CHAPTER

7

Motion

THRILLS AND SPILLS IN MOTION

The context of this chapter involves high-risk stunts. In Course 1, Chapter 4 (*Animation*) you described the motion of an object as it moves along a line. In this chapter you develop methods for real motion such as that of toy cars, falling balls, or stunt performers. You use a motion detector to gather data and draw distance-versus-time graphs. From the data and graphs, you find speed (velocity) and acceleration. To apply what you learn, you design and model high-risk stunts and then use small-scale versions to test your models.

Touring stunt shows became popular in the 1970s. Among the most famous of the early stunt drivers is Evel Knievel. He and car drivers such as Joie Chitwood became famous by stunt jumping with motorcycles and cars. According to Knievel's Website, 52% of U.S. households watched his 1975 jump over 14 buses. That is an *ABC Wide World of Sports* record.

Of course, such stunts require careful planning or the results can be tragic. In 1997, Corey Scott died during a jump at the Orange Bowl. He drove his motorcycle off a ramp and into a net. He was supposed to grab the net when he hit it, but missed and fell 60 feet to his death.

Mathematical modeling helps make the stunts performed today more predictable and much better than those of a few decades ago. Robbie Maddison's New Year's Eve 2007 record motorcycle jump of 322 feet 7 inches is over twice Evel Knievel's longest.

In this chapter, you won't perform stunt jumps, but you will plan a scaled-down version of a jump for a toy car.

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LESSON ONE

Learning Your Lines

Key Concepts

- Interpreting graphs
- Average velocity
- Slope
- Linear equations
- Piecewise-defined graphs

Jack be Nimble, Jack be Quick

You have no doubt seen stunts in movies in which a car speeds down a street, just missing a truck at an intersection. For stunt drivers, trial-and-error alone is not a useful method. A mistake could cost them their lives. Careful mathematical modeling is essential to successful stunts.

Early in his career, Evel Knievel relied on "gut-level instincts" to design stunts. They did not always go as planned. In one show, he put a row of open crates of rattlesnakes between two ramps. He sped up one ramp, sailed over the snakes, but fell short of the landing ramp. He hit a crate, freeing the snakes. Unharmed, Evel sped off as the crowd quickly dispersed. In another failed stunt, he tried to jump over a motorcycle speeding toward him. His feet hit the handlebars, hurling him 15 feet into the air. He broke most of his ribs on landing.

Jeff Lattimore's stunt in Chitwood's Thrill Show was "The Leap for Life." In his stunt, Jeff stood atop an eight-foot stool. A car sped toward the stool, and Jeff jumped a moment before impact. The stool was snapped from under him, the car whizzed beneath, and he landed safely on the ground.

Near collisions, ramp-to-ramp jumps, and leaps over oncoming vehicles are staples of motorcycle and car thrill shows. In this chapter, you plan and, in some cases, execute small-scale versions of these stunts. In this lesson, you take the first steps toward developing the mathematical models that you need.

DISCUSSION/REFLECTION

1. Which of the three stunts described in the preparation reading, the car-truck, near-collision stunt, Knievel's ramp-to-ramp motorcycle jump, or Lattimore's "Leap for Life," do you think is easiest to design? Which is hardest to design? Why?

Activity 7.1: Coordinated Efforts—Stunt Design

In this activity, you begin to develop plans for modeling stunts like those in the preparation reading.

Questions 1–3 ask you to think about how *you* would design stunts. (Refer to the Preparation Reading for examples of each of these stunt types.)

1. Suppose that you are designing a two-vehicle near-collision stunt.

   a) What information would you include about the vehicles and about the stunt area?

   b) What kind of instructions would you give to the drivers?

2. Suppose that you are designing a ramp-to-ramp motorcycle jump.

   a) Draw a sketch of the set-up for the two ramps.

   b) What information would be useful in planning the stunt?

   c) How might you get this information?

