Chapter 1

Major Contextual Themes

Government

Public Policy

Major Mathematical Themes

Optimization

Distance

Related Disciplines

Business and Finance

Chemistry

English

Environmental Science

Urban Planning

Scope and Sequence Chart

	Lesson 1	Lesson 2	Lesson 3	Lesson 4	Lesson 5
Distance in firetruck geometry	•				•
Circles in firetruck geometry	•				•
Total and average distance in two dimensions	•				•
Total and average distance in one dimension		•			
Addition of functions		•			
Slope and rate of change		•			
Absolute-value equations and inequalities		•	•		
Absolute-value graphs		•	•	•	
Piecewise-defined functions		•	•	•	
Median		•			•
Transformation of functions			•		
Maximum distance in one dimension				•	
Midrange				•	•
Maximum distance in two dimensions					•

Context Overview

Gridville, a city laid out in a rectangular grid of streets, needs a fire station. Students are challenged to find the optimum placement for this fire station. Throughout the chapter, the role of mathematics and the role of community values are considered together in the search for the best location. Lessons follow the modeling process of simplifying the problem to study the essential conditions and mathematics in detail. Finally, students return to the original problem and use their new mathematical understanding to provide optimal solutions.

Mathematical Development

In order to attach the two-dimensional problem, students first consider a simplified model, a one-dimensional village with houses spread along one street. Despite its simplicity, the linear model gives rise in a natural way to important mathematical ideas of absolute value, average, median, and midrange.

Students explore the problem numerically, graphically, symbolically, and logically--first seeking locations for which total distance to all houses is minimized. Absolute-value notation is introduced as a symbolic way to represent distances between locations in one dimension. Students determine that a median location must minimize the total distance and average distance.

Sine the use of the absolute-value function to represent distance is essential throughout the chapter, the absolute-value function is studied in detail. Students learn to recognize the graph of the basic function $y = |x|$ and of simple transformations of that function, including $y = a|x - h| + k$.

Because the graphs of the absolute-value function and of sums of absolute-value functions resemble combinations of linear functions, students are introduced to piecewise descriptions of functions. Students develop equations to match each portion of a graph and identify, using inequalities, the interval for which each equation is valid.

Still within the context of a linear village, students find locations minimizing the maximum distance a fire truck must travel to reach all the houses, noting that the midrange determines minimax locations in one dimension.

With two models and two examples of optimization, students consider the issue of fairness. Which location is “best” becomes “best of whom?” or “best under what circumstances?”

Students return to the two-dimensional context of Gridville and apply what they learned in the linear village. “Firetruck” geometry, introduced in the first lesson, is further developed as students explore distance in two dimensions. Students discover that the method to minimizing total distance two dimensions is to find the solutions to two one-dimensional problems. The search for minimax in two dimensions leads to methods for finding the centers of the smallest circles that contain all the houses in Gridville.

Students are now prepared better to address the key question, “What is the best location for the fire station in Gridville?” Given a map of Gridville, students can find locations that minimize total distance and locations that minimize maximum distance. Finding the best location involves establishing criteria that depend on community values. Once the criteria have been determined, students may use mathematics to help find and evaluate locations based on the criteria.

Related Disciplines

The real-world contexts of this chapter provide many opportunities for students to apply what they are learning to other subject areas and to apply knowledge from other areas to the development of new mathematical ideas.

Business and Finance

Optimizing routes for delivery services.

Lesson 1

Chemistry

Using absolute value to represent fluid and temperature levels.

Lesson 3

English

Preparing, writing, and presenting reports.

Lessons 1, 2, 3, 4, 5

Environmental Science

Determining the best location for a recycling center.

Lesson 1

Urban Planning

Determining the best location for a fire station.

Lessons 1 and 2

Locating a fire station or regional mall.

Lesson 5

Lesson 1: In Case of Fire

2–3 Days

This chapter explores the problem of optimizing the position of an object on a grid. The first lesson establishes the context for the chapter, poses the key question, and sets the stage for the modeling process. Gridville, a town whose streets are laid out in a grid-like manner, needs to determine where to build a fire station. A group activity challenges students to determine the best location for the fire station.

Lesson 2: Linear Village

4–6 Days

The purpose of this lesson is to begin the modeling process and to introduce multiple representations to solving the location problem. Students begin the modeling process by simplifying the two-dimensional Gridville model to the one-dimensional Linear Village. The first activity challenges students to use tables and graphs to observe patterns and develop a procedure for finding locations that minimize the total distance a fire truck would travel to reach each house in the community. The second activity introduces students to absolute-value notation as a symbolic method for exploring the location problem in one dimension.

Lesson 3: Absolute Value

2–3 Days

The purpose of this lesson is to study the absolute-value function in detail. In the context of a game, students transform absolute-value function graphs by changing the control numbers a, h, k in the general equation $y = a|x - h| + k$. Piecewise descriptions are used to specify the linear components of absolute-value graphs.

Lesson 4: Minimax Village

2–3 Days

While still in the context of Linear Village, students are introduced to another condition for optimization. The purpose of this lesson is to develop procedures for finding locations that minimize the maximum distance a fire truck must travel to reach any house in the community. The issue of fairness is raised as the competing criterion, leading to different choices for best location.

Lesson 5: Return to Gridville

3–5 Days

In search of the best location to build the fire station in Gridville, students investigate finding both the median and the minimax locations in two dimensions. The purposes of this lesson are to apply the concepts learned in the one-dimensional Linear Village to the two-dimensional Gridville and to develop procedures for easily finding locations that optimize distance.

Lesson 2: Chapter Review

1–2 Days

This section provides exercise to review concepts taught in the chapter, a chapter summary, and a glossary.

Teacher Provided Materials

Geometric drawing utility software

Spreadsheet software

Chart paper (with grids, if possible)

Markers or colored pencils

Materials Provided

Lesson 1.1

Video, Handout H1.1, and Video Support

Handouts H1.2–H1.4

Transparencies T1.1–T1.3

Assessment Problems A1.1, A1.2

Lesson 1.2

MINTOTAL.83P, MINTOTCL.XLS, MINTOTGC.XLS, MINTOTIF.XLS

Handouts H1.5–H1.13

Transparencies T1.4–T1.12

Assessment Problems A1.3–A1.6

Lesson 1.3

BIRD.83P, PIC1.83I –PIC6.83I

Transparencies T1.13–T1.20

Supplemental Activities S1.1 and S1.2

Assessment Problems A1.7–A1.9

Lesson 1.4

MINIMAX.83P ,MINMAXGC.XLS, MINMAXIF.XLS, MINTOTCL.XLS

Handouts H1.14–H1.18

Transparencies T1.21–T1.26

Assessment Problems A1.10–A1.12

Lesson 1.5

GRIDBASC.EXE, GRIDGSP.GSP, GRIDSS.XLS

Handouts H1.19–H1.33

Transparencies T1.27–T1.43

Assessment Problems A1.13–A1.15

Chapter Review

Supplemental Activity S1.3

Teaching Suggestions

This is a modeling unit. The lessons and activities may be sequenced in one of several possible ways. Three possible sequences are summarized below. If you choose another sequence other than “A” you will need to choose carefully the questions you assign from Individual Work.

