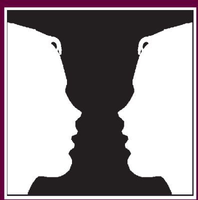
CHAPTER

2



Strategies

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EVERYDAY STRATEGIES

Strategic situations occur daily. They involve people like you, businesses, and governments. Often you don't think of them as strategic situations, and you certainly don't think about using mathematics to analyze them. However, after working through this chapter, you might find yourself viewing such situations differently.

This chapter is about one kind of strategic decision-making, and it introduces an important (and fairly new) area of mathematics called game theory. First developed during the 1940s by mathematician John von Neumann and economist Oscar Morganstern, game-theoretic modeling is widely used today, and has even received Nobel prizes. All you need to know in order to get started are a few concepts from algebra, such as matrices and systems of linear equations, and basic probability.

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LESSON ONE

Decisions

Key Concepts

Strategic situations (games)

Elements of a game: players, strategies, payoffs (or rankings)

Opposing interests

Payoff matrix

Endless Decisions

Making decisions is one aspect of life that we all have in common. Many decisions involve another person whose goals conflict with yours. For example, in the game of Monopoly, if you don't buy Boardwalk, another player may do so and bankrupt you with rent. If you don't put a quarter in a parking meter, the police may give you a $25 ticket.

There are many situations in which you must choose from one of several courses of action while one or more other parties are trying to do the same. In this lesson, you learn how mathematics can help you organize the key features of a situation. Later, you will discover that mathematics can help you select a best strategy.

Individual Work 2.1: Conflicts, Crises, and Games

In this Individual Work, you consider key elements of strategic situations and how to organize them.

A **strategic situation** exists when you must make a decision that affects one or more other parties who must also make decisions that affect you. The parties involved in a strategic situation are called **players** because strategic situations often resemble games. In fact, strategic situations are commonly called **games**. The actions the players can choose to take are called their **strategies**.

FYI

Game theory is a tool for analyzing strategic situations. The mathematician John von Neumann is considered the founder of game theory.

Here are some strategic situations.

Scenario #1: The Zairian Conflict of 1997

In 1997, Zaire was involved in a conflict. At one point, rebels had captured three-quarters of Zaire and were advancing on the capital. Rebel leader Laurent Kabila demanded that President Mobutu step down and leave the country. The president refused and demanded that the rebels cease their hostilities. The international community, aided by South African President Nelson Mandela, pressured the two leaders to meet and negotiate a peaceful resolution.

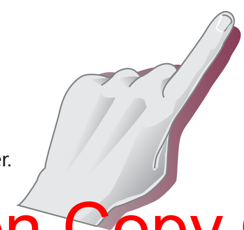
In order to begin to model a situation, you need to simplify it by identifying and isolating key factors.

1. Who do you think are the players in the Zairian conflict? What are their choices or strategies? What might be the consequences of the strategies?

Scenario #2: Simple Matching Game

In a simple two-player matching game, one player is the even player and one is the odd player. Each player has a choice of two strategies (see **Figure 2.1**).

Strategy #1:
Show one finger.



Strategy #2:
Show two fingers.

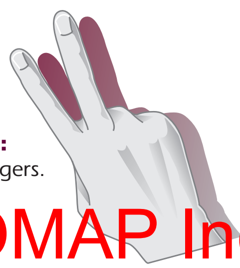


Figure 2.1.
Strategies for matching game.

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- The even player wins a point from the odd player if the sum is even.
- The odd player wins a point from the even player if the sum is odd.
- If a player wins a point, the payoff to that player is 1; if the player loses a point, the payoff to that player is –1.

You can organize the players, their strategies, and the results of the strategies, which are sometimes called **payoffs**, in a **payoff matrix**, as shown in **Figure 2.2**.

Column player

Row player		One finger	Two fingers
	One finger	1	–1
	Two fingers	–1	1

Figure 2.2. Payoff matrix for simple matching game.

In a payoff matrix:

- The choice of which player to assign to the rows is open. This player is called the row player. In this case, the even player is assigned to the rows.
- The payoffs in the matrix are for the row player. Sometimes, the payoffs to both players are listed as ordered pairs. If so, the row player's payoff should be first in each pair. **Figure 2.3** is a partial payoff matrix in this form.

Figure 2.3. Payoff matrix for simple matching game.

Column player

Row player		One finger	Two fingers
	One finger	(1, –1)	(–1,)
	Two fingers	(–1,)	(1,)

2. What are the payoffs to the column player in Figure 2.3?

Quite often players have opposing interests.

3. What do you think it means to say that the row player and column player in the simple matching game have opposing interests?

4. The players in a simple matching game may play it more than once. Do you think the game in Question 1 will be played more than once? Explain.

Scenario #3: The Cuban Missile Crisis of 1962

The Cuban Missile Crisis of 1962 is a strategic situation in which the interests of the players are not completely opposed. The United States discovered that the USSR was building a missile base in Cuba, and President Kennedy ordered a naval blockade. The USSR had to decide whether to challenge the blockade or back down. The United States would then have to decide how to respond.