3. "The Leap for Life" is a successful leap-over-a-vehicle stunt. Evel Knievel's attempt to leap over a motorcycle failed.

   a) What made "The Leap for Life" a success while Evel's jump was not?

   b) What things would you need to consider in designing a leap-over-an-oncoming-vehicle stunt?

The goal of this lesson is to design a two-vehicle near-collision stunt using toy cars. Look back at your answer to Question 1(b). In your instructions, you may have told the drivers how fast to drive. But toy cars do not respond to such instructions. Instead, you need to design your stunt based on the speeds of the toy cars when they are turned on. Before planning this stunt, you will find it helpful to study the motion of each car.

4. Imagine that a battery-operated toy car is moving along a straight line across the floor. How could you get information about how fast the car is going? How could you tell if the car's speed is constant?

FYI

So far, you've looked at three types of stunts. You will study each one more closely later in this chapter. This lesson's focus is on near-collision stunts. Lessons Two, Three, and Four treat leap-over-an-oncoming-vehicle stunts. In Lesson Five, you design and perform ramp-to-ramp jumps.

Activity 7.2: Recording the Motion

In this activity, you use a motion detector to collect data from people walking in front of a motion detector.

FYI

An ultrasonic motion detector sends out a sound beam. If an object is in the beam, the sound reflects off the object back to the detector. Since sound travels at a constant speed, the detector can use the time between sending the signal and receiving it back to calculate the distance to the object.

With a motion detector, you can gather distance-versus-time data from a moving object.

Some motion detectors, such as a TI-CBR, can be connected directly to a calculator. Others must be connected to another device, such as a TI-CBL, which is then connected to a calculator. **Figure 7.1** shows a set up with a CBL.

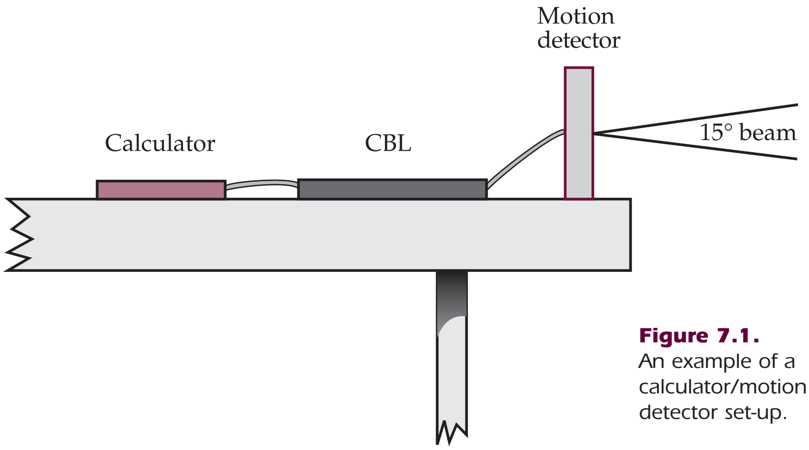

Figure 7.1. An example of a calculator/motion detector set-up.

TAKE NOTE

Motion detectors have a limited range. The beam from the detector in Figure 7.1 makes an angle of 15°. Unless your teacher tells you otherwise, assume that your detector can measure the distance to an object between 1.5 and 24 feet away.

To work with a motion detector, a calculator must run a program. This activity uses a program called HIKER.83P, but there are other programs that do the same thing. Your teacher will give you details about how to use the program and how to set up the equipment. Here is an example of how a detector and calculator program work together:

- A battery-powered toy car is placed in the motion detector's beam three feet away.
- The car is turned on and released when the detector begins recording.
- Two seconds later, the motion detector locates the car five feet away.
- Based on the readings from the motion detector, the calculator program plots two data points: (0, 3) and (2, 5). The times, 0 and 2, are stored in one list and the distances, 3 and 5, in another.

FYI

HIKER collects data every tenth of a second for six seconds. Some programs, such as CBL1, give you more control by letting you set the time interval, the number of readings, and the distance unit (feet or meters).

Walking

Your group should set up a motion detector as described in Handout H7.2 or as described by your teacher.