Sequence A

Proceed through the activities as they appear in the text. Two criteria are introduced in Linear Village and then adapted to Gridville.

Activities 1, 2, 3, 4, 5, 6, 7, 8

Sequence B

Introduce one criterion, minimizing the total distance in Linear Village, and then extend the investigation of the one criterion to Gridville. Then introduce a second criterion, minimizing the maximum distance in Linear Village, and extend the investigation to Gridville.

Activities 1, 2, 3, 4, 6, 7, 5, 6, 8 (Activity 6 is repeated for each new criterion.)

Sequence C

Reverse the order in which the criteria are introduced in sequence B. This may be the most difficult sequence to adapt from the textbook sequence.

Activities 1, 5, 6, 8, 2, 3, 4, 6, 7 (Activity 6 is repeated for each new criterion.)

Throughout the chapter, encourage students who use different methods to explain their procedures. You can build productive discussions around how to choose the best or most efficient representation. The contextual problem of finding the best location in Gridville can be extended to the pedagogical problem of determining the best representation or procedure for solving the problem. Students need to see, over and over again, that multiple representations are possible and useful. For this reason, students practice and become skillful at using different representations in various contexts. The greater skill may be the skill involved in making the decision to use one representation rather than another and the ability to defend the decision.

The Activities and Individual Works follow a familiar pattern. In an Activity, students explore a concept or attempt to solve a problem with minimal guidance. The Individual Work that follows an Activity usually contains a step-by-step introduction of many of the same concepts and skills that students may generate on their own. Many questions are provided in Individual Work assignments so you may choose the questions most appropriate for your class, or for individuals within the class. There is no need to assign these questions to students who understand the concepts and the procedures as a result of doing the Activity. You may, however, wish to assign them for review or assessment rather than for developing a concept.

Several transparency masters and handouts are provided with this unit. You will need to use some of them; others are available for use if you decide to assist a class during an activity or check their understanding of a concept. Ideally, students will discover rules and procedures on their own by using multiple representations and observing patterns. Encourage students to share discoveries with one another.

Chapter 2

Major Contextual Theme

Decision Making

Major Mathematical Themes

Chance
Optimization

Related Disciplines

Economics
Business
Military Strategy
Sports

Scope and Sequence Chart

	Lesson 1	**Lesson 2**	**Lesson 3**	**Lesson 4**	**Lesson 5**	**Lesson 6**
Payoff Matrices	•	•	•	•	•	•
Average (mean)		•	•	•	•	
Expected payoff		•	•	•	•	•
Zero-sum games		•	•	•	•	
Fair games		•	•	•	•	
Simulations		•	•			•
Strictly-determined games			•	•	•	•
Value of a game			•	•	•	
Linear functions & models				•	•	
Systems of equations				•	•	
Probability distributions					•	
Tree diagrams					•	
Non-zero-sum games						•

Context Overview

Game theory, the topic of this chapter, unlike the topics of many *Mathematics: Modeling Our World* chapters, is both a context and a branch of mathematics. Its origins can be traced to the work of mathematician John von Neumann and economist Oscar Morganstern who, in 1944, published their analysis in *Theory of Games and Economic Behavior*. In the fifty years that followed, game theory found numerous applications in business, economics, political science, military strategy, and other areas. The 1994 Nobel Prize in economics was awarded for work in game theory.

Game theory studies situations in which two or more parties have conflicting or partially conflicting interests. Those interests can be quantified by some measure of utility (dollar value, for example) or by ranking. The scope of this chapter is limited primarily to situations involving two parties, commonly called two-person games. Both games in which the interests of the players are completely opposed (zero sum) and games in which the interests of the players are partially opposed (non-zero sum) are considered.

Mathematical Development

Since game theory is both a branch of mathematics and a context, the lines dividing the development of the context and the development of the mathematics are blurry, at best.

The chapter begins with a discussion of strategic situations and basic terminology. Matrices are introduced as an organizational tool to help students think about games of strategy. Once students are comfortable identifying key features of a game and organizing the features into a matrix, the emphasis switches to experimenting with strategies in a simple matching game. Students play several specific games using various strategies, then use calculator simulations to speed the gathering of data. These games provide the backdrop for much of the exploration throughout the chapter, as each contributes to the successive methods of analysis. Students see that some strategies are better than others, but some in particular are very desirable because they create a kind of stability in the game. These “optimal strategies” assure a player of a particular payoff, independent of the strategy pursued by the opponent. This payoff is known as the value of the game, and initially students determine experimentally the strategies that produce it.

The experimental approach changes to an algebraic one after the data produced by the simulations are shown to contain linear patterns. Further analysis shows that the equations of two members of a family of concurrent lines can be found directly from the game matrix, and the system of equations formed by this pair of equations can be solved to find the optimal strategy for one of the players as well as the value of the game. A similar system can be solved to find the other player’s optimal strategy.

A generalization of the algebraic approach to finding optimal strategies produces deeper insights into two-person, zero-sum games. The expected payoff is examined as a function of two variables: the strategy pursued by one player, and the strategy pursued by the other.

The chapter concludes with a consideration of non-zero-sum games. Because some of these games present dilemmas for which well-defined solutions do not exist, the approach again involves actual play of the games and calculator simulations.

Related Disciplines

The real-world contexts of this chapter provide many opportunities for students to apply what they are learning to other subject areas and to apply knowledge from other areas to the development of new mathematical ideas.

Military Strategy

Cuban Missile Crisis, Zairian Conflict.
Lessons 2.1, 2.3, 2.6

Economics

Investment choices.
Lessons 2.1, 2.3

Business

Competition (chain stores, gas stations).
Lessons 2.1, 2.3, 2.6

Sports

Pitching (baseball) and serving (tennis) strategies.
Lessons 2.1, 2.3, 2.5

Lesson 2.1: Decisions

2–3 days

The purpose of this lesson is to introduce key features of a game: the players, the strategies, and the payoffs. The lesson includes basic game theory terminology and an organizational tool, the payoff matrix. Students model various strategic situations using games. They discover that games are common and begin to develop an approach to analyzing them.

Lesson 2.2: Changing Your Strategy

4–5 days

In this lesson, much of students' attention is focused on strategies. A game's average payoff per play becomes an important measure of the success of various strategies. In addition, the concept of a fair game is introduced. Because the definition of a fair game is connected to players using wise strategies, fair games and optimal strategies are linked.

One major purpose of this lesson is to demonstrate, in a hands-on fashion, that some strategies are better than others. The path to this result is a series of three simulations. In the first simulation, students plan a strategy and then play the simple matching game 60 times. They assess the success of their strategies by comparing their average points per play with their partner's and then with other members of their group. During the second simulation, the column player must follow a script. Students discover that they can capitalize on opponents who use patterned strategies or certain random strategies. In Simulation 3, students learn that there is an optimal strategy—one on which an opponent cannot capitalize.