It is hard to attach payoffs to outcomes in the Cuban Missile Crisis. Ranking the outcomes is easier. The United States, for example, felt that escalation on their part (continuing the blockade) and withdrawal of the missiles (backing down) on the part of the USSR was best, and ranked it first. Both sides probably ranked joint escalation last. **Figure 2.4** shows possible ranks from the row (U.S.) player's point of view, with 1 as the best outcome and 4 as the worst.

		USSR	
		Escalate	**Back down**
U.S.	**Blockade**	(4,)	(1,)
	Back down	(3,)	(2,)

Figure 2.4.
Payoff matrix with rankings.

5. Complete the payoff matrix in Figure 2.4 by adding the column (USSR) player's rankings. Justify your answer.
6. Suppose, instead, you use this system: rank from 0 to 3 with 0 the worst outcome and 3 the best.
   a) Write a payoff matrix for the Cuban Missile Crisis.
   b) How are the entries in this matrix related to the entries for your matrix in Question 5?
   c) Which ranking system do you like? Explain.
7. a) When rankings are used instead of payoffs, what do you think it means for players to have opposing interests?
   b) Based on your matrix in Question 5, do you think that the U.S. and the USSR had opposing interests? Explain.

> When the payoffs to one player are the opposite of payoffs to the other player, the game is called a **zero-sum game**. This term derives from the fact that opposite values have a sum of zero.

Look at the payoff matrix for the simple matching game (your answer to Question 2). Notice that for each set of row and column player strategies, the sum of the payoffs to the row and column players is zero. Thus, the simple matching game is a zero-sum game. Now look at the matrix of rankings for the Cuban Missile Crisis. This is not a zero-sum game since the sum of the rankings is always positive.

8. a) Return to the Zairian Conflict. Prepare a payoff matrix. Justify your choice for payoffs (or rankings). If you use rankings, explain your system.

b) Here are two new facts: (1) President Mobutu is dying of cancer and (2) Laurent Kabila says his troops will reach the capital in two weeks. In light of this new information, do you want to change your payoff matrix in part (a)?

MODELING NOTE

When you model a real-world situation, you must first decide which factors are most important and build your model based on those factors. If later you find new information that you think is important, you can adjust your model.

In Scenarios 1–3, there are two players. There are strategic situations that do not involve two "true" players. Here is an example.

Scenario #4: The Umbrella Decision

You are trying to decide whether to take an umbrella with you on a walk. Since rain may or may not happen, nature can be considered the second player.

9. a) How is nature like a player? How is nature different?

b) Prepare a payoff matrix for Scenario #4. Explain your choice of payoffs or rankings.

c) Does it make sense to list the payoffs or rankings for each outcome as an ordered pair? Explain.

d) Is this situation likely to repeat?

Activity 2.1: Exploring Strategic Situations

In this activity, you create payoff matrices for a number of different strategic situations. In each, you should decide if the situation is like the simple matching game, the Cuban Missile Crisis, the Zairian conflict, or the Umbrella situation.

MODELING NOTE

Remember that agreeing on payoffs or rankings is often the hardest part of the analysis of a strategic situation. The "right answer" is not always obvious, and often does not exist. A model seldom describes the real world perfectly.

In each of Questions 1–7,

- Prepare a payoff matrix that shows the players, their strategies, and the payoffs (or rankings) for the row player.
- If your matrix shows payoffs, label the units (dollars, for example). If you use rankings, describe your ranking method.
- Answer each of these after you finish the matrix:
  - (a) Are both players true players, or is one of them like nature in the umbrella situation?
  - (b) Do the players have opposing interests, or might it be to their advantage to cooperate?
  - (c) Is this a one-time situation, or can the players repeat it?

1. A baseball player is batting against a pitcher who throws fastballs and curve balls. If the batter guesses the correct pitch, he has a 40% chance of getting a hit; if he guesses the wrong pitch, he has a 10% chance.

2. Sarah has $1000 to invest. She can put it in a savings account or the stock market.
   - If she invests it in the stock market and the economy does well during the next year, her investment will gain about 20% in value. If the economy does poorly, her investment will lose about 10% of its value.
   - A savings account will pay her about 7% interest if the economy does well, and 5% if it does poorly.

3. You and a friend decide it would be cool if you both come to school tomorrow with weird haircuts. But you begin to worry. It would be embarrassing to come to school with a wild haircut if your friend does not.

4. Maka Deel sells cars for a dealer who drops a car's price $200 if a customer shows a competitor's business card when a transaction ends. Maka is close to a deal with a customer that will earn her $500. However, her boss says that any further reduction in price—

including $200 if the customer has a competitor's card—will come out of the $500. Maka feels that if she drops the price another $100, the customer will agree, but is unsure if the customer has a card.

5. Two competing chains plan to open new stores in a city that has two malls. Likely monthly profits are $20,000 at the larger mall, and $15,000 at the smaller mall. If the chains choose different malls, each store will capture all of the profits from the mall where it is located. If they are at the same mall, Store A can expect 60% of the profits from that mall, and Store B can expect 40%.

