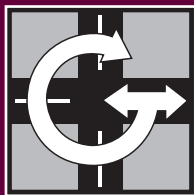


CHAPTER

1



## **Gridville**

LESSON ONE

### **In Case of Fire**

LESSON TWO

### **Linear Village**

LESSON THREE

### **Absolute Value**

LESSON FOUR

### **Minimax Village**

LESSON FIVE

### **Return to Gridville**

### **Chapter 1 Review**





## WHAT IS THE BEST LOCATION?

**L**ocating a fire station is an important decision for a town. Towns must also decide where to put other services, such as a police station, hospital, or school. Any location benefits some people more than others. A town must decide which factors are most important.

In this chapter, you make decisions about finding a best location and give reasons for your choices. When your reasons or criteria can be measured, mathematics can help you find a best location. There may be more than one best location depending on the criteria you use and how you decide what is fair.

You will use mathematical modeling to simplify a problem and discover methods for finding locations that satisfy criteria such as maximizing or minimizing quantities. Many people make important decisions with the goal of minimizing or maximizing something.

The primary focus of this chapter is on measuring and representing distance in order to minimize time and cost. You investigate the geometry of distance on a line and in a plane using absolute value functions. Through tables and graphs, you observe patterns that result from adding functions. You discover the usefulness and the limitations of extending concepts and strategies from one dimension to two.

Gridville is a good place to blend the mathematical tools of measurement, multiple representations, and the modeling process together with a town's values.



## LESSON ONE

# In Case of Fire

## Key Concepts

Distance in  
firetruck geometry

Circles in firetruck geometry

Total and average distance  
in two dimensions

## PREPARATION READING

## A Model Village

The focus of this lesson is on deciding what is meant by “best” as you search for a location for a fire station. You decide which criteria are most important, then use mathematics to find a location that meets the criteria.

It is not easy to find a best location for something in a town with many streets, traffic patterns, and buildings. Also, all towns are different. You may develop a method for finding a best location in your town or in the one in **Figure 1.1**, but a good method should be useful in other towns.



**Figure 1.1**  
Aerial view of town.

Start by designing a simple model—one that has the key elements of a typical town. Use your model town to develop strategies for finding the best location for a fire station.

At the end of this chapter, you will return to the town in **Figure 1.1**, or maybe your own town, and choose the best location for a fire station based on local values and the mathematics you learn in Gridville.

### DISCUSSION/REFLECTION

1. What are important criteria when deciding a place for a fire station?
2. Discuss the essential features of a town. How would you arrange them to make the simplest model for a town? In particular, what features of the town in **Figure 1.1** would you simplify, how would you simplify them, and what features would you keep?
3. Describe how you would find the best location for a fire station in a model town.