Lesson 2.3: Changing the Payoffs

2–3 days

The purpose of this lesson is to demonstrate that regardless of whether a game is fair or unfair, there may be an optimal strategy that prevents an opponent from capitalizing. Demonstration of this concept requires that many games be played, so calculator or computer simulations are used to speed the process. The results from simulation experiments are used to generate a table of data for many different row- and column-player strategy combinations. From these data students create tables of average points per play for several games and discover that some rows and columns in their tables are nearly constant. This indicates that the game is in a state of equilibrium; the average payoffs do not depend on the “other” strategy. The strategy that produces the state of equilibrium is the optimal strategy. The constant expected value of the payoff is called the value of the game. Students also encounter a strictly-determined game and learn that for these games a mixed strategy is not the best strategy.

Lesson 2.4: Optimal Strategies

2–3 days

The purpose of this lesson is to develop a procedure for finding the optimal strategy in a game in which a random mixture of the two options is appropriate. This is done by observing linear patterns in the data generated in Lesson 3. To visualize these patterns, students hold the column player’s strategy constant and use their data from Lesson 3 to graph the relationship between the expected payoffs and the row player’s strategy. This process is repeated for six different column player strategies. The result is a family of concurrent lines. The point of intersection of this family represents the row player’s optimal strategy and the value of the game. However, the results based on data are only approximations. Finding equations of two of the strategy lines directly from the payoff matrix leads to exact solutions for the optimal strategy and the value of the game without the aid of simulation data.

Lesson 2.5: Optimal Strategies Revisited

3–4 days

This lesson revisits the problem of finding optimal strategies in zero-sum and constant-sum games and the resulting equilibria from a probabilistic viewpoint. Students use tree diagrams to prepare distribution tables for the games’ payoffs. Then they determine an algebraic expression for the expected payoff in terms of the row and column player’s strategies. From here, students are able to find a general solution for the players’ optimal strategies and the value of the game. This results in new insights. For example, from the general solution students learn why mixed strategies in strictly-determined games are not necessary and determine what conditions are necessary for a game to be fair or for a 50-50 random strategy to be optimal.

Lesson 2.6: Games That Are Not Zero Sum

3–4 days

The purpose of this lesson is to consider games that are neither zero sum nor constant sum. These situations are characterized by interests that are not completely opposed. In the simplest of these situations, both players achieve the highest payoff by cooperating, but in others a better payoff can be obtained by defecting from a cooperative agreement. The most interesting of these situations pose a dilemma for the players: the best payoff is achieved by pursuing a course of action that would be disastrous if pursued by both players.

Students consider two versions of a game first introduced in Lesson 1. In one situation, the players both rank the cooperative option the highest. In the other, the players both rank the non-cooperative option highest. The activity shows that non-zero-sum games have different structures and introduces the concept of a dilemma. Throughout the lesson, students compare other strategic situations, including Prisoner's Dilemma and Chicken games, to the two situations in the game from Lesson 1.

Students return to the problem of finding optimal strategies for this new type of game. They brainstorm strategies and then test them first against other students and then against two calculator programs. For these games, however, optimal strategies remain the focus of current mathematical research.

Chapter Review

1–2 days

This summary provides exercises to review concepts taught in the chapter, a written summary of the mathematical concepts, and a chapter glossary.

Teacher Provided Materials

Coins
Envelopes (for student scripts)

Materials Provided

Lesson 2.1

Video, Handout H2.1, and Video Support

Supplemental Activity S2.1

Teacher Background Reading 2.1

Lesson 2.2

GAME1.83P

Handouts H2.2–H2.7

Transparencies T2.1–T2.5

Supplemental Activity S2.2

Assessment Problems A2.1 and A2.2

Lesson 2.3

GAME2.83P

Handouts H2.8–H2.11

Transparencies T2.6 and T2.7

Supplemental Activity S2.3

Lesson 2.4

Handouts H2.10–H2.13

Transparency T2.8

Supplemental Activities S2.4–S2.6

Assessment Problems A2.3 and A2.4

Lesson 2.5

Handouts H2.9–H2.12

Supplemental Activities S2.7 and S2.8

Assessment Problems A2.5–A2.7

Lesson 2.6

PD1.83P and PD2.83P

Handout H2.14

Chapter Review

Handout H2.15

Chapter 3

Major Contextual Theme

Relationships Among Discrete Objects

Major Mathematical Themes

Optimization
Algorithmic Thinking

Related Disciplines

Business/Management
Computer Science
Medicine/Education

Scope and Sequence Chart

	Lesson 1	**Lesson 2**	**Lesson 3**	**Lesson 4**	**Lesson 5**	**Lesson 6**
Elementary graph theory	•	•	•	•	•	•
Optimization	•	•	•	•	•	•
Algorithms		•	•	•	•	•
Minimum spanning tree			•			
Vertex coloring				•		
Traveling salesperson problem					•	
Stable matching						•

Context Overview

The contextual concern of this chapter is connections (or relationships) among discrete objects. The situations involve cities that are connected by airline flights or roads, events that are connected because they conflict, and people who must be paired and are connected by their preferences for one partner over another. Although there is considerable variety in the situations, they share a common structure that can be modeled with graphs in which the vertices represent the objects and the edges represent the connections among the objects.

The situations require optimal solutions to problems such as finding the most economical way to plan a trip, the fewest time slots needed for meetings or activities, and a stable way of matching partners. Solving these problems requires procedures (algorithms), and an important goal of the chapter is developing and critiquing these algorithms. Because the problems have widespread application in fields such as business and industry, this goal is particularly important.

Mathematical Development

The chapter begins with an introduction to the use of graphs to model a variety of real-world situations. Modeling diverse situations with graphs helps students to appreciate the common structure of the situations while also learning basic graph-theoretic terminology.

Since solving graph-theoretic problems requires algorithms, the introduction to graphs is followed by an introduction to writing and critiquing algorithms. To make the experience seem less foreign to students, many of the algorithms are related to procedures from previous chapters in the *Mathematics: Modeling Our World* program. An initial experience in this chapter with graph-theoretic algorithms involves minimum spanning trees.

The remainder of the chapter examines algorithms for solving specific graph-theoretic problems in this order: minimum spanning trees, vertex coloring, traveling salesperson, and stable matching.

In several lessons students are shown well-established algorithms such as Kruskal's and Gale-Shapley and asked to compare these algorithms to those the students have developed. The main goal is not for students to apply traditional algorithms blindly. Rather, the goal is an understanding of the algorithm design process, and a deep understanding of how and why an algorithm works. When an algorithm that always provides an optimal solution does not exist (i.e., coloring and traveling salesperson problems), students are told that the type of problem is a topic of current research.

