be is $2.35 \times 4.65 = 10.9275 \text{ cm}^2$.

Assuming you measured properly, the area is between 10.2375 cm$^2$ and 10.9275 cm$^2$. 

---

Measure carefully!

Place the ruler’s 0 at the vertex of the triangle. To be sure the edge of the ruler is perpendicular to the base of the triangle, align the ruler’s markings with the base.
1. On a piece of paper or paperboard, draw a net that folds into a square prism. That is, the two bases are squares. Make the sides of the squares 4 cm. Make the long sides of each rectangle 9 cm.
   a) Use numbers to label the edges that come together when the net is folded into a prism.
   b) Calculate the area of each square and rectangle and use these areas to find the total area of the prism’s surfaces. Write each measurement you make and each area you calculate on your net.

2. Cartons such as the Toblerone package and the FedEx Tube use equilateral triangles for bases. An equilateral triangle is one with edges that are the same length. The net in Figure 16 is different. The triangles are isosceles. That is, only two of each triangle’s edges are the same length.
   a) Number the edges to show how this net folds into a prism.
   b) Use Handout 2 to calculate the area of each triangle and rectangle and use these areas to find the total surface area of the prism. Write each measurement you make and each area you calculate on the net.

---

**Figure 16.**
A net for a triangular prism. A reduced version of Handout 2.
3. **Figure 17** is a regular hexagon. It is called regular because all of its sides are equal and all of its angles are equal.

a) There are several ways to divide this hexagon into triangles. Show one way.

![Hexagon](image)

b) Find the area of each triangle you drew and use these areas to find the area of the hexagon. Write all measurements you make on the hexagon.

c) This hexagon can be used to make a hexagonal prism similar to the Droste package in Figure 4. Draw a net that will fold into a hexagonal prism. Number the edges to show how the net folds into a prism.

d) If your net uses the hexagon in Figure 17 for bases and rectangles with lengths of 10 cm, what is its total surface area?

---

**Have you ever wondered what your area is?**

There are several formulas for a person’s surface area. Most of them use weight (W) and height (H). The following formula uses weight in kilograms and height in centimeters. It estimates area in square meters.

\[
\text{Body surface area} = (W^{0.425} \times H^{0.725}) \times 0.007184.
\]

The Internet site [www.halls.md/body-surface-area/bsa.htm](http://www.halls.md/body-surface-area/bsa.htm) calculates body surface area by several different formulas. To find other sites, do a search on "body surface area."
4. **Figure 18** shows an unusual carton. **Figure 19** is a net that can be folded into a carton similar to the one in Figure 18. One difference is that the triangular pieces are not pushed in as they are in Figure 18.

![Figure 18. A Biscotti carton.](image)

![Figure 19. A reduced version of Handout 3.](image)

a) Number the edges to show how this net folds into a carton.

b) Use Handout 3 to calculate the area of each triangle and rectangle and use these areas to find the area of the paperboard in the carton. Write each measurement you make and each area you calculate on the net.

5. Surface area is important in fields other than packaging. A painter has been asked to give a cost estimate for painting the building shown in **Figure 20**. The building has a shingled roof. The area to be painted includes the gable ends. The building requires two coats of paint. One gallon of paint covers about 350 square feet. To give an estimate, the painter must know the number of gallons of paint required. Show how the painter can calculate the number of gallons needed.

![Figure 20.](image)
6. Some cartons are shaped like cylinders. Cylinders are not prisms, but they have a lot in common with prisms. The main difference is that the bases of cylinders are circles. The Pringles package in Figure 21 is an example of a carton that is cylindrical.

Figure 22 is a net that folds into a cylinder. As you can see, the net is two circles and a rectangle.

![Figure 21. A cylindrical Pringles carton.](image)

![Figure 22. A net for a cylinder. A reduced version of Handout 4.](image)

In some ways, calculating surface area of a cylinder is simpler than it is for a prism. There are only two areas to calculate.