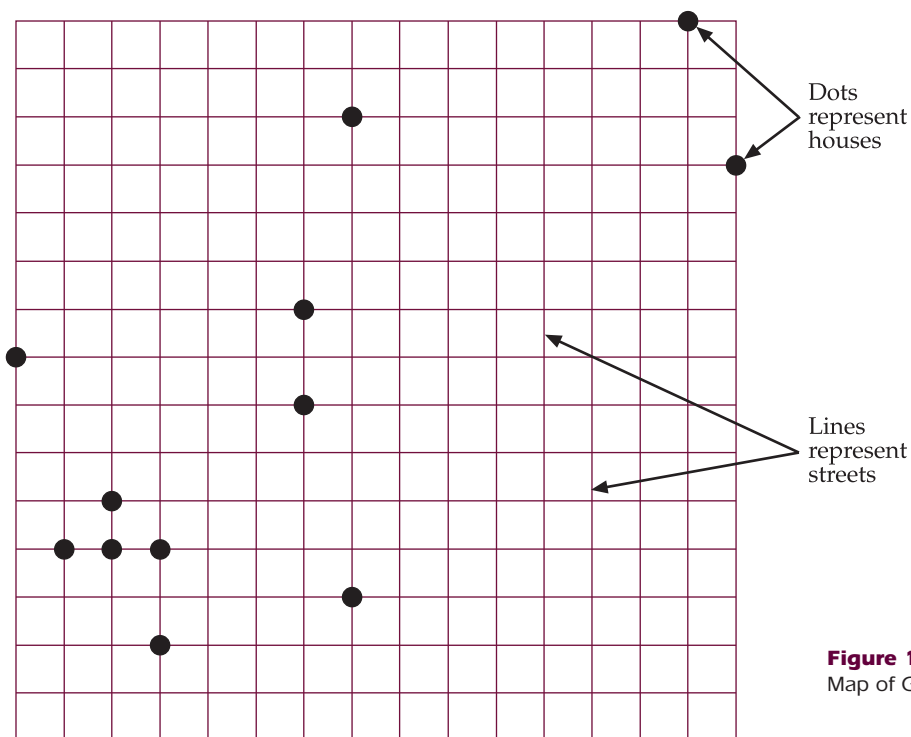
### Activity 1.1: The New Fire Station

*In this activity, you begin the modeling process by examining a simple town: Gridville.*

- Gridville has streets running parallel and perpendicular to one another.
- The streets of Gridville are two-way streets represented by lines of the grid.
- Houses are represented by points and are located at the intersections of grid lines.
- Fire trucks travel only on grid lines. Diagonal movement is not allowed.

#### The Task:

The map of Gridville in **Figure 1.2** shows all houses. Decide the best location for a fire station. Write an argument defending your choice.



**Figure 1.2**  
Map of Gridville.

**Guidelines:**

- Begin your argument by answering the questions, “What are important factors to consider in deciding a best location?” and “What does ‘best’ mean?”
- You are encouraged to use charts, diagrams, tables, graphs, equations, calculations, and logical reasoning.
- Clearly state your choice of best location. Your written summary should include arguments and mathematics that support your decision. The summary should also explain how your charts, diagrams, tables, graphs, equations, calculations, and logical reasoning relate to the factors you considered and led to your decision.
- Be prepared to give an oral presentation of about two minutes. You should summarize your arguments and explain the reasons for your decision.

**DISCUSSION/REFLECTION**

1. Which criteria were used most often in your class?
2. Which factors invite mathematical investigation?
3. Even Gridville can be simplified in order to study key elements of the problem. What are some ways you can simplify Gridville to investigate distance relationships?

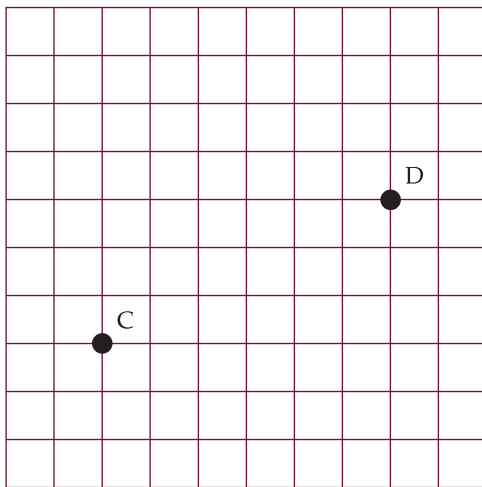
# Individual Work 1.1: Keeping Your Distance

*In this Individual Work, you consider several types of distance that might be used to decide a best location for a fire station.*

1. Gridville is a model for a town. In some ways it is like a real town or village. It differs in many ways.
  - a) How is Gridville different from your town?
  - b) How is Gridville like your town?

**Firetruck distance** is the shortest path along grid lines between two points.

2. a) What is the length of the shortest path from C to D in **Figure 1.3**? Explain how you can be sure it is the shortest.



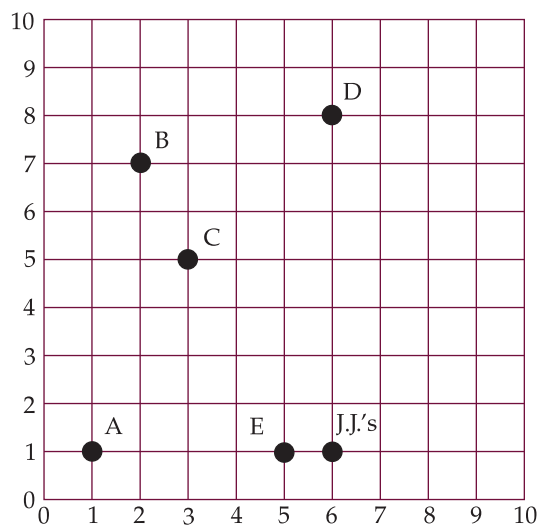
**Figure 1.3.**  
Map to determine shortest path.