Since the chapter does not develop around a single contextual problem, it is possible to change the sequencing of lessons to suit your needs. For example, students who have substantial previous experience with graphs as models can skip most of Lesson 1. Students who have previous experience writing and critiquing algorithms may be able to skip Lesson 2. The amount of time spent on any of Lessons 3–6 can be adjusted with little harm to the development of the other lessons. However, since new terminology is scattered throughout the lessons, you should examine any omitted material carefully.

Related Disciplines

The real-world contexts of this chapter provide many opportunities for students to apply what they are learning to other subject areas and to apply knowledge from other areas to the development of new mathematical ideas.

Business / Management

Allocating Tasks and Scheduling Events to Minimize Costs
Lessons 1, 4

Designing Networks to Minimize Costs
Lessons 1, 2, 3

Minimizing the Cost of a Trip (Traveling Salesperson Problem)
Lesson 5

Computer Science

Writing / Critiquing Algorithms
Lessons 2, 3, 4, 5

Medicine / Education

Matching medical students with hospitals
Lesson 6

Lesson 3.1: Connections

2–3 days

The purposes of this lesson are to introduce graphs as ways of representing a variety of situations and to give students practice with graph representations. An activity introduces graph models and basic terminology. Students apply graphs to a variety of situations in the individual work that follows the activity and several additional terms are introduced.

Lesson 3.2: Procedures

2–4 days

The purposes of this lesson are to establish the need for procedures (algorithms) to solve problems represented by graphs and to begin the development of algorithm-critiquing and algorithm-writing skills. An activity guides a first attempt at writing an algorithm for minimum spanning tree problems. In an individual work, students critique algorithms for solving problems from several previous chapters in this program.

Lesson 3.3: Minimum Spanning Tree Algorithms

2–4 days

The purpose of this lesson is to develop an algorithm for solving minimum spanning tree problems. Trees are introduced in the preparation reading, and students develop and refine an algorithm in an activity. An individual work provides practice using and critiquing minimum spanning tree algorithms.

Lesson 3.4: Coloring to Avoid Conflicts

4–6 days

The purpose of this lesson is to develop algorithms for solving vertex coloring (conflict resolution) problems and to determine the minimum number of colors by identifying common types of subgraphs. An activity uses graphs to model conflict-resolution situations and asks students to begin development of an algorithm. An individual work provides practice modeling conflict-resolution situations with graphs and introduces the strategy of identifying common subgraphs as a way of determining a minimum number of colors before applying an algorithm. Since no perfect algorithm for solving such problems exists, this strategy is an important one. A second activity asks students to complete the development of an algorithm, and a second individual work asks students to find counterexamples to show that various algorithms do not always produce an optimal solution.

Lesson 3.5: Traveling Salesperson Problems

2–4 days

The purpose of this lesson is to develop, critique, and apply algorithms for solving traveling-salesperson problems. An activity asks students to model a traveling salesperson problem with a graph and develop an algorithm for solving it. Again students are faced with a situation in which no perfect algorithm exists. The individual work that follows the activity asks students to critique algorithms. Since a common proposal for solving these problems is to list all possible solutions and pick the optimal one, consideration is given to the number of different solutions and the time it would take a computer to list them all.

Lesson 3.6: Matching

4–6 days

The purpose of this lesson is to develop a procedure for solving stable matching problems. An activity introduces matching problems, asks students to evaluate ways of representing these problems, and begins consideration of the meaning of stable matching. A definition of stable matching is developed in the individual work that follows the activity and students are asked to develop an algorithm for solving stable matching problems.

Chapter Review

1–2 days

The summary provides a set of review problems, a summary of the mathematical concepts in the chapter, and a glossary.

Teacher Provided Materials

D-Stix
Geometric drawing utility
Blank transparencies and transparency pens
Colored pencils
Large pieces of paper (newsprint or butcher paper)

Materials Provided

Lesson 3.1

Video, Handout H3.1, and Video Support

Assessment Problems A3.1–A3.4

Lesson 3.2

Assessment Problems A3.5 and A3.6

Lesson 3.3

Transparencies T3.1–T3.3

Assessment Problems A3.7–A3.9

Lesson 3.4

Handouts H3.2–H3.4

Assessment Problems A3.10–A3.13

Lesson 3.5

Transparency T3.4

Assessment Problems A3.14 and A3.15

Lesson 3.6

Handouts H3.5–H3.9

Transparency T3.5

Supplemental Activity S3.1

Assessment Problem A3.16

Chapter Review

Handout H3.10

Chapter 4

Major Contextual Theme
Efficient Packaging

Major Mathematical Themes
Optimization
Space

Related Disciplines
Environmental Science
Economics and Consumer Science

Scope and Sequence Chart

	Lesson 1	Lesson 2	Lesson 3	Lesson 4	Lesson 5
Measurementbt	•	•	•	•	•
Area	•	•	•	•	•
Volume		•			
Perimeter		•			
Similarity		•	•	•	•
Surface area of solids		•	•	•	•
Proof		•	•	•	•
Symmetry			•	•	•
Tangents			•	•	•
Congruence			•	•	•
Polygons			•	•	•
Right triangles			•	•	•
Equilateral triangles			•	•	•
Circumference				•	•
30°–60° right triangle relationships			•		
Deductive reasoning				•	•
Inductive reasoning				•	
Parallel lines				•	
Alternate interior angles				•	
Corresponding angles				•	
Vertical angles				•	
Diagonals of polygons				•	
Pythagorean formula				•	•
Aspect ratio					•
Quadratic equations					•

Context Overview

The primary contextual concern of this chapter is efficient packaging. This broad concern is initially focused on the secondary packaging of soft drinks. (Primary packaging is the cans; secondary packaging is the material that holds several cans together.) Students examine ways to package soft drink cans with the goal of optimizing either the use of package space or packaging material. After examination of the soft drink packaging problem, students turn their attention to the problem of designing a package to hold melons of more than one size.

Mathematical Development

Mathematical modeling is a theme that runs throughout this course, and the initial mathematical concern of this chapter is the modeling process. The problem of designing efficient packaging for soft drink cans is posed, and a search for efficiency criteria and factors that affect the criteria begins.

Two important efficiency criteria emerge: the percentage of space used by cans in the package and the amount of package material per can. Factors that affect one or both of these criteria include the number of cans in the package, the shape of the package, and the arrangement of cans in the package.

Analysis of the efficiency of package designs requires application of area and volume formulas. Students initially analyze designs by making direct measurements from drawings and models and applying basic area formulas they have used in previous courses. They also consider whether simplification of the problems to cross-sectional (two-dimensional) counterparts affects their conclusions.

Calculations made from direct measurements are imprecise, which motivates students to consider ways to improve precision. Technology in the form of an electronic drawing utility gives excellent precision if drawings are made accurately. However, some aspects of drawings of package designs, such as tangencies of circles and package boundaries, can be difficult to achieve by visual means alone. Simple transformation techniques based on the symmetry of package designs are used to achieve accurate drawings.