**Useful circle formulas:**

- **Area** = \( \pi r^2 \)
- **Circumference** = \( 2\pi r = \pi d \)

The area formula for a circle requires the radius. But the radius can be hard to measure if you don’t know the exact location of the circle’s center. So many people measure the diameter and divide by 2.
a) Describe how the net in Figure 22 folds into a circle.

b) Use Handout 4 to make and record the measurements you need to calculate the surface area of the cylinder, and then show how to calculate the surface area.

c) You may have measured both the length and the width of the rectangle to calculate its area. You don’t have to measure both because you can use the diameter of the circle to calculate one side of the rectangle. Explain how to do this.

7. Part of the packaging of a roll of paper towels is a paperboard tube that runs down the inside of the roll (see Figure 23).

In one brand of towels, this tube measures 27.9 cm high. The circular base has a diameter of 4.4 cm. Calculate the number of square centimeters of paperboard in the tube. Show your calculations.
The Right Package: Teacher Notes

**Prerequisites**
Basic calculator skills
The ability to round decimal numbers
The ability to use a ruler to measure lengths
Previous experience with basic area formulas (i.e., rectangle, triangle)

**Technology Note**
This module assumes that students use calculators. Any basic calculator will suffice. The best calculator is one that displays an entire expression before execution.

This module offers opportunities to use geometric construction software such as *Geometer's Sketchpad*. For example, students can use software to design their own nets. Another possibility is to have students use software to construct altitudes and take measurements on existing sketches.

Accompanying this module is a folder of *Geometer’s Sketchpad* sketches that match several of the nets shown in the student materials or solutions. The following is a list of the file names and the associated components.

- Activity1.gsp  Activity 1 and Handout 1
- ExSet1P1.gsp  Exercise Set 1, Problem 1
- ExSet1P2.gsp  Exercise Set 1, Problem 2 and Handout 2
- ExSet1P3.gsp  Exercise Set 1, Problem 3
- ExSet1P4.gsp  Exercise Set 1, Problem 4 and Handout 3
- ExSet1P6.gsp  Exercise Set 1, Problem 6 and Handout 4
- Activity2.gsp  Activity 2 and Handout 5
- Triang6pack.gsp  Project
- Hex6pack.gsp  Project
- Hex7pack.gsp  Project
- Cylind7pack.gsp  Project

Note that a free Java geometry construction program is available at http://mathsrv.ku-eichstaett.de/MGF/homes/grothmann/java/zirkel/index_en.html.
Internet Sites
There are many Internet sites on geometric solids that you and your students may find interesting while doing this module. Here are a few that were active at the time this draft was written.

http://www.korthalsaltes.com/ has printable nets for many geometric solids.
http://www.ex.ac.uk/cimt/res2/trolqc.htm has calendars in a variety of solid geometric shapes that can be downloaded and printed.
http://mathworld.wolfram.com/topics/SolidGeometry.html has Java applets that allow the user to grab, rotate, and tilt a variety of geometric solids. Prisms are filed under polyhedra.
http://www.paperzone.com/Crafter_Corner/cornerboxes.htm has several gift box nets that can be printed free of charge.
http://www.peda.com/poly/ has Mac and Windows trial versions of a program called Poly, with which the user can construct solids and print nets.
http://www.macdirectory.com/reviews/quarkwrapture/ has information on Quark Wruption, a professional program for designing cartons.

Background Reading

Lesson 1

Materials List
Copies of Handouts 1–4 printed on card stock or cover stock (available from office supply stores such as Staples)
Scissors
Tape
(Optional) Geometer’s Sketchpad files: Activity1.gsp, ExSet1P1.gsp, ExSet1P2.gsp, ExSet1P3.gsp, ExSet1P4.gsp, ExSet1P6.gsp

Many students will benefit from seeing the actual cartons that were photographed for this module. Here is a list of them and the source from which each was obtained:
Toblerone: Walgreen’s
Droste: Trader Joe’s
FedEx Tube: found behind a FedEx drop in an office building
Biscotti: Trader Joe’s
Pringles: Any supermarket and many other stores (i.e., Wal-Mart, drug stores)
Paper towel tube: ditto

**Preliminary Reading**
The purposes of this reading are to define the term prism and to review basic area concepts and formulas.

Note that this module does not consider oblique prisms: those in which the faces are parallelograms rather than rectangles. Thus, the type of prism defined here is really a right prism.

Also note that the term surface as applied to prisms in this module means a polygonal or circular (in the case of cylinders) region.

