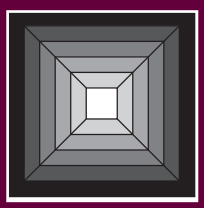
CHAPTER

4



The Right Stuff

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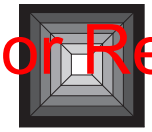

IT'S A PACKAGE DEAL

It is hard to imagine life without packaging. Indeed, packaging seems more a necessity than a convenience. Packages that contain food keep the food from spoiling, thereby reducing the risk of disease. But packaging causes problems. It accounts for about 30% of the waste in U.S. landfills, many of which are overburdened. By adding to the volume of goods, packaging increases the need for space in warehouses, in delivery trucks, and on store shelves. To minimize these problems, packages must be well designed.

Packages are geometric. For example, soft drinks, vegetables, and soup are packaged in metal cylinders. Sometimes packages that are geometric shapes hold packages that are other geometric shapes. For example, cylinders that contain soft drinks are often sold in rectangular boxes of six or more.

Package design requires knowledge of geometry. Since packages are three-dimensional objects with two-dimensional sides, volume and area play important roles. In this chapter, you consider how geometry can be used to create a definition of efficient packaging and improve package design.

LESSON ONE

Packaging Models

Key Concepts

Mathematical modeling

Modeling criteria

Modeling factors

PREPARATION READING

The World of Packaging

Nearly everything that is manufactured and sold is packaged in some way. Many products undergo several levels of packaging. Foods, for example, may be packaged in cans, which in turn are packaged in cartons that are stacked on pallets.

Because packaging is common, it affects many people. Some people may have concerns that conflict with the concerns of others. If these concerns can be quantified, perhaps they can be modeled mathematically.

In this lesson, you consider the impact of packaging and ways in which efficient packaging can be defined. Your definition of efficiency will be used in subsequent lessons of this chapter to consider ways in which packaging can be improved.

Graphic Packaging International's Cap-It paperboard secondary packaging for bottled soft drinks received an Eco Award from the Paperboard Packaging Council in 2009.

Photo used with permission of Graphic Packaging International.

Activity 4.1: Packaging Concerns

In this activity, you consider who might be affected by soft drink packaging and what some of their concerns might be.

MODELING NOTE

Optimizing packaging schemes is a broad problem. Here, as in all modeling efforts, it is best to start by narrowing the focus and considering just a single type of package.

Soft drinks, like many products, have several levels of packaging. The primary package is a can or bottle. Secondary packaging holds several of the cans or bottles. Secondary packaging is often made of a material called paperboard.

Consider the issue of secondary packaging of soft drinks. Secondary packaging is used to hold 6, 12, 24, or 30 cans. (Unless otherwise indicated, in this chapter "soft drink packaging" refers to secondary packaging. The cans contained in the secondary packaging are standard 12 oz. can.)

In order to use mathematics to improve packaging efficiency, you must first consider the concerns of people affected by packaging. Those concerns will help you choose criteria you can then use to create a definition of efficient packaging.

1. Make a list of some people who are affected by the way soft drink cans are packaged.
2. List some concerns of each of the people you listed in Question 1. (You may want to interview people in your town.)
3. Use the list of concerns you made in Question 2 to make a list of criteria that could be used to create a mathematical model. For example, your criteria might include an optimization statement such as, "Good soft drink packaging should minimize (or maximize) _____ ."

Individual Work 4.1: Packaging Criteria

In this Individual Work, you consider criteria that could be used to define efficient packaging.

1. When you begin the task of applying mathematics to a real-world problem, it is interesting to ask whether the problem is worth solving. In 2009 Americans consumed over 80 billion cans of soft drinks. One criterion for efficient packaging is that it be economical. Suppose you find a way to save one-tenth of a cent ($.001) on the cost of packaging of each six-pack sold.

   a) Estimate the total annual savings to the soft drink industry.

   b) If you receive royalties worth 10% of the savings to the industry, estimate your annual income for the use of your innovation in the United States.

   c) Can you think of anyone who might be opposed to decreasing the cost of packaging?

   d) What are some factors that could affect the cost of soft drink packaging?

2. One criterion that could be used to judge soft drink packaging efficiency is that the packaging should minimize the amount of space used to store the soft drinks.

   a) In what units might storage space be measured?

   b) Who would approve of a reduction in storage space?

   c) Who might disapprove of such a reduction?

   d) What are some factors that could affect the space needed to store packages of soft drinks?