Another way to achieve precision is to obtain exact answers through the application of geometric knowledge. Deductive reasoning (proof) is used to establish several important geometric facts that are then applied to the calculation of package efficiency. Among these facts are the Pythagorean formula, the sum of the measures of the angles of a triangle and other polygons, and 30°-60° right triangle relationships. Properties of similar triangles are also used in conjunction with (or instead of) the Pythagorean formula for some calculations.

Once the analysis of soft drink package designs reaches a fairly advanced state, the contextual concern turns to the problem of designing an efficient carton for melons of different sizes. Analysis begins with consideration of a carton that holds just four melons. Determining the size of the largest melon so that four such melons fit a carton of given size requires solving a quadratic equation. An investigation of the relationship between efficiency and aspect ratio (of the carton) yields data that appear quadratic, but are better modeled by a piecewise algebraic function. As a chapter project, students are challenged to design a carton that holds four large melons or six smaller ones. Since the optimal carton for four melons is not optimal for six, the project is open to a variety of approaches.

Related Disciplines

The real-world contexts of this chapter provide many opportunities for students to apply what they are learning to other subject areas and to apply knowledge from other areas to the development of new mathematical ideas.

Environmental Science

Designing packages to minimize waste in landfills.
Lessons 4.2–4.4

Economics and Consumer Science

Designing packages to use storage space efficiently.
Lessons 4.2–4.5

Lesson 4.1: Packaging Models

3–5 days

The purposes of this lesson are to introduce the context of packaging, to focus the context on the secondary packaging of soft drinks, and to develop a definition of efficient soft drink packaging. Students develop a list of people affected by soft drink packaging and their concerns. From this list they develop one or more definitions of efficient packaging, and they explore factors that might affect the efficiency. Two definitions are prominent in the lessons that follow: percentage of package space used by the cans and amount of package material used per can. The optimization problem that results is one of maximization in the former case and one of minimization in the latter case.

Lesson 4.2: Designing a Package

2–4 days

The purposes of this lesson are to design a new soft drink package and to apply basic area formulas to the problem of calculating efficiency. Students make linear measurements directly from drawings and models. The area calculations that result suffer from lack of precision, which sets the stage for consideration of ways to improve precision.

Lesson 4.3: Technological Solutions

4–6 days

The purpose of this lesson is to introduce geometric drawing utilities and to apply them to the problem of improving the precision of efficiency calculations. After a basic introduction to the utility, students consider the symmetry of soft drink packages and apply related transformational techniques to construct accurate electronic sketches of package designs. Measurements provided by the drawing utility are used to calculate efficiency more precisely.

Lesson 4.4: Getting the Facts

4–6 days

The purpose of this lesson is to apply deductive reasoning to the problem of calculating efficiencies. The deductive process resolves the problem of improving the precision of calculations because it produces exact answers. An initial activity discusses deductive reasoning, contrasts it with inductive reasoning, and gives a proof that the sum of the measures of the angles of a triangle is 180°. Other results that are established include the Pythagorean formula and the fact that a tangent and radius drawn to the point of tangency are perpendicular. Examination of triangular and hexagonal package designs applies and extends geometric results developed in the lesson. At the end of the lesson, students are asked to prepare a report recommending a soft drink package design.

Lesson 4.5: Packaging Spheres

5–7 days

In this lesson, the contextual concern changes to the problem of designing an efficient carton for melons of more than one size. An examination of four-melon packages involves solution of a quadratic equation to determine the largest melon that will fit in a carton of given dimensions. The examination also shows that the carton's aspect ratio determines efficiency and produces data for the relationship between efficiency and aspect ratio that appear quadratic. A closer examination shows that the relationship is not quadratic. After examination of the four-melon situation, students undertake an examination of a similar six-melon situation. As a chapter project, students prepare a report recommending a carton to hold either four melons or six melons.

Chapter Review

1–2 days

This summary provides exercises to review concepts taught in the chapter, a written summary of the mathematical concepts, and a chapter glossary.

Teacher Provided Materials

Geometric drawing utility (e.g., *Geometer's Sketchpad, Cabri*)

Card stock or heavy paper

Coins: pennies, nickels, dimes, and quarters

Graph paper or dot paper

Plexiglas® mirrors

Rulers

Scissors

Six soda cans

Small blocks such as pattern blocks (or copies of Handout H4.2 printed on card stock)

Tape

Materials Provided

Lesson 4.1

Video, Handout H4.1, and Video Support

Handout H4.2

Assessment Problem A4.1

Lesson 4.2

Handouts H4.3 and H4.4

Transparencies T4.1–T4.3

Assessment Problems A4.2–A4.7

Lesson 4.3

RECT6PK.GSP (rectangular 6-pack)
SQUAR9PK.GSP (square 9-pack)
TRIAN3PK.GSP (triangular 3-pack)
CYL7PK.GSP (cylindrical 7-pack)
HEX7PK.GSP (hexagonal 7-pack)

Handout H4.5

Transparencies T4.4–T4.10

Lesson 4.4

SIMDEMO.GSP (similarity demo)
RDTNDEMO.GSP (radius & tangent demo)
PYTHDEMO.GSP (Pythagoras demonstrator)
TRIAN6PK.GSP (triangular 6-pack)
MODTR6PK.GSP (mod triangular 6-pack)
HEX7PK.GSP (hexagonal 7-pack)
TRIANGCS.GSP (triangular case)
MODTRCS.GSP (mod triangular case)

Transparencies T4.11–T4.16

Supplemental Activities S4.1 and S4.2

Assessment Problems A4.8–A4.13

Lesson 4.5

MLN4PKR.GSP (four melon packer)
MLN5PKR.GSP (five melon packer)
MLN6PKR.GSP (six melon packer)
MLN8PKR.GSP (eight melon packer)
LSQDEMO.GSP (least squares demo)
LSQDEMSQ.GSP (least squares demo with squares)

Supplemental Activities S4.3 and S4.4

Assessment Problem A4.14

Chapter 5

Major Contextual Theme

Proximity Relationships

Major Mathematical Theme

Optimization

Related Disciplines

Archeology

Environmental Science / Meteorology

Robotics

Scope and Sequence Chart

	Lesson 1	Lesson 2	Lesson 3	Lesson 4	Lesson 5
Simple arithmetic mean	•				
Voronoi diagram	•	•	•	•	•
Area	•		•	•	
Volume	•		•		
Weighted average	•		•		
Algorithm		•		•	
Geometric construction		•			
Polygons		•	•	•	
Perpendicular bisector		•	•	•	
Heron’s formula			•		
Pick’s formula			•		
Coordinate geometry				•	•
Systems of equations				•	
Reflection					•
Iteration					•

Context Overview

The central contextual problem with which this chapter opens is that of estimating rainfall in the state of Colorado from rainfall measurements made at eight gauges at various locations in the state. The model that is developed constructs a polygon around each of the gauges so that every point in a gauge’s polygon is closer to that gauge than to any other gauge. The rainfall at a gauge is weighted according to the area of its region. The weighted-average model is applied to several other situations including estimation of the volume of water in a lake, drawing school attendance boundary lines, and choosing a location for a restaurant. These situations all involve drawing boundary lines for a given set of centers. The chapter also examines situations in which centers must be found from a set of boundaries. One of these situations is locating archeological dig sites.