Members of your group should take turns walking in such a way that the calculator produces a graph. Keep in mind the length of time your program runs. For example, if your program runs for six seconds, you may want to have a someone count, "Go, 1, 2, 3, 4, 5, 6" during the data collection.

1. Select a member of your group to be the first walker. Watch as this walker walks. On Handout H7.3, describe what you see the walker do. Then make a sketch of the calculator's graph. Repeat the process for each group member.

Talking

2. Your group should select two graphs that you like, sketch them on large pieces of paper, and post them for the rest of the class to see.

3. After all graphs have been posted, look them over and sort them into groups. Explain why you put graphs in the same group.

IN THE NEWS

Aussie Breaks World Motorcycle Jump Record

Associated Press

January 2, 2008

Robbie Maddison broke the world motorcycle jumping record Monday night, soaring 322 feet, 7-1/2 inches at the Rio All-Suite Casino and Hotel.

Maddison broke the Guinness World Records-certified mark of 277-6 set by Trigger Gumm in 2005 in Australia, and the non-certified mark of 310-4 set by Ryan Capes in Kent, Washington, also in 2005.

During practice Sunday night, Maddison jumped 350 feet. It was unclear if that jump would be certified as the record.

On Monday night, Maddison landed in perfect control, sailing over a drawn-in football field and coming down on a large dirt landing area.

"I'd like to think I'm a calculating mathematician for sure."
—Robbie Maddison

Individual Work 7.1: Equations of Motion

In this Individual Work, you consider several situations involving motion detector data.

FYI

You might have first studied average velocity in Course 1, Chapter 4 (*Animation*).

The **average velocity** of an object between time 1 and time 2 is given by this ratio:

average velocity from time 1 to time 2

$$= \frac{\text{change in location}}{\text{change in time}}$$

$$= \frac{\text{distance 2} - \text{distance 1}}{\text{time 2} - \text{time 1}}.$$

Notice this is the same formula as the one used to find the slope of the line joining the points (time 1, distance 1) and (time 2, distance 2).

1. **Figure 7.2** displays the distance between you and a toy car moving away from you.

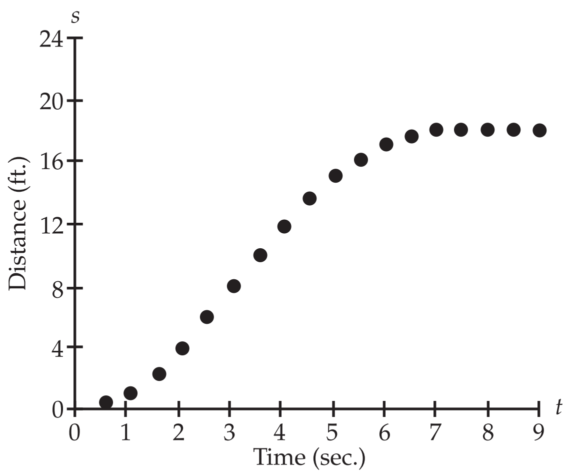

Figure 7.2. Distance-versus-time graph for a toy car.

a) What is the car's average velocity from $t = 0$ to $t = 9$? (Be sure to include units.)

b) What is average velocity from $t = 0$ to $t = 2$? What is it from $t = 2$ to $t = 4$? Is the car going faster during the first two seconds or the next two seconds? How can you tell from the graph?

c) Find a two-second interval over which the car appears to travel at constant velocity. What is this velocity? Can you find more than one such two-second interval?

d) Describe in words what happens to the velocity during the trip.

2. Scott started moving as soon as a motion detector began recording. At $t = 0$ he was five feet from the detector. At $t = 6$ he was 23 feet from it. He walked at constant velocity.

   a) What was Scott's velocity?

   b) Complete the table in **Figure 7.3**.