You may find that some students confuse the term *bases* with top and bottom. Sometimes these students are more comfortable with the informal term *ends*, especially when the prism is fairly elongated. For example, a triangular prism has triangles for ends.

Note that this preliminary reading suggests a triangulation procedure for finding areas of polygons. A single versatile procedure such as this is likely to serve many students better than a battery of special formulas, which are often memorized and soon forgotten. For more advanced students, you may want to discuss special area formulas such as \( \frac{s^2 \sqrt{3}}{4} \) for equilateral triangles and the related \( 1.5s^2 \sqrt{3} \) for regular hexagons or the more general Heron’s formula: \( \sqrt{s(s-a)(s-b)(s-c)} \), where \( a, b, \) and \( c \) are the side lengths, and \( s \) is half the perimeter.

As you progress through this module, you may want to show your students a variety of cartons and ask them to decide whether each is a prism. For example, consider the Trader Joe’s Truffles carton in **Figure 1.**
At first glance, the carton may seem to have a prism shape. However, the edges that appear vertical in Figure 1 are not. If extended upward far enough, they would meet, as shown (not to scale) in Figure 2. Thus, this carton is based on a rectangular pyramid. (It is a frustum of a pyramid, to be more accurate.)

![Diagram of a rectangular pyramid](image)

**Figure 2.**

No matter which two faces students claim are bases, they are not. The top and bottom as shown in Figure 1 are not prism bases because the top is slightly smaller than the bottom. In each of the remaining two pairs of opposite faces, the polygons are congruent, but they are not parallel.

**Activity 1**

The purpose of this activity is to provide practice with visualizing prisms and calculating their surface areas.

Have copies of Handout 1, scissors, and tape available for this activity.

In this activity and the exercise set that follows, advise students to keep a record of their answers and calculations. Doing so will save them time in Lesson 2, in which volumes of many of the same objects are considered.

This module does not discuss construction methods for drawing altitudes of triangles. The decision to do so is left to the teacher. Students should be able to measure altitudes to the nearest millimeter without constructing them. To help students make such measurements, the first sidebar in this activity includes a photograph. Circulate among your students and check for good measurement technique. The ruler’s 0 should be over the vertex, and the ruler’s markings should align with the triangle’s base or be parallel to it.

Note that the second sidebar in this activity contains information on the precision of area calculations. This module does not require students to give an interval for their area answers. Again, the decision to do so is left to the teacher.
Exercise Set 1
The purpose of these exercises is to provide additional practice with visualization and area calculation for prisms and to extend these concepts and skills to cylinders.

Have copies of Handouts 2–4 available for these exercises. If students do the exercises in class, provide scissors and tape.

Exercises 1 and 2 are similar to Activity 1. Handout 2 is helpful in Exercise 2.

Exercise 3 uses the triangulation technique discussed in this lesson’s Preliminary Reading.

Note the sidebar in Exercise 3. You may want to mention to your students that knowing the surface area of a person is important in certain kinds of medical treatment such as caring for burn patients.

Exercise 4 considers a solid that is not a prism. However, the net given in Figure 19 actually is prismatic since its triangular faces are not pushed inward. Each base is composed of one of the long narrow rectangles and the adjacent triangle. Students may think this is not a prism because the bases are pentagons that are not regular. Handout 3 is helpful for this exercise.

Exercise 5 considers the surface area of an object that also has a prismatic shape. Again, students may not recognize the shape as a prism. In this case, the bases are the ends with gables. These bases are pentagons, but not regular ones.

Exercises 6 and 7 involve cylinders, which of course are not prisms, but for which similar area procedures apply. Handout 4 is helpful for Exercise 6.

Lesson 2

Materials List
Copies of Handouts 5–6 printed on card stock or cover stock (available from office supply stores such as Staples)

Scissors
Tape

(Optional) Geometer’s Sketchpad file: Activity2.gsp

Preliminary Reading
The purpose of this reading is to review basic concepts of volume.