- b) In Figure 1.3, is there more than one shortest path? If your answer is “no,” explain why there is only one. If your answer is “yes,” find one or more paths that are the same length as the shortest path.
3. Suppose you use a coordinate system to identify locations in another Gridville, one with three houses. House A is at (1, 4), house B is at (2, 7), and house C is at (8, 5).
    - a) Two locations are proposed for a fire station: (3, 5) and (5, 4). How do you decide which is better?
    - b) Find a location that you think is better. Explain.

4. A town might decide that the best location is one that minimizes total distance. The total distance is the sum of the lengths of the shortest paths from a fire station to all of the houses.

Suppose a fire station F is at (5, 5), house A is at (3, 8), and house B is at (10, 4).

- Find the distance from F to each house and the total distance?
  - Suppose the fire station moves from (5, 5) to (8, 2). Find the change in total distance.
  - Miles claims he has a location for the fire station that makes the total distance 10. Do you believe him? Explain.
5. Carla delivers pizza for J.J.'s Pizzeria. J.J.'s is at (6, 1) on a grid. The houses in Carla's delivery area are at (1, 1), (2, 7), (5, 1), (3, 5) and (6, 8) (**Figure 1.4**).



**Figure 1.4.**  
J.J.'s Pizzeria map.

- For which house is firetruck distance from J.J.'s largest? Explain how you got your answer. (Measure distance using units on the grid.)
- Find the total distance from J.J.'s to the five houses.

The **average distance** is the total distance divided by the number of houses or trips.

- Find the average distance from J.J.'s to the five houses.

Round-trip distance is the length of the shortest path from a fire station to a house plus the length of the shortest path from the house back to the fire station.

- d) What is the round-trip distance between J.J.'s and A?
  - e) If Carla makes a trip from J.J.'s to each house, what is the total round-trip distance?
  - f) Do you think total round-trip distance is always twice total distance? Explain.
  - g) Carla gets orders from all five houses and decides to deliver them all on one trip. What is the shortest total distance she can travel? Explain.
6. Suppose J.J.'s Pizzeria moves from (6, 1) to (5, 4) (Figure 1.5).

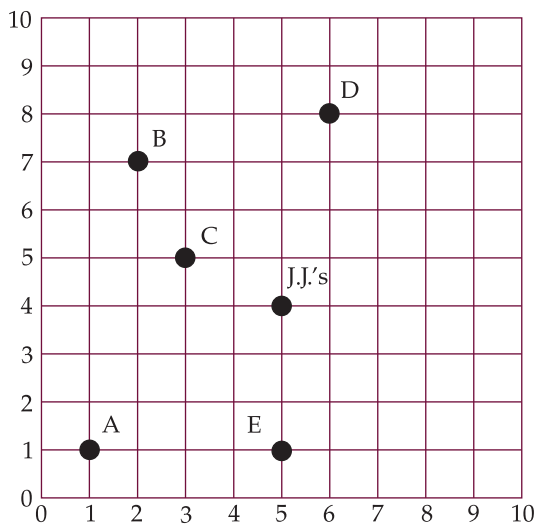
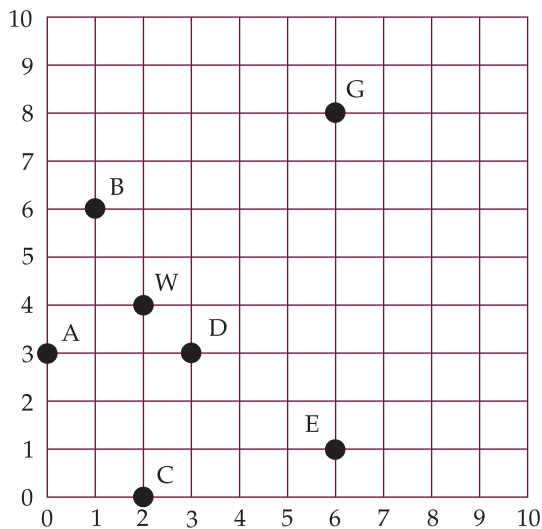


Figure 1.5.  
New map for J.J.'s Pizzeria.