Elapsed time (sec.)	Scott's distance from the motion detector (ft.)
0	5
1	
2	
3	
4	
5	
6	23

Figure 7.3. Table recording Scott's motion.

   c) Draw a distance-versus-time graph for Scott's motion.

   d) Write a closed-form equation for Scott's distance from the detector in terms of elapsed time. Interpret the numbers in the equation in the context of Scott's walk.

   e) Use your equation from part (d) to find Scott's distance from the detector at $t = 2.5$ seconds?

3. Imagine that you have set up a motion detector to track the motion of a friend walking at constant velocity *toward* the detector.

   a) Draw a distance-versus-time graph for your friend's walk. Assume that you monitor the walk for six seconds.

   b) What is your friend's average velocity during the walk? Your answer should be negative. Why?

   c) Write a closed-form equation for the relationship that you graphed in part (a).

4. **Figure 7.4** is a time-lapse graph of the motion of a toy car over six seconds. Unlike the lines in distance-versus-time graphs, the solid line here shows the car's actual path. Sample times have been added to show when the car passed certain locations.

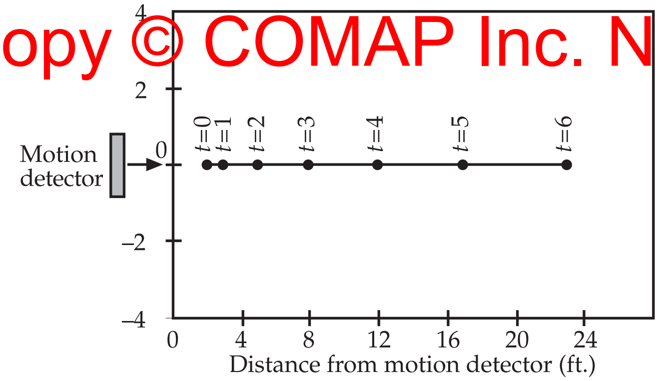

Figure 7.4. Time-lapse graph of the motion of a toy car.

a) Is the car moving at constant velocity? How can you tell?

b) What is the car's average velocity over the six-second trip? Over what one-second interval is the car's average velocity closest to its average velocity for the whole trip?

c) Draw a distance-versus-time graph for the car.

d) What type of a function describes the relationship between distance and time? How do you know?

FYI

If you have trouble answering part (d), recall from Chapter 6, *Growth*, what first or second differences tell you about a function.

5. The first graph in **Figure 7.5** was made when Carol walked in front of a motion detector. The program recorded time in seconds and distance in feet. Carol used TRACE to find the points in the other three graphs.

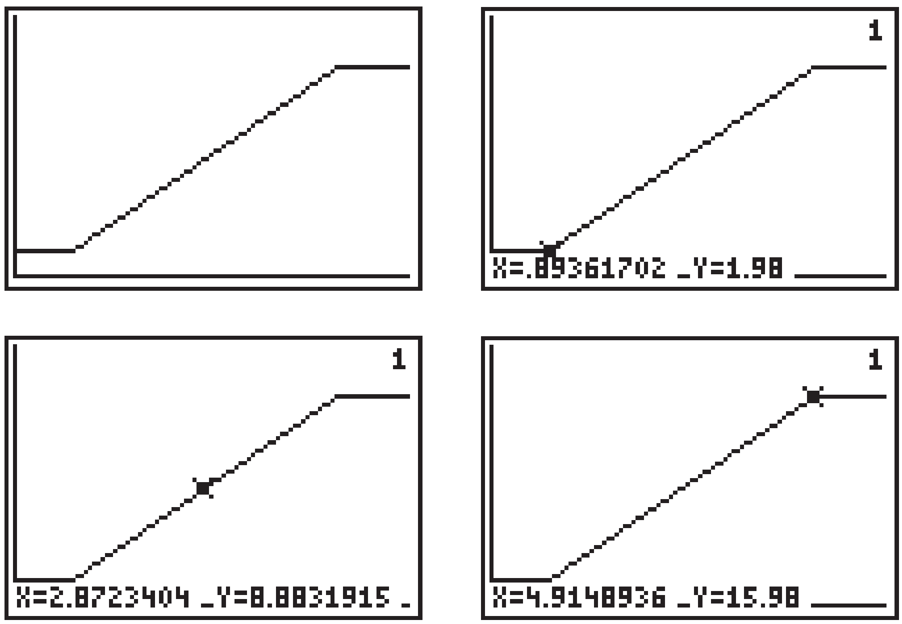

Figure 7.5. Graph of Carol's walk.