The primary objects of discussion in this reading are the prism and the cylinder. At the end of the reading, there is a brief discussion of cones and pyramids, which are used in a few of the supplemental problems only.
Handout 1
Handout 3
The Right Package: Answers

Lesson 1

Activity 1

1. Answer:

3. The total surface area is $72 + 72 + 72 + 15.6 + 15.6 = 247$ cm$^2$. Measurements and areas are shown in the following figure.
4. Sample answer: It’s probably low. One reason is that an actual carton needs flaps to glue together. Also, there might be waste if the carton is cut from a larger piece. If the waste is recycled, maybe it’s not important.
Exercise Set 1

1. a) Sample answer:

Note that the positions of the squares in the sketch can vary: the top square can share an edge with any of the four rectangles, as can the bottom square.

b) The individual areas are shown in the answer to part (a). The total surface area is $36 + 36 + 36 + 36 + 16 + 16 = 176 \text{ cm}^2$. 
2. a) Sample answer:

\[ 96 \text{ cm}^2 + 96 \text{ cm}^2 + 72 \text{ cm}^2 + 22 \text{ cm}^2 + 22 \text{ cm}^2 \approx 308 \text{ cm}^2. \]

b) The individual areas are shown in the answer to part (a). The total surface area is \( 96 + 96 + 72 + 22 + 22 = 308 \text{ cm}^2. \)
3. a) Sample answer:

Note that in this division, there appear to be two pairs of congruent triangles. The measurements students take in part (b) can confirm that there are two pairs of equal areas.

b) Sample answer based on the answer to part (a):
Two of the triangles have area $0.5 \times 2.5 \times 8.7 = 10.9 \text{ cm}^2$ each. The other two each have area $0.5 \times 4.3 \times 10.0 = 21.5 \text{ cm}^2$. Thus, the total area is $2 \times 10.9 + 2 \times 21.5 = 65 \text{ cm}^2$.

c) Sample answer:

d) There are six rectangles, and each has area $5 \times 10 = 50 \text{ cm}^2$. So the total surface area is $6 \times 50 + 2 \times 65 = 430 \text{ cm}^2$. 

4. a) Sample answer:

\[ 18 \text{ cm}^2 \quad 45 \text{ cm}^2 \quad 10 \text{ cm}^2 \quad 2.0 \text{ cm} \quad 5.0 \text{ cm} \]

b) Measurements and areas are shown in the answer to part (a). There are three rectangles each with an area of 10 cm\(^2\), two rectangles each with an area of 18 cm\(^2\), two rectangles each with an area of 45 cm\(^2\), and two triangles each with an area of 1.7 cm\(^2\). Therefore, the total area is 
\[ 3 \times 10 + 2 \times 18 + 2 \times 45 + 2 \times 1.7 \approx 159 \text{ cm}^2, \text{ or about } 160 \text{ cm}^2. \]

5. There are two rectangular surfaces each with an area of \(32 \times 8 = 256 \text{ ft}^2\). Each end is composed of a rectangle with a triangular gable. The rectangle’s area is \(24 \times 8 = 192 \text{ ft}^2\). The triangle’s area is \(0.5 \times 24 \times 5 = 60 \text{ ft}^2\). Thus, the area to be painted is \(2 \times 256 + 2 \times 192 + 2 \times 60 = 1016 \text{ ft}^2\). This requires \(1016 \div 350 \approx 2.9\) gallons. But two coats are required, which means about \(5.8\) gallons are needed. Therefore, 6 gallons are needed.
6. a) Sample answer: the rectangle rolls into a tube shape. Each long side of the rectangle meets the circumference of a circle.

b) Each circle's diameter measures 6 cm, so the radius of each circle is 3 cm. Each circle's area is \( \pi \times 3^2 = 28.3 \text{ cm}^2 \). The rectangle measures 12 cm by 18.8 cm, so its area is 12 \( \times \) 18.8 = 225.6 cm\(^2\). The total surface area is 2 \( \times \) 28.3 + 225.6 = 282 cm\(^2\).

c) The top and bottom sides of the rectangle in Figure 22 must be the same length as the circumference of the circle. So each of them must be \( \pi \times 6 \approx 18.85 \text{ cm} \).

7. If cut from top to bottom and laid flat, the tube is a rectangle. One side measures 27.9 cm. The length of the other side is the same as the circumference of the circle: \( \pi \times 4.4 \approx 13.82 \). So the area of the tube is 27.9 \( \times \) 13.82 \( \approx \) 111 cm\(^2\).
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