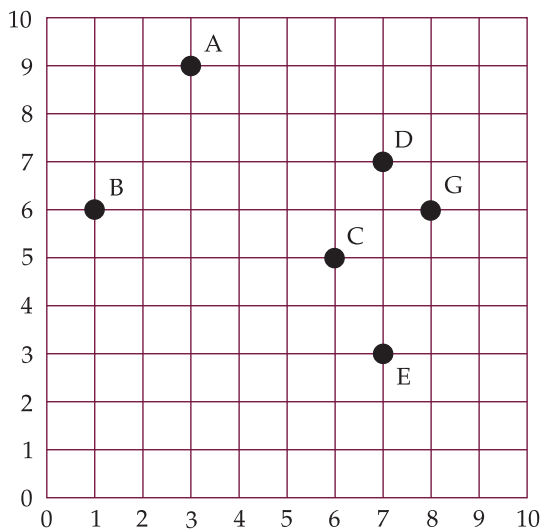
- a) If Carla makes separate round-trips from the new location to each house, what is the total round-trip distance? How much does the total round-trip distance change?
- b) Calculate the change in total distance and average distance.
- c) Explain how the total distance changes if J.J.'s moves from (5, 4) to (5, 3).
- d) You found the total distance when J.J.'s is at (5, 4). Find a new location for J.J.'s that makes total distance smaller. Justify your answer.

c) Explain how to find a place for J.'s that makes total distance as small as possible.

7. **Figure 1.6** shows locations of several stores and a warehouse. The stores are at A, B, C, D, E, and G. The warehouse is at W.
  - a) Find total distance, average distance, and total round-trip distance if a truck makes a separate trip from the warehouse to each store.
  - b) What is the maximum distance a truck travels to go from the warehouse to the farthest store?
  - c) Suppose the warehouse moves one unit to the right. For which stores does distance to the warehouse increase? For which stores does it decrease? For which stores does it stay the same?
  - d) Describe how total distance changes when the warehouse moves one unit to the right.
  - e) Describe how distance to the farthest store changes when the warehouse moves one unit to the right.
  - f) Describe how total distance changes when the warehouse moves up one unit.



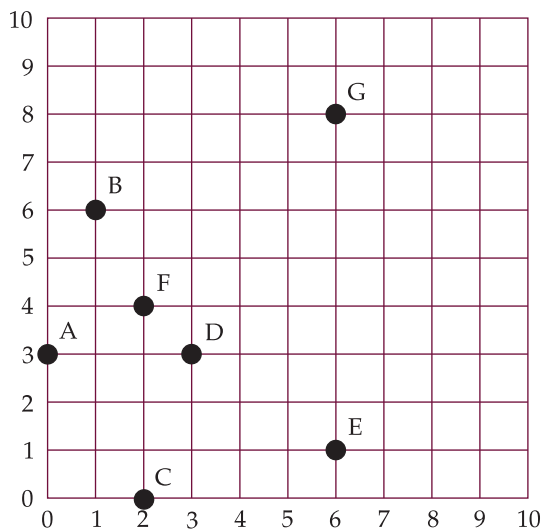
**Figure 1.6.** Map of stores and warehouse.