a) How far was Carol from the motion detector when the program began?

b) For how much time did Carol stand still before moving?

c) For how much time did Carol walk?

d) How far did she walk?

e) How fast did she walk?

f) Write an equation for the relationship between distance, d, and time, t, during the period that Carol moved. State the domain.

g) Does the d-intercept in your equation in part (f) have meaning in this context? What about the slope? Explain.

6. Select two of the walks your group made in Activity 7.2. For each walk, describe the average velocity over some interval.

7. In previous chapters you fit a line to data and then used residuals to decide if your linear equation was a good model. What are residuals? How are they found? What does a residual plot for a "good-fitting" model look like?

8. Look back at Question 5. **Figure 7.6** shows an attempt by one of Carol's friends to use a line to describe the distance-versus-time data.

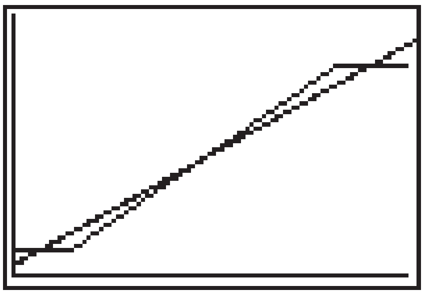

Figure 7.6.
An attempt to fit a line to Carol's data.

a) Is the line a good fit?

b) How well does the line's slope estimate Carol's velocity?

c) Make a rough sketch of a residual plot. Explain how the plot confirms your answer in part (a).

Activity 7.3: Walking the Walk

In this activity, you match verbal descriptions of walking motions with graphs.

Set up a motion detector and get ready to conduct experiments!

Part I: Graph to Walk

1. Walk in such a way that the distance-versus-time graph is a straight line with positive slope. When you are happy with the graph, copy the data to lists that are not used by the program. Then link calculators with the other members of your group and share the data.

   a) Sketch your graph. Label the distance between the walker and the detector when the detector began recording. Also label the distance between the walker and motion detector when the detector stopped recording.

   b) Find a linear model for the relationship between distance and time. Then make a residual plot. Does your model describe the data well? Explain.

   c) If your model in part (b) does not describe the data well, either adjust the model or gather new data. Then repeat part (b). Was your walker able to walk a good line?

   d) What do the slope and y-intercept of your model in part (c) tell you about the walk?

MODELING NOTE

Modelers sometimes adjust a model by removing outliers and refitting. Removing outliers is reasonable when, for example, they are due to data collection errors such as a walker moving outside a detector's beam.

2. a) Plan four new graphs with different shapes. Sketch each graph on a separate set of axes on the left side of Handout H7.4. Add scales and labels to the axes.

   b Write instructions on how to walk in order to make each graph in part (a). Include details such as where to start, which direction to go, and how fast to walk.

   c) Members of your group should take turns using the instructions for each graph in part (a). Use a motion detector to record data from each walk. Sketch the actual graph to the right of the planned graph. Again, add scales and labels.

   d) Describe how the planned and actual graphs differ and give reasons for the differences.

Part II: Walk to Graph

3. a) Plan four more walks. Write instructions on how you want a walker to move. Include things like where to start, which direction to go, and how fast to walk. Record the instruction on a second copy of Handout H7.4.

   b) Predict what the graphs will look like. Sketch each predicted graph on a separate set of axes on the left side of Handout H7.4. Add scales and labels.

   c) Try the walks. Follow your instructions and record the actual graph on the same axes as your sketch of the predicted graph. Label which graph is the predicted one and which graph is the actual one.

   d) Describe how the planned and actual graphs differ and give reasons for the differences.

