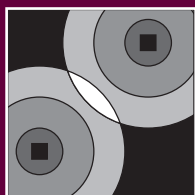


CHAPTER

5



Proximity

LESSON ONE

Colorado Needs Rain!

LESSON TWO

Neighborhoods

LESSON THREE

Rainfall

LESSON FOUR

**A Method of
a Different Color**

LESSON FIVE

Digging for Answers

Chapter 5 Review





HOW CLOSE?

Many real-world situations involve questions about proximity. For example, a robot traversing an unexplored planet must decide how close it can safely come to rocks and craters. A manager of a restaurant chain wants to know how close to their existing franchises they can open a new one. Police, fire, ambulance, and other emergency services must decide which unit is closest to the site of a crime, fire, or accident.

Many mathematical modeling problems are geometric in nature.

Proximity problems are geometric because:

- They involve objects like rocks or locations like fire stations and hospitals that can be represented by points.
- Boundaries that help people decide which of several facilities is closest to them can be represented by lines.
- The part of a region that is closest to a given fire station or hospital is often a triangle, quadrilateral, or other type of polygon.

In this chapter, you apply geometric concepts from Chapter 4, *The Right Stuff*, and also develop new geometric tools to help you create mathematical models to solve proximity problems.



LESSON ONE

Colorado Needs Rain!

Key Concepts

Samples

Mean

Area

Volume

Weighted average

PREPARATION READING

Colorado Needs Rain!

An accurate estimate of rainfall over an area is important because many decisions are based on rainfall estimates. For example, allocation of water resources and disaster relief funds is based, in part, on the amount of rainfall a region receives.

Consider estimation of rainfall for the state of Colorado. The state has 240 stations that measure rainfall. The measurements at these stations are samples of the rainfall for the state. From these samples the state must estimate the total amount of rainfall for all of Colorado. If rainfall is inadequate, the state may need to obtain water from other sources.

The state could ensure an accurate estimate of rainfall by using thousands of rain gauges. Doing so is impractical, however, because of the cost of distributing the gauges and hiring staff to check them.

In this lesson, you develop a first model for estimating rainfall in a region based on readings from a few gauges.

Activity 5.1: A First Estimate

Since the primary goal of this chapter is to develop a model for estimating rainfall in Colorado, in this activity, you will make your first attempt to develop such a model.

MODELING NOTE

The mathematical modeling process often involves several cycles: A model is created, tested against reality, found wanting in some way, and the process is repeated. The knowledge a modeler gains from creating a model, even if the model isn't a good one, helps perfect the solution.

An important strategy in developing a mathematical model is to start simply. Therefore, suppose that the state of Colorado uses only eight rain gauges to estimate rainfall. Colorado is nearly rectangular in shape. The rectangle in **Figure 5.1** has an aspect ratio of 1.36, which makes it similar in shape to Colorado.

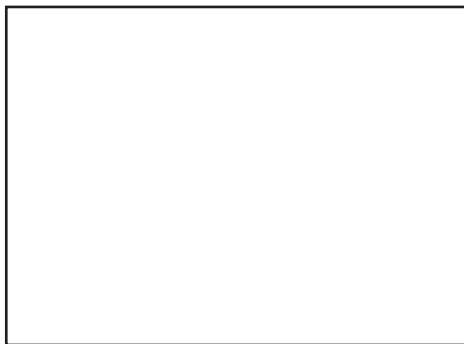


Figure 5.1.
A rectangle with aspect ratio 1.36.

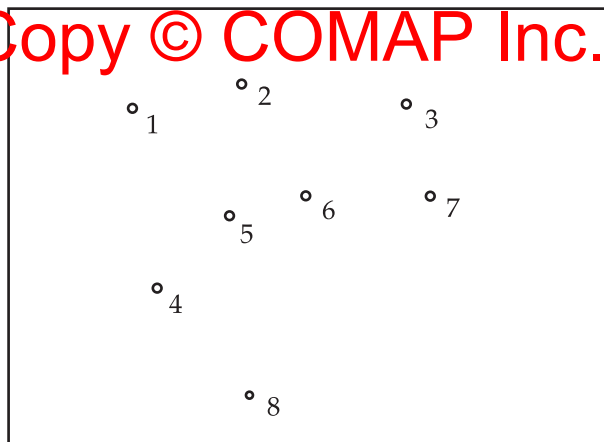
1. Sketch a distribution of eight rain gauges that you think would give an accurate estimate of rainfall for the state. Number the gauges to make it easy to refer to them.
2. Rainfall is commonly measured in inches. For example, a city gets an inch of rain, then the entire volume of rain would be an inch deep if spread equally across the city. If the rainfall at each of the eight gauges is as shown in **Figure 5.2**, estimate the depth of rainfall for the entire state.