**Figure 1.7.** Boxville.

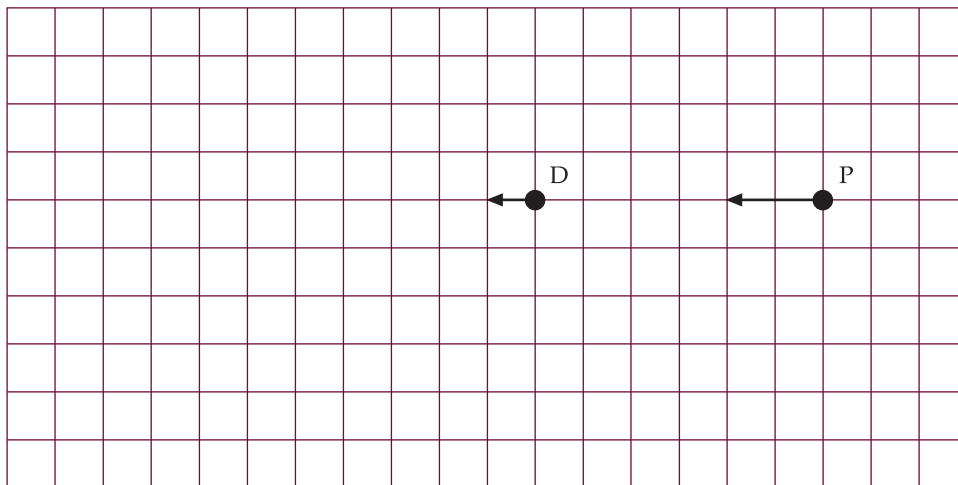
8. Boxville (**Figure 1.7**) is building a recycling station. They are considering two locations: (7, 5) and (4, 6).
  - a) Which of these two is a better place for the station? Explain.
  - b) What location is better than (7, 5) and (4, 6)? Explain.
  - c) Is it easier to find a best location in Boxville or Gridville? Explain.

9. Figure 1.8 represents a town. Houses are at A, B, C, D, E, and G. A school is at F.



**Figure 1.8.**  
Map of a town.

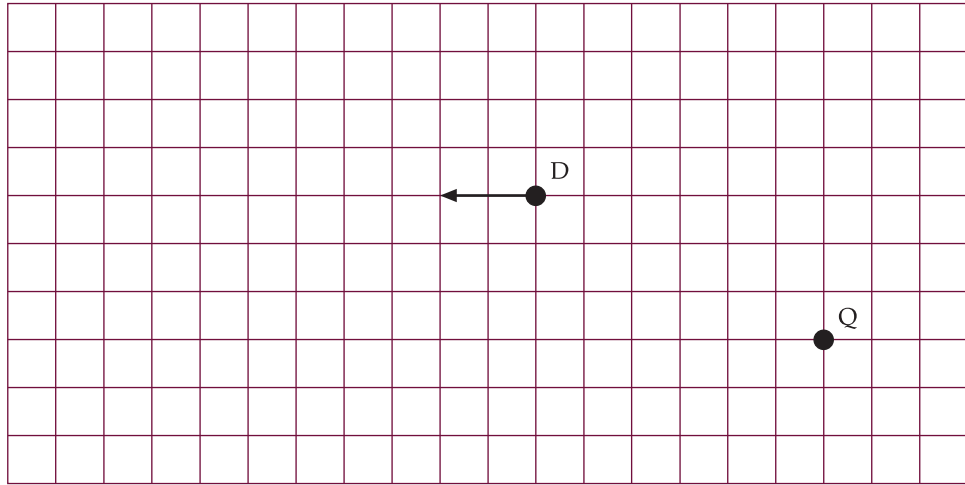
- a) Which houses benefit most when the school is at F? Does the placement of the school seem fair to you? Explain.
  - b) Suppose you live in house G. You want the school to be at a location that is best for you. Where would you put it? Explain.
  - c) Suppose you live in house G and a member of your family lives in house C. You want the school to be at a location that is best for both houses. Where would you put it? Explain.
  - d) Suppose you want the school to be at a location that is best for houses A, C, and D. Where would you put it? Explain.
10. a) In Figure 1.9, a police car at P is chasing a driver at D who is driving west. If the police car goes twice as fast as the driver, how far will the police car travel before it catches up? Why?



**Figure 1.9.**  
Pursuit map (a).

b) Suppose the police car begins at Q (Figure 1.10). The police car goes twice as fast as the driver. The driver is at D when the chase begins and is heading west. How far must the police car travel to catch up? Describe at least two paths the police car can follow.