Mathematical Development

A Voronoi diagram divides a region with various centers of influence into smaller regions around each center of influence. Every point in a given region is closer to the center of influence of that region than to any other center of influence.

The mathematical modeling in this chapter begins by considering how one might estimate rainfall in the state of Colorado from measurements made at eight gauges. Students propose a first model that uses a simple arithmetic mean, and the model is rejected because of the non-uniform distribution of the gauges. A weighted-average model is developed in which the areas of Voronoi regions around the eight gauges are weights.

Students explore a variety of methods for constructing Voronoi regions. Tools used include compasses, Plexiglas® mirrors, paper folding, and a geometric drawing utility. Coordinates are introduced as a convenient way to transfer essential information to the drawing utility.

After construction techniques are developed, the focus shifts to ways to measure areas of Voronoi regions. Again, a variety of techniques are considered, including Pick's formula, subdivision of the polygonal regions into triangles and application of Heron's formula, simulation, and measurement by a drawing utility.

A coordinate geometry approach to the boundary and area problems produces equations of boundary lines and coordinates of regions' vertices. Polygons are divided into triangles, the lengths of the sides of the triangles are found by application of the distance formula, and Heron's formula is used to find areas. Coordinate techniques are used to develop calculator programs for these tasks.

The chapter ends with an examination of the opposite problem--that of locating Voronoi centers if the boundaries are known. An iterative approach is developed in which a center is guessed and reflected through boundaries and back to its own region. Introduction of a rectangular coordinate system enables students to write a calculator program to implement the iterative solution. The problem is also examined deductively and two other solutions are developed.

Related Disciplines

Archeology

Locating dig sites.

Lesson 5

Environmental Science / Meteorology

Estimating rainfall in a region.

Lessons 1-4

Robotics

Determining a path to avoid obstacles.

Lessons 2, 4

Lesson 5.1: Colorado Needs Rain!

3-5 days

The purposes of this lesson are to develop a first model for estimating the rainfall in a region, to show the limitations of the model, and to improve the model by incorporating weighted averages. The weighted average model raises questions about how boundaries of regions can be established and how areas can be measured.

Lesson 5.2: Neighborhoods

4-6 days

The purpose of this lesson is to develop methods of constructing boundaries of regions. Students use compasses, Plexiglas mirrors, and paper folding to investigate a simple diagram with two centers and a single boundary. They then use their discoveries to propose methods for constructing boundaries. Drawing utilities are also introduced for those students who have access to them. An algorithmic approach to situations with several centers is encouraged.

Lesson 5.3: Rainfall

2-4 days

The purposes of this lesson are to establish boundaries for eight Colorado rain gauges, to develop methods for finding the area of polygonal regions, and to complete the weighted-average model's estimate of Colorado's rainfall. Students establish the Colorado boundaries with a method of their choice. Heron's formula, Pick's formula, and simulation are discussed as methods of calculating area. Students select a method to determine areas of the Colorado regions and use the results to complete the weighted-average estimate of Colorado's rainfall.

Lesson 5.4: A Method of a Different Color

4-6 days

The purposes of this lesson are to develop coordinate methods for determining boundaries, vertices, and areas, and to apply this approach to the development of calculator programs that perform the method's routine tasks. Students develop coordinate methods that find the midpoint of a segment and the equation of the segment's perpendicular bisector, find the point of intersection of two boundary lines, and determine the distance between two vertices. Calculator programs that are written include one to find the midpoint and the perpendicular bisector and one to determine the area of a triangle from the lengths of its sides or from the coordinates of its vertices.

Lesson 5.5: Digging for Answers

2-4 days

The purpose of this lesson is to develop methods of locating centers of regions when the boundaries are known. Students experiment with compasses, Plexiglas mirrors, and paper folding to check whether a guessed center is in the correct location. If it is not, an improved guess is made and the process repeated. A coordinate geometry version of the iterative approach allows it to be implemented on a calculator. The lesson also includes a deductive investigation of the problem that leads to two additional solutions.

Chapter Review

1-2 days

This summary provides exercises to review concepts taught in the chapter, a written summary of the mathematical concepts, and a chapter glossary.

Teacher Provided Materials

Compasses

Geometric drawing utility (e.g., Geometer's Sketchpad)

Paper (dot or graph)

Plexiglas® mirrors

Rain gauge
Rice (one-pound bag)
Rulers

Materials Provided

Lesson 5.1

Video, Handout H5.1, and Video Support
Transparencies T5.1-T5.9

Lesson 5.2

Handout H5.2
Transparencies T5.10-T5.16
Assessment Problems A5.1-A5.4

Lesson 5.3

COLORADO.83p
LAKE.GSP
Handouts H5.3 and H5.4
Transparency T5.17
Assessment Problems A5.5-A5.7

Lesson 5.4

PERP.83p
HERON.83p
POLYAREA.83p
Handouts H5.5–H5.7
Assessment Problem A5.8

Lesson 5.5

VITERATE.83p
Handouts H5.8 and H5.9
Transparency T5.18
Assessment Problem A5.9

Chapter Review

Handout H5.10
Supplemental Activity S5.1

Chapter 6: Growth

Contextual Theme

Growth and Decay

Major Mathematical Theme

Sequences and Series

Related Disciplines

Environmental Science
Family and Consumer Science
Medicine
Population Biology

Scope and Sequence Chart

	Lesson 1	Lesson 2	Lesson 3	Lesson 4	Lesson 5
Additive growth	•	•			
Multiplicative growth	•	•			
Linear equations	•	•			
Exponential equations	•	•	•		
Recursive equations	•	•	•	•	•
First differences	•	•		•	•
Relative rate of growth	•	•		•	•
Growth factor	•	•		•	•
Arithmetic sequences		•			
Geometric sequences		•	•		•
Recursive graphs		•		•	•
Time-series graphs		•	•		•
Web diagrams		•	•		•
Properties of exponents		•	•		
Inverse of a function			•		
Logarithms			•		
Series				•	
Quadratic functions				•	
Second differences				•	

Mixed growth					•
Equilibrium					•

Context Overview

People make plans, adjust policies, and establish regulations based on the expectation that a pattern of growth will continue as it has in the past or as it has in a similar situation. Rangers replenish the fish in lakes, communities pass laws to regulate the number of homes built, doctors determine the proper dose of a medicine, and waste management engineers monitor the space available in existing landfills.