IN THE NEWS

Skateboarder Clears Great Wall of China

July 9, 2005

USA Today

Daredevil skateboarder Danny Way rolled down a massive ramp at nearly 50 mph and jumped across the Great Wall of China on Saturday, becoming the first person to clear the wall without motorized aid.

Way botched the landing on his first attempt but then successfully completed the jump across the 61-foot gap four times, adding 360 degree spins on his last three tries.

Way made the jump on an adaptation of the so-called mega ramp that he helped create near his home in the Southern California desert. He set a skateboard jump world record for distance (79 feet) on a mega ramp at last summer's X Games, and in 2003 set the height record of 23 feet at the desert ramp.

"Math is becoming a very serious factor in what I'm doing these days … I never would have thought that math equations would have been the deciding factor in my career."

—Danny Way

Individual Work 7.2: Interpreting Motion Graphs

In this Individual Work, you interpret distance-versus-time graphs.

1. **Figures 7.7(a)–(e)** are graphs of walks. In each, the *y*-axis represents distance (feet) from detector to walker and the *x*-axis represents time (seconds). For each graph, describe the motion.

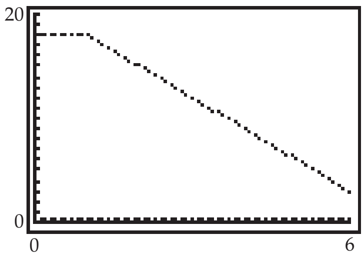

Figure 7.7(a).

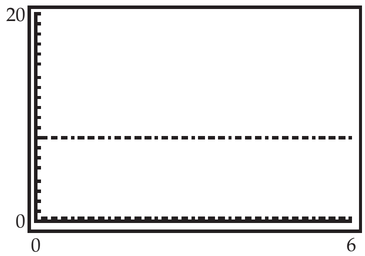

Figure 7.7(b).

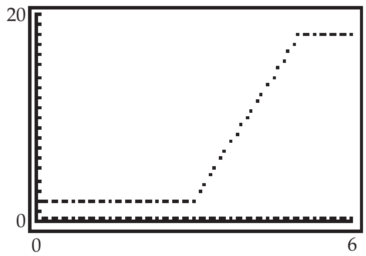

Figure 7.7(c).

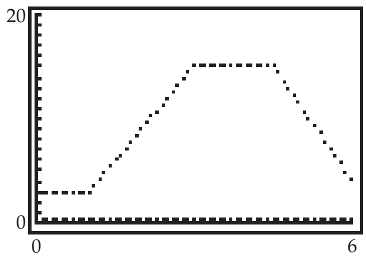

Figure 7.7(d).

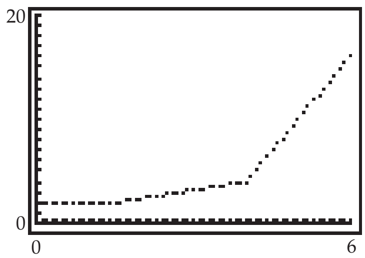

Figure 7.7(e).

2. **Figures 7.8 and 7.9** are graphs of walks. Assuming the scales are the same, in which is the walker moving faster? Explain.

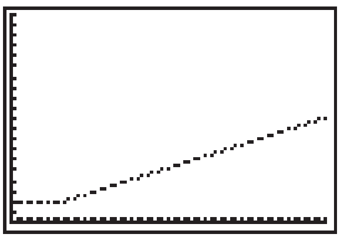

Figure 7.8.

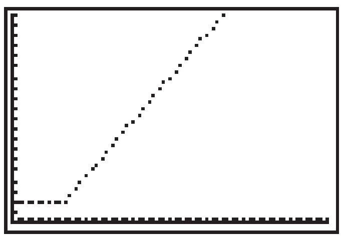

Figure 7.9.

3. **Figure 7.10** is a distance-versus-time graph for the "walk-a-positive-slope" data in Question 1 of Activity 7.3.

   a) What is the distance between walker and detector when recording began?

   b) How far did the walker walk during the time that the detector recorded?

   c) What was the walker's average velocity over the entire time that the detector recorded?