Gauge	1	2	3	4	5	6	7	8
Rainfall	2.11"	2.15"	1.21"	4.45"	2.67"	2.51"	3.11"	2.43"

Figure 5.2.
Rainfall at eight sites in Colorado.

3. The terrain of a region and its population distribution are two factors that can prevent placing rain gauges in ideal locations. **Figure 5.3** shows locations of eight rain gauges in Colorado. Discuss strengths and weaknesses of the model you used to make your estimate in Question 2. Suggest ways that you could improve your estimate.

Figure 5.3.
Rain gauges
in Colorado.



**TAKE
NOTE**

Recall that to get a simple arithmetic mean of a data set, add the data and divide by the number of items.

DISCUSSION/REFLECTION

1. Does the mean of the rainfall measured at several gauges give an accurate estimate of the rainfall over a region? Explain.
2. What are some examples of situations in which the mean is useful? What are some examples of situations in which it is not?

Individual Work 5.1: Rainfall and Other Averages

Since it is not possible to measure the rainfall at all points in a region, models for estimating rainfall often use some type of average. In this Individual Work, you consider several aspects of rainfall measurement and the use of averages.

- Colorado is nearly rectangular, with a length of 375 miles and a height of 275 miles.
 - Estimate Colorado's area.
 - What is Colorado's aspect ratio?
- Rainfall is sometimes measured in the volume of water that falls over a region.
 - Convert Colorado's dimensions to feet and find the number of cubic feet of water that falls on Colorado if the state gets one inch of rain.
 - What is the change in the total volume of water if the rainfall increases from 1 inch to 1.1 inches?
 - A flat roof must have adequate drainage so that it does not collapse from the weight of rain or snow. Find the volume and weight of the water that falls on a 100 ft. \times 100 ft. roof when one inch of rain falls.
 - Find the volume of water in gallons that falls on a 100 ft. \times 100 ft. roof during a one-inch rain.
- Figure 5.4** shows eight rain gauges from Question 3 of Activity 5.1 and fourteen cities in Colorado. For each rain gauge, list the cities for which you think the gauge provides the best estimate of rainfall.
 - Give one or more ways to use these gauges to estimate rainfall in Boulder.
 - Should gauge 7 be used to find Denver's rainfall? Explain.



Recall that
1 mile =
5280 feet.

A cubic foot of water weighs
approximately 62.5 lbs.

There are about 7.5 gallons
of water in a cubic foot.

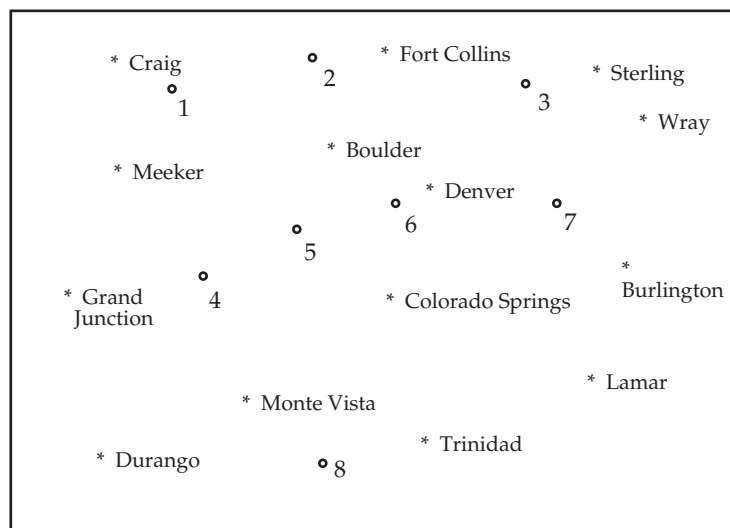


Figure 5.4. Eight rain gauges and fourteen Colorado cities.