6. Droughtsville has a severe water shortage and asks residents to practice water-saving measures. But you have been planning to clean house, wash cars, and water your lawn. You wonder if you should go ahead with your plans, and you wonder what will happen if you and the rest of the town hog water.

7. Here is another "one-finger, two-finger" matching game. If on any play the two players show an even number of fingers, then the even player wins; otherwise the odd player wins. The winning player is awarded points equal to the total number of fingers showing; the losing player loses as many points.

A game in which the sum of the payoffs to the row and column players is the same regardless of the players' choices of strategies is called a **constant-sum game**. (A zero-sum game is a type of constant-sum games because the sum of the row and column player's payoffs is always zero.)

8. Which of the games in Questions 1–7 are constant-sum games?

A **pure strategy** is a player's choice of play on a single game.

9. Pair off with someone and play the game in Question 7. Decide who is the even player and who is the odd player. Before playing the game, each player should mentally pick a pure strategy: Decide whether to show one finger or two.

   a) Play the game once using the strategy that you picked. Did the even player or the odd player win? What are the players' scores after one play?

   b) If you were to play this game 9 more times using your pure strategy each time, what would be the players' scores after the 10 games?

c) Next, you don't have to stick with a pure strategy but can mix strategies. Play the game for a few minutes or until your teacher says to stop. What are the players' scores?

DISCUSSION/REFLECTION

1. Do you think the game that you have just played is fair?
2. What do you think it means for a game to be fair?
3. Which students in the class are the best players? How did you decide?
4. Do you think the best players are lucky or did they use a winning strategy? Explain.

Individual Work 2.2: Show Me the Money

In this Individual Work, you look at two other ways to represent key features of games.

In Course 1, Chapter 6 (*Imperfect Testing*), you used tree diagrams. For example, suppose you want to know if the percentage of female students with driver's licenses at your school differs from the percentage of male students with licenses. After collecting data, you might make a tree diagram like **Figure 2.5**.

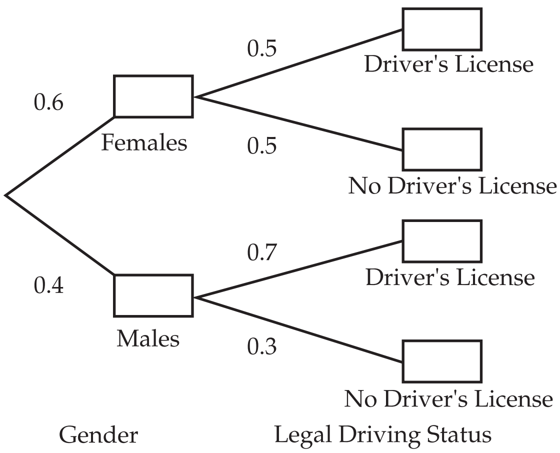

Figure 2.5.
Tree diagram of gender and driving status.

1. Two people are playing a game with pennies.
   - Each starts with 50 pennies.
   - Each player takes one or two of her pennies and hides them in her hand.
   - Both players open their hands at the same time and show their pennies.
   - If the total number of pennies is even, the even player gets them. If the total is odd, the odd player gets them.

   a) Write a payoff matrix for this game.

   b) Use a tree diagram to show the information in your payoff matrix.

   c) Do you think this game is fair? If not, would you rather be the row or the column player? Why?

   d) Sasha and Joel played the game 25 times. Jan and Dana played it 50 times. **Figures 2.6 and 2.7** show the results. Who do you think is the best player? Explain.

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		Joel	
		One penny	Two pennies
Sasha	One penny	5	7
	Two pennies	9	4

Figure 2.6. Number of times each outcome was observed in 25 plays.

		Dana	
		One penny	Two pennies
Jan	One penny	12	12
	Two pennies	8	18

Figure 2.7. Number of times each outcome was observed in 50 plays.

You have used graphs made up of edges and vertices as models in other chapters. For example, in Course 1, Chapter 0 (*Pick a Winner*), you used a graph like **Figure 2.8** to show election results.

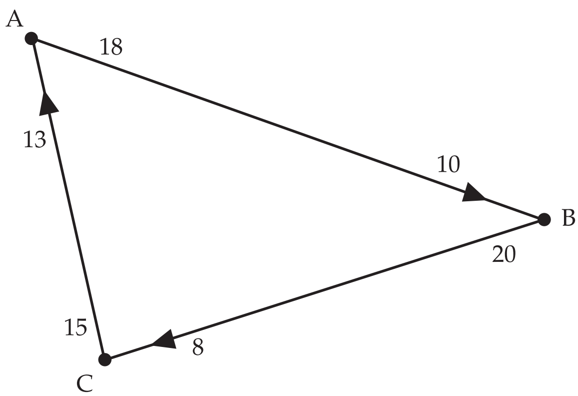

Figure 2.8. Graph made up of edges and vertices.

- The arrow on the edge from A to B means that A beats B. The 18 and 10 mean that A got 18 votes and B got 10.
- B beats C by 20 to 8.
- C beats A by 15 to 13.

2. Suggest a way to use a graph to model the key features of one of the strategic situations in Questions 1–7 of Activity 2.1. What useful information can you get from your graph?