about how our world works can and should be an enjoyable and rewarding experience." The rest of the book, comprising seven units, supports that statement.

Unit 1, called "Gridville," takes its title from the name of an imaginary town whose thirty-two streets form a square grid, and whose citizens live in twelve widely scattered houses. COMAP's writers direct their readers to work in groups and to attack the problem of picking the best location for a fire station that the citizens of Gridville have decided to build. This is a real problem -- as real as the next emergency vehicle you will see on your own street -- and it embodies some important ideas. One of these is the principle that mathematical reasoning can be used to meet social needs. Another is the fact that "best" is a slippery word. Should the "best location" for the fire station be taken to mean the location that will minimize the average time for answering a call? Should it be the location that will minimize the median time? Should it be the location that will minimize the worst-case time? Whatever criterion a group of students may choose, the students have to be prepared to explain it and justify it. COMAP's writers tell the students to "Determine the best location for Gridville's fire station, and write a persuasive argument defending your choice" while paying attention to these guidelines:

> *1. Begin your argument by answering the questions, "What are important factors to consider in deciding the best location?" and "What does 'best' mean?"*
>
> *2. You are encouraged to use charts, diagrams, tables, graphs, equations, calculations and logical reasoning in making your decision.*
>
> *3. Clearly state your choice of best location. Your written summary should include the arguments and mathematics that support your decision. The summary should also explain how your charts, diagrams, tables, graphs, equations, calculations, and logical reasoning relate to the factors you considered and led your group to your choice.*

So merely throwing numbers and graphs around, in the way that is favored by writers of consumer-product advertisements, won't suffice here. The numbers and graphs must have clear meanings, and they must be relevant to the argument that the students are erecting. Furthermore, the students learn to distinguish matters of mathematical analysis from matters of policy or preference, and they learn that having a good idea is not enough. If an idea cannot be presented effectively, its value may never be realized.

The fire-station problem introduces the students to piece-wise functions which represent, for different locations of the fire station, the distances that a fire truck will have to travel in responding to a call. Even when the streets form a perfect grid of identical blocks, variations can arise. A problem on page 12 requires the students to deal with blocks that are elongated rectangles (like the blocks in New York City) rather than squares. And in a problem on page 88, one of the structures that must be protected from fire is a six-unit apartment building while the rest of the structures are single-family dwellings. Should the same importance be assigned to a single-family house as to the apartment building, or should the apartment building be equivalent to several houses?

I have been mildly annoyed by the COMAP writers' coining of two odd terms. They use the phrase "firetruck distance" to signify the length of the shortest path that connects two points while following the streets on a grid, and they use "helicopter distance" to signify the length of the path that connects two points directly, irrespective of where the streets are. Both of those measurements, however, already have well established names -- "Manhattan distance" for the former, and "Euclidean distance" for the latter.

On the other hand, I have been much pleased to find a problem which requires students to plot some points that are equidistant from a given line and an existing point. This problem doesn't look special, but it introduces a concept that the students may use later, when they study analytic geometry and encounter the parabola.

Playing Games

Unit 2, "Strategies," deals with some uses of game theory for analyzing strategic situations. COMAP's writers define a strategic situation as one in which "you must make a decision that affects one or more other parties who must also make decisions that affect you," and then they illustrate what they mean by furnishing some scenarios. The first scenario comes from a civil war that was fought in Zaire in 1997: Rebels advance on the capital; their leader, Laurent Kabila, demands the resignation of Zaire's president, Mobutu Sese Seko; the president refuses; and other parties, led by Nelson Mandela, try to induce Kabila and Mobutu to meet and resolve the situation peacefully. The second scenario is the common two-player game in which each player "throws" his fist with some number of fingers extended, and the winner is decided by whether the number of extended fingers thrown by both players is odd or even. The third scenario is the Cuban Missile Crisis, in which "the interests of the two players were not completely opposed."

In presenting these scenarios, COMAP's writers develop the concepts of strategy and payoff, lead the student through the building of payoff matrices, and point out the difference between a zero-sum game and a non-zero-sum game. A little later in the unit, they elaborate on these basic ideas in engaging ways: For example, they revisit the civil war in Zaire and provide some new information that could well change the anticipated payoffs. This may be the most gripping problem that I ever have seen in a textbook. It deals with a real war that took place within the students' own lifetimes, and it enables the students to experience (albeit on a small scale) a kind of analysis that often is invoked during the evaluation and application of military intelligence.