4. A weather office serves three counties, each of which is roughly rectangular in shape. Each county has a single rain gauge at its center. **Figure 5.5** shows the dimensions of the counties and the rainfall at each gauge during a recent month.

Figure 5.5.
Countries
and rainfall.

Dimensions	Rainfall
11 mi. × 15 mi.	1.25"
12 mi. × 8 mi.	2.17"
14 mi. × 12 mi.	1.63"

- Use a simple arithmetic mean to estimate the rainfall for the tri-county region.
 - Do you think the county's gauges should be considered equal for estimating the region's total rainfall? Explain.
 - Estimate the total volume of the rainfall in the region.
 - Show how the volume you found in part (c) can be used to calculate the average depth of rainfall for the region. How else can the depth be calculated?
5. As every student knows, averages are used to decide report card grades. Here is a list of a student's grades in the four categories a teacher uses to decide report card grades:

Tests: 89%

Quizzes: 85%

Homework: 99%

Class participation: 75%

- Find a simple mean of these grades.

A **weighted average** is an arithmetic mean that places more importance on some items than others. To find a weighted average:

- Decide a decimal (or percentage) weight for each item.
- Multiply each item by its decimal weight.
- Find a total of all the products in the previous step.

- The teacher counts tests as 50% of the final grade, quizzes as 20%, homework as 20%, and class participation as 10%. The teacher calculates the report card grade by computing a weighted average

- of the four category grades. Find the weighted average for these four grades.
- c) What is the sum of the weights used to find the weighted average in part (b)?
 - d) Which category has the greatest impact on the weighted average? Which category has the least impact? In other words, to which category is the report card grade most sensitive, and to which category is it least sensitive?
 - e) Could a weighted average be more useful than a simple mean in finding a rainfall estimate? Explain.
6. A family has three children. The parents have budgeted \$20 weekly for the children's allowances.
- a) If the allowances are divided equally among the children, how much does each get?
 - b) The ages of the children are 5, 7, and 8. If the parents distribute the \$20 so that the allowances are proportional to the ages, how much does each child get?
 - c) What portion of the \$20 does each child get in part (b)? How are these portions similar to weights?
 - d) If the amount budgeted for allowances stays the same, find each child's allowance one year from now. Explain how you got your answers.
 - e) Examine the long-term implications of this policy.
7. Can weighted averages be used to improve your model for finding rainfall in Colorado? Explain.

Activity 5.2: Divide and Save

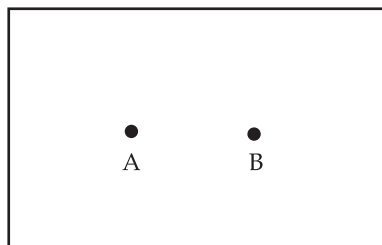
If rain gauges are not uniformly distributed, some of them are useful for estimating rainfall in much larger portions of the state than others are. In this activity, you consider how to find the part of a region to assign to a given gauge.

Again, it is best to start with a simple situation. Consider a region much smaller than Colorado: a beach. Instead of eight rain gauges, use lifeguard stands. Each lifeguard is assigned the part of the beach that is closest to that lifeguard's stand.

Figures 5.6–5.11 show arrangements of two or three lifeguard stands. In each, draw lines to show the part of the beach assigned to each lifeguard. In Questions 1–6, estimate the decimal portion of the beach that each lifeguard guards.

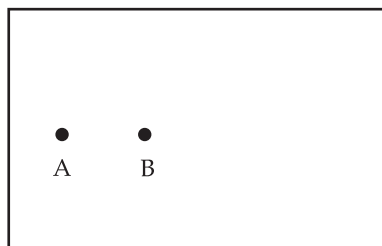
1. See Figure 5.6.

Figure 5.6.



2. See Figure 5.7.

Figure 5.7.



3. See Figure 5.8.

Figure 5.8.