Multiple contexts are introduced throughout the chapter to facilitate the study of patterns of growth and challenge students to match known models to unknown collections of data. The contexts for growth are diverse: the accumulation of money in a savings account, the concentration of a drug in the body, the increase or decrease in an animal population, the spread of a disease, the consumption of natural resources, employment contracts, and the buildup of trash in a landfill. Students identify patterns in one context and apply the patterns to other contexts.

Mathematical Development

This chapter continues the study of concepts introduced in Course 1, Chapter 5, *Wildlife. Wildlife* introduces the assumptions behind additive and multiplicative growth models and followed with a development of closed-form and recursive equations. Growth is an extensive study of patterns revealed by recursive graphs and patterns associated with differences and ratios of consecutive terms in sequences.

Throughout the chapter, students examine situations involving the growth or decay of quantities. They analyze the situations for characteristics of familiar growth models. If a familiar model "fits" a new situation, then the model is fine-tuned. If a familiar model cannot be found, then a new model is proposed and developed. Then characteristics of new models are identified for future reference.

The chapter opens with a pharmacological problem and a challenge for students to apply concepts and skills developed in previous chapters. Its solution involves a mixed-growth model; students revisit it throughout the chapter and complete its solution in Lesson 5.

The first sequences studied are arithmetic and geometric sequences. Students learn to recognize additive and multiplicative growth by comparing consecutive terms in a sequence and by identifying linear patterns in recursive graphs. Once a model is matched to a situation, students practice writing and solving equations. Logarithms are introduced to provide a symbolic method for solving the exponential equations related to geometric sequences.

Next, sequences of partial sums, or series, are studied. This new growth model, created by summing the terms of sequences, is examined to determine which characteristics it shares with multiplicative and additive growth. Students discover that a series created by summing the terms of an arithmetic sequence is quadratic. Characteristics of quadratic functions are studied in the context of growth so students can identify similar patterns when presented with data that approximate quadratic growth.

Then students study sequences that combine or “mix” multiplicative and additive growth. They identify similarities and differences between a sequence formed by a mixed-growth model and sequences that are strictly arithmetic or geometric. Recursive graphs provide clues for identifying mixed-growth characteristics in unknown situations. Study of mixed sequences leads to characterization of geometric series.

Related Disciplines

The real-world contexts of this chapter provide many opportunities for students to apply what they are learning to other subject areas and to apply knowledge from other areas to the development of new mathematical ideas.

Environmental Science

Examining limits to new development.

Lesson 2

Modeling the accumulation of trash in a landfill.

Lesson 4

Family and Consumer Science

Comparing contracts with various options for raises.

Lesson 4

Modeling the accumulation of money in a savings account.

Lesson 5

Medicine

Modeling drug concentrations in the blood.

Lessons 1,2,4,5

Modeling the spread of a disease.

Lesson 2

Determining the time of death for a corpse.

Lesson 5

Population Biology

Writing equations for unrestricted growth of mice in a field; modeling the decline in the population of winter-run Chinook salmon in California.

Lesson 2

Modeling the growth of a guppy population in a pond.

Lesson 3

Modeling the population of trout in a pond.

Lesson 4

Lesson 6.1: Growing Concerns

2–3 days

The purposes of this lesson are to connect the study of growth in Course 1, Chapter 5, *Wildlife*, with the study of growth in this chapter and to introduce the central problem, which is to determine the proper drug dose for a patient.

Lesson 6.2: Double Trouble

5–7 days

The purpose of this lesson is to develop a checklist of characteristics that may be used to identify additive and multiplicative growth situations. Students examine the relationship between consecutive terms for linear and exponential models. They prepare recursive graphs using known functions so they know what to look for in unknown or approximate situations.

Lesson 6.3: Finding Time

2–3 days

A primary purpose of this lesson is to introduce logarithms as a symbolic method for solving exponential equations. A review of inverses serves as an introduction to the logarithm function.

Lesson 6.4: Sum Kind of Growth

2–3 days

The purpose of this lesson is to identify the characteristics of sequences of partial sums. Students discover that the sequence of partial sums for an arithmetic sequence is quadratic. Further study reveals that constant second differences are characteristic of quadratic sequences.

Lesson 6.5: Mixed Growth

2–3 days

The purpose of this lesson is to identify the characteristics of mixed-growth sequences. Students discover that recursive graphs may be used to identify arithmetic, geometric, and mixed sequences.

Chapter Review

1–2 days

This summary provides exercises to review concepts taught in the chapter, a written explanation of the mathematical concepts, and a chapter glossary.

Teacher Provided Materials

Graphing calculators with a LIST feature and SEQUENCE mode capabilities.

Spreadsheet software, including a graphing utility.

Materials Provided

Lesson 6.1

DOSEEVTH.XLS (dose every twelve hours)

Video, Handout H6.1, and Video Support

Handouts H6.2–H6.3

Assessment Problem A6.1

Teacher Background Reading 6.1

Lesson 6.2

SEQFTEST.XLS (sequence form tester)

Handout H6.4

Transparency T6.1

Supplemental Activities S6.1 and S6.2

Assessment Problems A6.2–A6.4

Lesson 6.3

Assessment Problems A6.5–A6.7

Lesson 6.4

Assessment Problems A6.8–A6.16

Lesson 6.5

Handouts H6.5 and H6.6

Transparency T6.2

Supplemental Activities S6.3 and S6.4

Assessment Problems A6.17–A6.22

Chapter Review

Handout H6.7

Chapter 7: Motion

Major Contextual Theme

Modeling Motion of Objects

Major Mathematical Themes

Linear and Quadratic Functions
Rate of Change

Related Disciplines

Physics
Stunt Design

Scope and Sequence Chart

	Lesson 1	Lesson 2	Lesson 3	Lesson 4	Lesson 5
Rate of change	•	•	•	•	•
Slope	•	•			
Linear equations	•				
Data analysis & residuals	•	•	•	•	
Piecewise equations	•			•	
Quadratic equations		•	•	•	
Linear transformations		•	•	•	
Instantaneous slope		•	•	•	
Symmetric difference quotient		•	•	•	
Systems of equations				•	
Parametric equations					•

Context Overview

The study of motion serves as the context for the investigations in this chapter. In particular, this study is directed toward understanding some of the mathematics involved in planning successful car and motorcycle stunt jumps. Students model four different stunts during the chapter: two near-collision stunts, one intentional collision, and a ramp-to-ramp jump.

In this chapter, students use motion detectors to collect distance-versus-time data on moving objects. Most of the work in the chapter deals with the relationships between location and time for objects in motion along a straight line, in either the horizontal or vertical direction. The chapter concludes with a look at motion in a plane.

Students begin by studying their own motion as they walk in a straight line in front of a motion detector. Then they study the motion of battery-operated toy cars. After completing their study of motion along a line in the horizontal direction, students focus on the effect of gravity (as well as air resistance) on falling objects. In the final lesson, students extend what they have learned about motion in a single direction (horizontal or vertical) to motion in a plane.