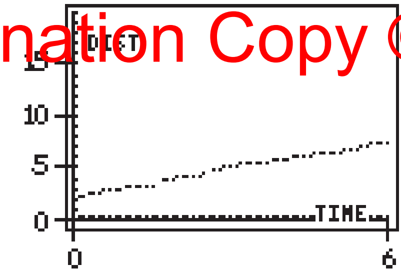

Figure 7.10.
"Walk-a-positive-slope" data.

d) Is the velocity you found in part (c) a good measure of the walker's velocity at each instant of the walk? Explain.

e) Find a linear model for the relationship between distance, *d*, and time, *t*.

f) What is the meaning of the *d*-intercept in the equation you found in part (e)? What is the meaning of the slope?

FYI

Questions 4 and 5 help you prepare for a near-collision stunt in the next activity.

4. A toy car moves along a straight line in front of a motion detector and its distance-versus-time graph is a line.

a) What does this tell you about the toy car's velocity?

b) How could you find the toy car's velocity?

c) Suppose that the distance-versus-time graph has a negative slope. What does that tell you about the car's motion?

5. A distance-versus-time graph for a toy car moving along a straight line in front of a motion detector is curved. What does this tell you about the car's velocity? Explain.

6. Henry and George each felt that he had walked the better line. Data from their walks are in **Figure 7.11**.

a) Make a distance-versus-time scatter plot for each walk.

b) Fit a least squares line to each walker's data. Sketch the lines on the plots you made in part (a).

c) Assume that your equations in part (b) are good models for the walks. According to these models, about how fast was Henry walking? What about George? (Remember to include units.)

d) Make a residual plot for each model. Based on these plots, who do you think walked a better line? Explain.

Elapsed time (sec.)	Henry's distance from the motion detector (ft.)	George's distance from the motion detector (ft.)
0.5	6.0	6.0
1.0	7.5	7.0
1.5	7.9	8.1
2.0	8.8	9.2
2.5	9.5	10.6
3.0	11.0	12.2
3.5	12.5	13.2
4.0	13.5	15.0
4.5	15.5	16.8
5.0	16.5	18.6
5.5	16.8	20.5
6.0	17.4	22.4

Figure 7.11. Data from Henry and George's walks.

Activity 7.4: I Miss You Nearly

In this activity, you plan and stage a near-miss stunt with toy cars.

Two toy cars are moving toward each other along straight roads that intersect.

1. **The Roadway**. Decide on a layout for the intersection: right angles or some other angles. Make a rough sketch of the intersecting roads.
2. **The Cars**. You need two battery-operated toy cars (or trucks or robots).
   a) Describe the cars that you will use.
   b) Use the motion detector to gather data about how your cars move. Write equations for the distance-versus-time data collected by the detector. Defend your equation.
   c) Based on your work in part (b), estimate each car's velocity?
3. **The Stunt**. Describe how you will stage this stunt.
   - Where on the roadway will you put the cars?
   - Will you start the cars at the same time? If not, how will you stagger the starts?

Remember, this stunt should be exciting—the vehicles should pass as closely as possible without colliding. Include in your description mathematics that supports your design.

4. **The Proof**. Perform the stunt. Did it go as planned? If not, why?

Individual Work 7.3: Moving Cars

In this Individual Work, you reconsider some of the important things you have learned about motion.

1. How would you describe distance-versus-time graphs for battery-powered toy cars? What does this tell you about the velocities of these cars?

2. A battery-powered toy car moves along a straight highway until its batteries die.

   a) Draw a sketch of how you think its distance-versus-time graph looks. Assume that distance is measured from the car's starting position. Be sure to label units on the axes.

   b) Describe what your graph tells about the car's velocity during its trip.