4. See Figure 5.9.

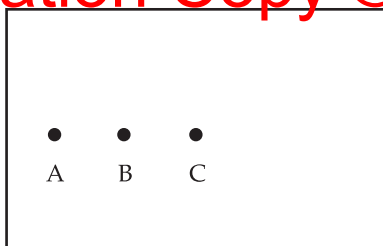


Figure 5.9.

5. See Figure 5.10.

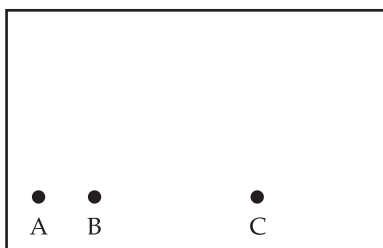


Figure 5.10.

6. See Figure 5.11.

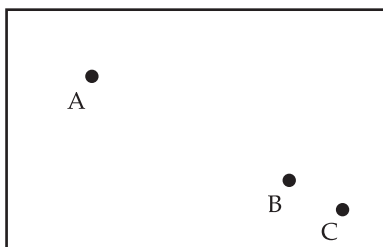


Figure 5.11.

- In Questions 1–6, suppose that the beach is the state of Colorado and the lifeguard stands are rain gauges. The rainfall measured at gauges A, B, and C is 1.25", 1.80", and 2.27", respectively. Show how a weighted average could be used to estimate Colorado's rainfall in each of Questions 1–6.
- Reconsider the map of Colorado in Figure 5.3 or Figure 5.4. Rank the gauges according to which you think should be given the greatest weight in making a rainfall estimate for the state. Explain your ranking.

Individual Work 5.2: Regional Rainfall in Colorado

FYI

The answers you get are likely to be imprecise. Ways to improve precision are considered in the lessons that follow. For now, you should concentrate on becoming comfortable with the models.

In this Individual Work, you make rainfall estimates for portions of Colorado that contain only two or three gauges. In doing so, you apply two models: One that uses a simple average and another that uses a weighted average.

Figure 5.12 is the table of rainfall measurements at eight gauges in Colorado that you first saw in Activity 5.1.

Gauge	1	2	3	4	5	6	7	8
Rainfall	2.11"	2.15"	1.21"	4.45"	2.67"	2.51"	3.11"	2.43"

Figure 5.12. Rainfall at eight sites in Colorado.

- Figure 5.13 shows northwest Colorado and two gauges in that region.

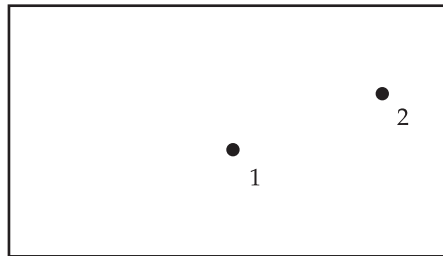


Figure 5.13.
Northwest Colorado.

- Use a simple average to estimate rainfall in the region.
 - Copy the drawing onto paper. Draw a line to divide the region into two parts so that the points in each part are closest to the corresponding gauge. Estimate the portion of the area in each part and use a weighted average to estimate the region's rainfall.
- Figure 5.14 shows northeast Colorado and two gauges in that region.

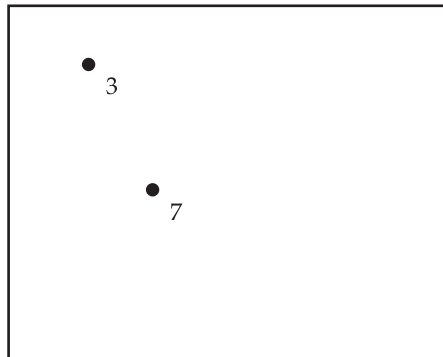


Figure 5.14.
Northeast Colorado.

- Use a simple average to estimate rainfall in the region.

b) Copy the drawing onto paper. Draw a line to divide the region into two parts so that the points in each part are closest to the corresponding gauge. Estimate the portion of the area in each part and use a weighted average to estimate the region's rainfall.

3. **Figure 5.15** shows central Colorado and two gauges in that region.