3. One criterion that could be used to judge soft drink packaging efficiency is that the packaging should maximize sales.

   a) How would you measure the ability of packaging to increase sales?

   b) Who is likely to benefit from a sales increase?

   c) Who might suffer adverse effects from a sales increase?

   d) What are some factors that might affect the ability of packaging to increase sales?

4. A criterion that could be used to judge soft drink packaging efficiency is that the packaging should minimize the amount of packaging material.

   a) How might the amount of packaging material be measured?

   b) Who might approve of a reduction in the amount of packaging material?

   c) Who might suffer adverse effects from a reduction in the amount of packaging material?

   d) What are some factors that might affect the amount of packaging material?

5. In Question 3 of Activity 4.1, you listed criteria that might be used to create a mathematical model for packaging efficiency. Select one or more of those criteria.

   a) Discuss how you would measure efficiency.

   b) Discuss people who would benefit and people who might object if efficiency improved.

   c) List one or more factors that might affect efficiency.

6. Designing efficient secondary packaging for soft drinks is similar to designing an efficient container to hold several honeydew melons. Explain.

7. How is the problem of designing efficient secondary packaging for soft drinks like the problem of designing cartons to fit in a moving van? How do the two problems differ?

Activity 4.2: Packaging Efficiency

In this activity, you play the role of a manager of a soft drink company who is trying to decide if an employee's suggestion has merit.

Your company pays bonuses to employees who find ways to save the company money. The bonus policy means that you have to consider employee suggestions and decide whether to enact them. Thus, you have to establish decision-making criteria for new suggestions.

Today you are considering a suggestion for the redesign of your most basic secondary package: the six-pack. An employee has suggested that the six-pack be based on a "staggered" arrangement as shown in **Figure 4.1**. Your company currently uses a conventional arrangement in which the cans are "stacked" against each other as shown in **Figure 4.2**. The employee thinks the new design is more efficient and will save the company money.

Figure 4.1.
A staggered six-pack.

Figure 4.2.
A conventional stacked six-pack.

Your task in this activity is to select a criterion for deciding whether the new six-pack design is more efficient than the old. You do not actually have to make the decision. When you have settled on a criterion, you can send the design and your criterion to the company's mathematicians for evaluation.

The following are two possible criteria:

- Packaging should minimize the total amount of package space not used by the cans.
- Packaging should maximize the percentage of package space used by the cans.

1. Note that the first criterion involves the total amount of space. List at least two other criteria that involve the total of some quantity.
2. Note that the second criterion involves a percentage, which in this case is the ratio of space used by the cans to space used by the package. List at least two other criteria that involve a ratio of two quantities.

3. Discuss the pros and cons of each criterion.
4. Write a memo to the company's mathematicians describing the new design and the criterion you chose for judging it.

MODELING NOTE

A good criterion must be clearly stated. It should be based on a quantitative definition of efficiency that can be explored mathematically.

DISCUSSION/REFLECTION

1. In general, do you prefer criteria that involve total amounts of a quantity or criteria that involve a ratio of two quantities? Explain.
2. If a package design makes efficient use of space in a six-pack, do the six-packs also make efficient use of space on a store shelf?
3. If six-packs make efficient use of space on a store shelf, do they also make efficient use of space in a delivery truck?

Individual Work 4.2: Packaging Factor

In this Individual Work, you consider two factors that that might affect the efficiency of soft drink packaging.

1. A criterion used to judge the efficiency of soft drink packaging can be based on the amount of storage space used by the cans or by the package. If space is used to define efficiency, then it is important to understand factors that have an effect on the amount of space used and how to measure their effect.

   a) Consider the shape of the package as a factor. Do you think changing the shape of the package affects how well space is used? For example, would a package shape that uses the staggered arrangement in Activity 4.2 use space more efficiently than a package shape that uses the conventional arrangement? Explain.

   b) How would you decide whether the staggered arrangement uses space more efficiently than the conventional arrangement? That is, how would you measure the way each arrangement uses space?

   c) Consider the number of cans in the container as a factor. That is, predict whether changing the number of cans in a conventional stacked arrangement might result in more efficient use of space.

   d) How would you decide whether changing the number of cans makes more efficient use of space? For example, is a design in which six cans use 200 units of space more efficient than a design in which eight cans use 250 units?