Still later in the unit, the writers painlessly expose the students to the tragedy of the commons and the prisoner's dilemma (though the writers don't call them by their formal names). Both of these are classic frameworks for scenarios in which individuals compete for material resources or for other payoffs. Here is the problem in which the students encounter the tragedy of the commons:

> *A week into a hot, dry spell, Droughtsville has announced that there is a severe water shortage and all citizens should practice water-conserving measures. But you have been planning to clean the house, wash cars, and do several other water-intensive activities. Will it matter that much if you go ahead with your plans, or should you conserve? What if you're a water hog and so is the rest of the community?*

The writers continually make sure that students recognize the limitations of their analytical tools. A problem on page 170, for example, deals with a driver who has scraped a parked car and now must decide whether he should report the incident or should sneak away. COMAP's writers tell that he will have to pay $500 in damages if he reports the accident, but he will incur costs of $1,000 if he sneaks away and later is identified to the police by a witness. These data, however, omit consideration of the driver's emotions. "It can be difficult to attach a dollar value to feelings," the COMAP writers say, "but estimate a dollar value on the guilty feelings a person would experience by leaving the scene. Does this change your analysis of the game?" Without pushing their point too far, the writers have hinted at what happens whenever personal preferences and feelings enter into decisions.

A Rich, Readable Presentation

Unit 3, called "Hidden Connections," addresses optimization, and it includes several items that COMAP's writers call "traveling salesperson problems." These are variants of the famous traveling salesman problem. (In adopting the contrived phrase "traveling salesperson," the writers have let political correctness override familiar, frugal terminology. However, they have committed this sort of folly only rarely.) Unit 3 also addresses the use of optimization in the allocating of resources, the resolution of conflicts, and the coloring of graphs. The common language that unites all of these is graph theory, one of the mainstays of applied math.

The writers give a rich, readable presentation, and they lard it with some surprisingly advanced material. An example is their exposition of Prim's algorithm, a technique that often is covered in college-level courses on Internet routing. COMAP's writers present it clearly, without fanfare, and they incidentally expose students to the common practice of identifying a mathematical entity (such as an algorithm, a theorem or a conjecture) by the name of the person who created it.

Unit 4 ("The Right Stuff") and Unit 5 ("Proximity") address geometric problems. Most of the examples are concrete and moderately realistic, but (in Unit 5) the discussion of the measurement of rainfall in Colorado is weak. Students are supposed to build Voronoi diagrams around the locations of rain gauges, and they are supposed to use these diagrams for identifying the gauges that most reliably indicate the supplies of rainwater that are available in different parts of the state. This approach, however, is too simple to give meaningful results. A Voronoi region is a purely geometric construction, so it doesn't take account of topography -- yet topography is what defines the watersheds that deliver rainwater to Colorado's various reservoirs. (I don't mean to suggest that the concept of a Voronoi region is unimportant. While it is inappropriate in the context COMAP's writers have chosen, it has many valid applications. Students someday may learn how it is used in the compressing of digital movies and in the preserving of faint signals that reach Earth from exploration vehicles in deep space.)

In Unit 6, "Growth," the students manipulate information that describes how things increase or decrease over time. The first few cases that the writers present deal with medicines -- specifically with the problem of maintaining effective and safe levels of medicines in the body. This material will be a real eye-opener for some students: Medication isn't merely a matter of administering a pill or an injection; it is a matter of balancing the delivery of the medicine against the metabolic degradation and elimination of the medicine, so that the medicine is present at a therapeutic concentration throughout the day.

As the COMAP writers continue their exposition of growth, they hark back to a scenario that they introduced in *Course 1* -- the expansion of a moose population in a state park. They use this for reviewing the fundamental difference between additive growth and multiplicative growth; then they offer problems in which students employ equations and graphs to describe the growth of other animal populations, the propagation of pathogenic microbes, and the accumulation of money in interest-bearing bank accounts.

The last unit in the book -- Unit 7, "Motion," -- deals with equations of motion, as applied to such things as stunt driving, the behavior of toy cars, and the use of motion-detection instruments. A significant portion of the material is oriented toward demonstrations and hands-on activities. Perhaps this reflects the COMAP writers' awareness that students will tackle Unit 7 when the school year is winding down and the students' minds are likely to wander. Even so, the material is good, and it includes an exercise in which students must fit curves to different sets of experimental data and must plot the residuals. Here is a fine example of how the writers teach that different mathematical endeavors can support each other: You use your knowledge of statistics even when you're dealing with velocity and acceleration.

As in *Course 1,* each unit of *Course 2* includes a few questions or problems that reach far beyond the unit's formal scope. (An example is problem 6 on page 372, which requires the student to evaluate an iterative geometric algorithm for locating Voronoi centers.) Such problems serve at least two purposes. First, they remind students that, in the real world, mathematical tasks don't come with all of the necessary information neatly packaged. Second, they provide advanced students with extra challenges and with opportunities to gain great satisfaction by meeting those challenges.

Course 2 is an outstanding textbook in every respect -- in its visual features, in its didactic approaches, and in its mathematical content. If you teach math to high-school students, please give this book the most serious consideration.

Notes

1. See "My Answer Is 'Yes' " in *The Textbook Letter,* Vol. 11, No. 5. [return to text]

2. For example, *Glencoe Pre-Algebra.* To read my reviews of the 1997, the 1999, and the 2001 versions of that book, see *TTL,* Vol. 10, No. 2; *TTL,* Vol. 10, No. 4; and *TTL,* Vol. 11, No. 4. [return to text]

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