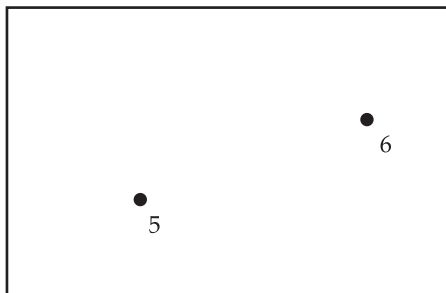


Figure 5.15.
Central Colorado.

- a) Use a simple average to estimate rainfall in the region.
- b) Copy the drawing onto paper. Draw a line to divide the region into two parts so that the points in each part are closest to the corresponding gauge. Estimate the portion of the area in each part and use a weighted average to estimate the region's rainfall.

4. **Figure 5.16** represents southern Colorado and two gauges in that region.

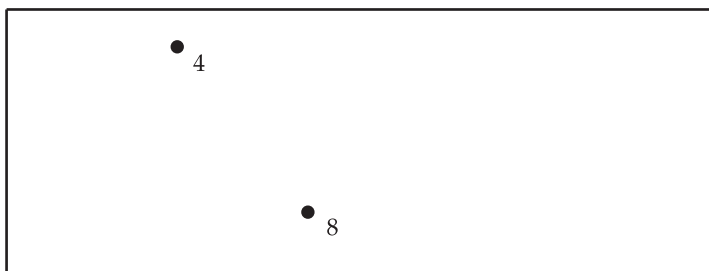


Figure 5.16.
Southern Colorado.

- a) Use a simple average to estimate rainfall in the region.
 - b) Copy the drawing onto paper. Draw a line to divide the region into two parts so that the points in each part are closest to the corresponding gauge. Estimate the portion of the area in each part and use a weighted average to estimate the region's rainfall.
5. Discuss at least two ways that the estimates you made with weighted averages in Questions 1–4 could be used to estimate rainfall for the entire state.
6. A region has three rain gauges as shown in **Figure 5.17**.
- a) Copy the figure onto paper. Draw boundaries to divide the region into parts so that each gauge is closest to the points in its part.

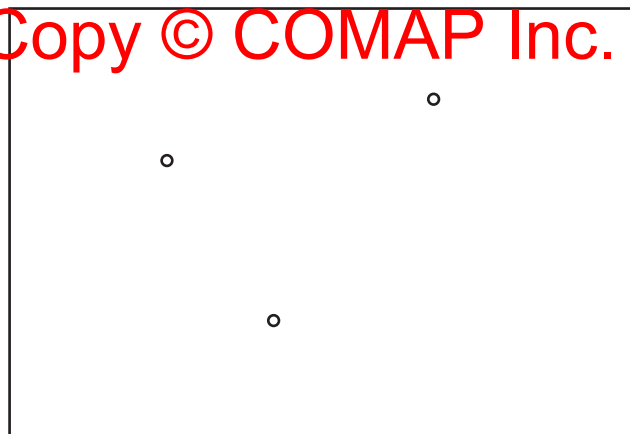


Figure 5.17.

- b) Estimate the portion of the total area of each part.
 - c) Compare the boundaries you drew in part (a) with those you drew in Questions 4–6 of Activity 5.2. How are the results alike? How do they differ?
7. Write a brief summary of what you have learned about the way rainfall is measured and the way rainfall for an area is estimated. Also discuss obstacles you need to overcome in order to make a good rainfall estimate for Colorado.



A New Kind of Weatherman

Brighton-Pittsford Post

May 13, 2009

Every day at 7 a.m., Scott Wagner walks outside of his home to measure precipitation.

He and more than 12,000 others are a part of a non-profit organization called the Community Collaborative Rain, Hail and Snow Network (CoCoRaHS).

CoCoRaHS started in 1998 at the Colorado Climate Center, after a flood that caused a lot of property damage had gone unrecorded by traditional weather observing methods.

Now the group has expanded to 39 states.

The information on its Website—www.cocorahs.org—is used by others, such as the National Weather Service, meteorologists, hydrologists, emergency managers, city utilities, insurance adjusters and farmers.

Wagner keeps a CoCoRaHS-certified rain gauge outside. It measures rainfall in 1/100-inch increments.



Dan Goldman/Messenger Post Media

Scott Wagner examines his rain gauge outside of his home in Corbett's Glen.