2. A criterion used to judge the efficiency of soft drink packaging can be based on the amount of packaging material used.

   a) Consider package shape as a factor. Do you think changing the shape of the package has an effect on how well packaging material is used? For example, does the staggered arrangement in Activity 4.2 use packaging material more efficiently than a conventional arrangement? Explain.

   b) How would you decide whether the staggered arrangement uses packaging material more efficiently than the conventional arrangement? That is, how would you measure the way in which each arrangement uses packaging material?

   c) Consider the number of cans in the package as a factor. That is, predict whether changing the number of cans in a conventional stacked arrangement might result in more efficient use of packaging material.

d) How would you decide whether changing the number of cans makes more efficient use of packaging material? That is, how would you compare the use of packaging material by a stacked arrangement of six cans with the use of packaging material by a stacked arrangement of, say, eight cans?

3. Consider a simplified version of the soft drink packaging problem. The problem is simplified in two ways. First, the cans are two-dimensional. Second, as shown in **Figure 4.3**, the cans are shaped like squares.

MODELING NOTE

Modelers often begin by considering a simplified version of a problem and building on it. Simplifying a problem can help clarify your thinking about it.

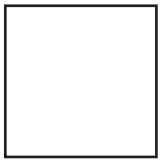

Figure 4.3.
A 2 cm square "can."

a) **Figure 4.4** is a 4×5 cm package that is intended to hold several of the cans. How many does it hold? Make a drawing to explain your answer.

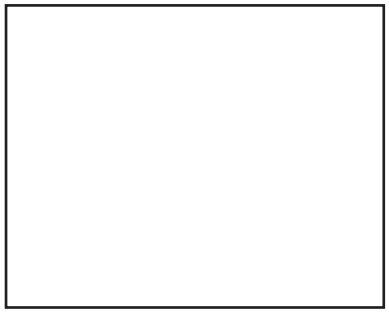

Figure 4.4.
A 4×5 cm package.

b) **Figure 4.5** is a 6×7 cm package that is intended to hold several of the cans. How many does it hold? Make a drawing to explain your answer.

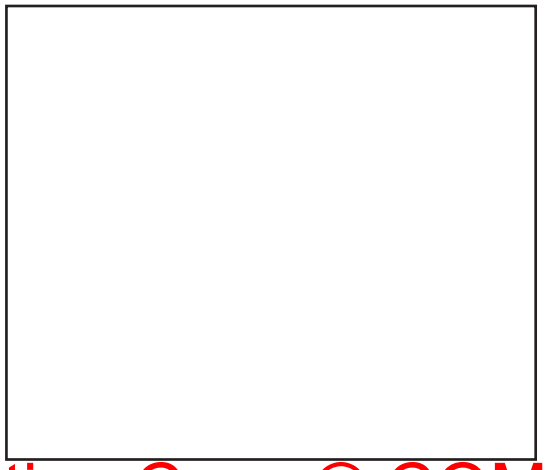

Figure 4.5.
A 6×7 cm package.

c) In which of the two packages do you think the cans use space more efficiently? Explain.

d) A display rack in a store is 12 cm wide and 35 cm long. Compare the number of Figure 4.4 packages and the number of Figure 4.5 packages that fit on the shelf. If necessary, make a drawing to support your answer. Which package lets the store have more soda on display?

e) How efficiently do the cans use space in a 6×6 cm pack? How well do the packs use space on the display rack?

4. This question is a simplification of the packaging problem in which the criterion is based on the efficient use of packaging material. Imagine that the packages in Figures 4.4 and 4.5 are made of wire.

   a) How much packaging material is used by the packages in Figures 4.4 and 4.5?

   b) In which of the two packages do you think packaging material is used more efficiently? Justify your answer.

5. A standard moving van is 8 feet wide, 40 feet long, and 8 feet high.

   a) How efficiently do cartons that are 16 inches wide, 16 inches long, and 8 inches high cover the floor of the van?

   b) How efficiently do the $16 \times 16 \times 8$ inch cartons use space in the van?

   c) Do cartons that are 9 inches high use space in the van as well as those in part (b)?

   d) How efficiently does a single layer of cartons that are 18 inches wide, 18 inches long, and 8 inches high cover the floor of the van?

   e) How efficiently do the $18 \times 18 \times 8$ inch cartons use space in the van?

   f) Is carton height a factor when deciding how well cartons cover the van's floor? Is it a factor when deciding how well the cartons fill the van?