3. A battery-powered toy car moving in front of a motion detector produced the graph in **Figure 7.12**.

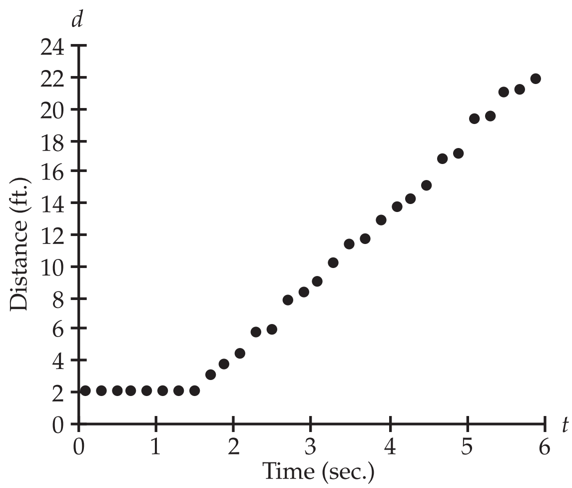

Figure 7.12.
Distance-versus-time graph for a toy car.

   a) About how far was the car from the detector when recording began? How far was the car from the detector when recording stopped?

   b) After recording began, how much time passed before the car began to move?

   c) Jason wanted to find a model for the car's motion and fit a least squares line to the data in Figure 7.12. A graph of his model is in **Figure 7.13**. According to Jason's model, what was the car's velocity? How did you get your answer?

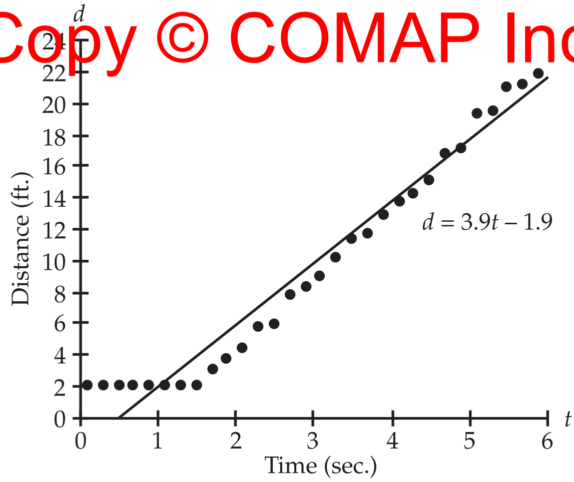

Figure 7.13. Least squares line superimposed on scatter plot.

d) Do you think your estimate of the car's velocity in part (c) is too high, too low, or about right? Explain.

e) Do you think Jason's model does a good job of describing the motion? If not, why?

4. **Figure 7.14** is a distance-versus-time graph for a wind-up toy car powered by a spring. Three traces show times, x, in seconds and distances, y, in feet.

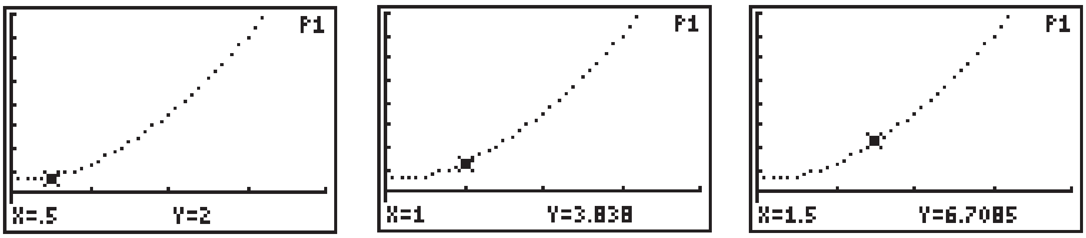

Figure 7.14. Distance-versus-time graph for wind-up car.

a) Is the car speeding up, slowing down, or moving at constant velocity? How can you tell from the first graph?

b) Find the average velocity for the first half-second. What are the average velocities for the second and third half-seconds? Based on these average velocities, is the car speeding up, slowing down, or moving at a constant rate?

c) Explain how to use TRACE to estimate velocity at a specific time.