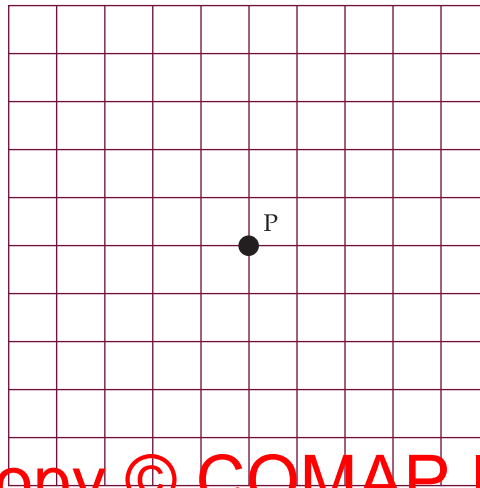
**Figure 1.10.**  
Pursuit map (b).



11. Distance is a factor in many decisions.

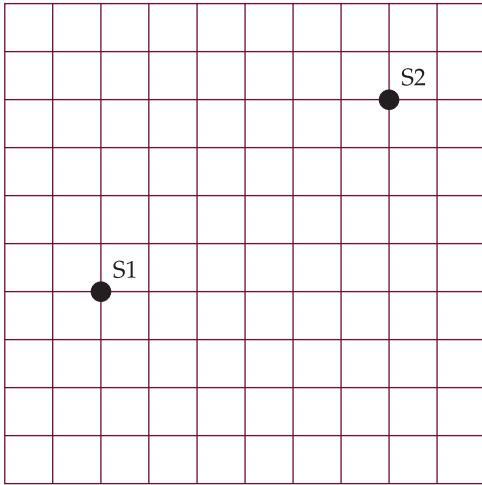
- a) Some businesses restrict delivery service to a certain number of blocks or miles.
- Draw a grid like **Figure 1.11** and mark Point P.
  - Mark all points at a firetruck distance of 2 from P. (Remember that houses are only at corners.)
  - Use a second color to mark all points at a firetruck distance of 3 from P.
  - Use a third color to mark all points at a firetruck distance of 5 from P.

Describe the patterns made by the colored marks on your grid.



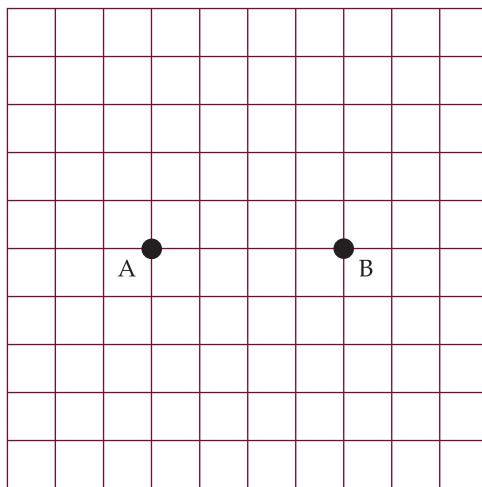
**Figure 1.11.**

b) Figure 1.12 shows a town with two schools, S1 and S2. Students attend the school closest to their homes, in firetruck distance. Draw a grid like Figure 1.12 and mark all points that are the same distance from both schools. (This is the boundary between the attendance areas.)



**Figure 1.12.**  
Map of two schools.

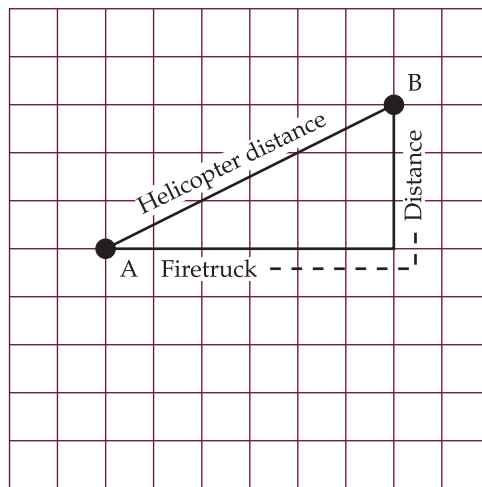
c) Mr. and Mrs. Walker work at A and B (Figure 1.13). They want to rent an apartment close enough to their jobs so that the total firetruck distance they walk to work is no more than 10 units. Draw a grid like Figure 1.13 and mark all possible places for an apartment.