The chapter is structured so that each lesson ends with the completion of one main stunt design. Thus there are several natural stopping places if time cuts the chapter short.

Mathematical Development

Students being the chapter with an introduction to the measuring equipment they will be using. Informal explorations of student motion lead to a review of velocity as rate of change and its representation as the slope of a line. Linear distance-versus-time graphs are quickly followed by graphs that curve. The notion of “local linearity”—that a curve looks like a line if you zoom in enough—is used as the basis for defining instantaneous velocity. The concept that a linear distance-versus-time graph means a constant velocity is extended to the concept that a quadratic distance-versus-time graph means constant acceleration.

The fact that the slope of a parabola changes at a constant rate underlies one method of identifying data as quadratic. However, because of variability in real data, perfectly quadratic data are rare. Therefore, quadratic regression is used to fit models to data that appear quadratic. The importance of using residuals as a determining factor in checking the fit is stressed. One quick and dirty method for finding a quadratic equation for data uses the solution of a 3 x 3 system of linear equations. This method proves useful when the motion detector picks up numerous stray readings that make regression ineffective.

Because real data are being collected and studied, the piecewise nature of real data becomes an issue early in the chapter.

One other major concept in the chapter falls into the category of physics. In order to predict the nature of the equations of motion of real objects, students come to understand that acceleration is caused by force. They also learn that if an object’s acceleration is not zero, then its velocity is non-constant, and if an object’s velocity is not zero, then its position changes.

The chapter closes with a study of motion in the *xy*-plane and uses parametric equations to model the motion.

Software note

Programs needed for a motion detector and TI-CBL are included with this course. Similar (updated) programs also can be downloaded from the Internet.

Related Disciplines

The real-world contexts of this chapter provide many opportunities for students to apply what they are learning to other subject areas and to apply knowledge from other areas to the development of new mathematical ideas.

Physics

Motion at a constant velocity; the effect of gravity and air resistance on falling objects; non-constant velocity and acceleration; horizontal and vertical components of motion in a plane.

Lessons 1–5

Stunt Design

Near-collision stunts, falls, and ramp-to-ramp jumpts.

Lessons 1–5

Lesson 7.1: Learning Your Lines

4-7 days

This lesson serves as both an introduction to motion detector equipment and to the study of motion. The lesson is built around activities in which students study the distance-versus-time graphs that result as they move within the field of a motion detector.

Students produce both linear and nonlinear graphs and use residual plots to judge the results of fitting linear equations. The concept of velocity as a rate of change emerges, and students review the relationship between velocity and the slope of the distance-versus-time graph.

In the final activity of this lesson, students apply what they have learned to the design of a near-collision stunt and test their design using toy cars or a calculator simulation program.

Most of the concepts studied in this lesson have been previously introduced in Course 1, Chapters 3 and 4, Prediction and Animation.

Lesson 7.2: Falling in Line

2-4 days

Students use motion detectors to study the motion of falling objects and discover that this type of motion can be modeled using quadratic equations. Students use quadratic regression to find equations of parabolic “pieces” of piecewise-defined, distance-versus-time curves and then apply transformations to shift the pieces to a convenient start time. They continue to use residuals as the primary tool for evaluating how well a particular equation describes given data.

Students learn that the parabolic distance-versus-time graphs produced by falling objects indicate that an object's velocity changes during its fall. This leads to a study of the meaning of "slope" for a curve and the interpretation of "slope" as instantaneous velocity. Students approximate instantaneous velocity using a zoom-in approach and a symmetric-difference-quotients approach. (In Lesson 7.4 they discover the exactness of the symmetric-difference-quotient approach for parabolas.)

Lesson 7.3: It Feels Like Fall

2-3 days

A definition of acceleration results from the effort to quantify the changing velocity of motion that has a parabolic graph. Students learn that constant acceleration characterizes parabolic motion.

Students apply what they have learned from this lesson to the design of a fall-from-a-building-into-the-back-of-a-pickup stunt.

Lesson 7.4: What Goes Up Must Come Down

4-8 days

In Lesson 7.4, students continue their study of the motion of falling objects. However, here the objects are first thrown vertically upward and then allowed to drop. Students discover that the distance-versus-time graphs produced by this type of motion are still parabolic.

Because the data collected from tossing a ball may have numerous outliers, students learn a quick and dirty method for fitting quadratic equations that involves solving systems of equations. Once students have determined a quadratic model for the motion, they interpret the constants in their model in the context of throwing a ball.

In the concluding activity for this lesson, students apply what they have learned in Lessons 2–4 to analyze Jeff Lattimore's stunt "The Leap for Life." In this stunt, Jeff leaps from his eight-foot stool to avoid injury when an oncoming car crashes into it. Students determine whether or not Jeff could still execute his stunt safely if he jumped from a shorter stool.

Lesson 7.5: The Grand Finale

2-4 days

In Lesson 7.5, students model the motion of an object in the *xy*-plane using parametric equations. Previous to this lesson, all work in the chapter has involved motion in one dimension only. However, students find that they can combine the constant-velocity models of Lesson 1 with the non-zero, constant-acceleration models in Lessons 7.2 and 7.3 to produce parametric equations describing a car's motion as it leaves a ramp.

This lesson consists of one activity in two lab segments. The second lab, setting up a safe landing area for a stunt jumper, is the culminating test of the mathematics of the entire chapter.

Chapter Review

1 day

The summary provides exercises to review concepts taught in the chapter, a written explanation of the mathematical concepts, and a chapter glossary.

Teacher Provided Materials

Basketballs (or other balls of similar size)
Battery-operated toy cars
Books (for dropping)
Calipers
Carbon paper (1 piece)
Catch ramp (coffee can or landing ramp)
Large beach ball or balloon
Large sheets of paper
Masking tape
Meter sticks (2)
Motion detector equipment
Photogate equipment
Protective frame
Shims
String and screw eye
Stunt vehicle (toy car, ball)
Take-off ramp (homemade or commercially made)

Materials Provided

Lesson 7.1

Video, Handout H7.1, and Video Support

HIKER.83P or CBL1.83P
Handouts H7.2–H7.4
Transparency T7.1
Supplemental Activities S7.1–S7.3
Assessment Problems A7.1–A7.2

Lesson 7.2

BALLDROP.83P or CBL2.83P
EDITPART.83P
Handout H7.5
Transparencies T7.2–T7.4
Supplemental Activity S7.4
Assessment Problem A7.3

Lesson 7.3

BALLDROP.83P or CBL2.83P
Handout H7.6
Transparency T7.5
Supplemental Activities S7.5 and S7.6
Assessment Problems A7.4 and A7.5

Lesson 7.4

BALLDROP.83P
BASKBALL.83P
Supplemental Activities S7.7–S7.10
Assessment Problems A7.6–A7.9

Lesson 7.5

BALLDROP.83P
GATE.83P or TIMER.83P (or equivalent)
Handout H7.7
Supplemental Activities S7.11–S7.13
Assessment Problems A7.10–A7.12