**Figure 1.13.**  
Walker house locations.

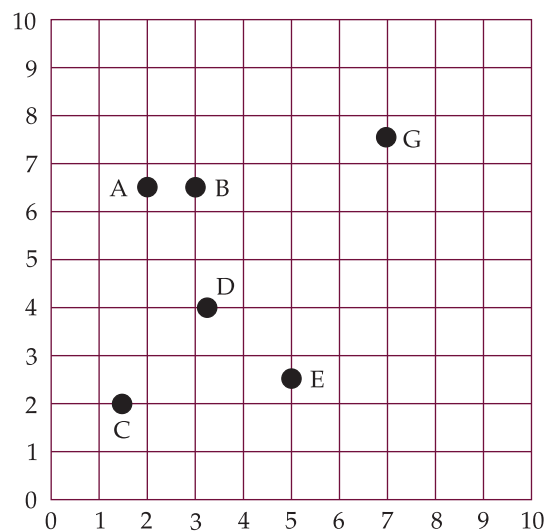
The helicopter distance is the direct (straight line) distance between two points.

12. a) In **Figure 1.14**, find the firetruck distance and the helicopter distance between A and B.
- b) Is helicopter distance between two points always less than firetruck distance? Explain.



**Figure 1.14.** Comparing firetruck distance to helicopter distance.

13. For these questions houses do not have to be at intersections (**Figure 1.15**).



**Figure 1.15.** Gridville with houses between the grid lines.

- a) Find the firetruck distance between A and B.
- b) Find the firetruck distance between C and D.

- c) Find the firetruck distance between A and T.
- d) Describe problems you might have if you try to find a location that minimizes total distance.

14. In Question 11, you found a boundary between two schools. To do this, you found all locations that are the same distance from the schools.

- a) Find similar boundaries for Gridville cities with 3, 4, or 5 schools (Figure 1.16). Copy each figure on paper. Draw or mark the boundary lines.

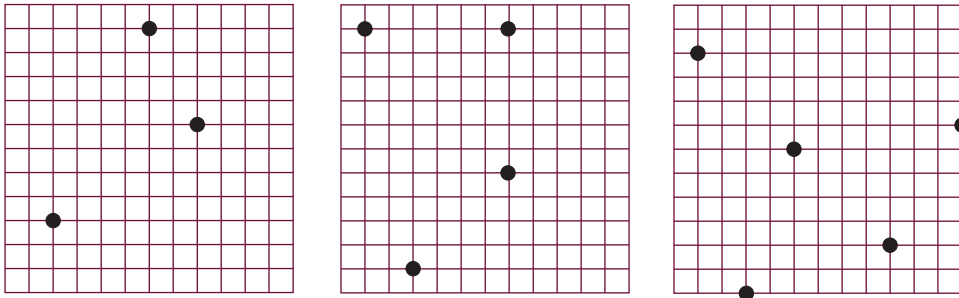


Figure 1.16. School locations are represented by dots.

- b) Draw your own map with six schools. Find the boundaries.



### Lower Macungie to Build Second Firehouse

*The Morning Call*

May 13, 2009

The Lower Macungie Fire Department, which for nearly two decades has talked about adding a second fire station, could have its new building by year's end.

The department's current station will remain in operation. The volunteer firefighters have been eyeing a second building to supplement service in what historically has been one of the faster-growing municipalities in the region.

The township now has about 29,000 people living in a 23-square-mile area.

"If you look at a map of the township and put our existing station on it, you see that it's tucked up in one of the corners," said Fire Chief Dave Nosal. "If you look at where the new station is going, it is smack dab in the center of the township."

He said the second station would provide better protection to more than 10,000 residents living in more than twenty developments in the center of the township.