MATHEMATICS: MODELING OUR WORLD
Mathematics: Modeling Our World is an integrated core curriculum for high school that is based on the premise that students learn best when they are actively involved in the process. In this program students do not first learn mathematics and then apply what they’ve learned. Rather, important questions about the real world come first. Students analyze situations and apply the mathematical concepts needed to solve problems. Contextual questions drive the mathematics. In each unit, students build, test, and present models that describe a real-world situation or problem, such as deciding where to build a fire station. Mathematical modeling is a central focus throughout the curriculum.

Each course covers the mathematical content found in the NCTM Standards. Each of the first three courses of Mathematics: Modeling Our World contains seven or eight units. Units are divided into four to seven lessons; each may take several days to complete. Each lesson contains a Lesson Opener, which provides the context for the lesson; Activities, which students work on in pairs or small groups using hands-on mathematical investigation; and Individual Work, items that review, reinforce, extend, practice, and foreshadow concepts developed in the lesson.

Course 4 is comprised of eight chapters and is intended to be a bridge between Courses 1, 2, and 3 and collegiate mathematics. The student text contains eight chapters divided into three to six lessons. Each lesson contains an activity designed for group work, expository readings, and exercises. Each chapter ends with a set of review exercises.

Assessment is an integral part of Mathematics: Modeling Our World. Both Activities and Individual Work offer embedded opportunities to assess student progress. The Teacher’s Resources provide Assessment Problems for use with each unit/chapter.

The units/chapters of Mathematics: Modeling Our World begin with a real situation or problem to be solved during the course of the unit. In Courses 1, 2, and 3, a short video segment may be used to introduce the theme or problem. Students use both graphing calculators and computers extensively throughout the curriculum to assist in carrying out computations of real problems and to enhance concept development. While it is strongly recommended that students use computers with this curriculum, material is provided to teach the lessons without computers as well. However, use of the graphing calculator is essential throughout the program.

Student materials for Mathematics: Modeling Our World are available in four hardcover texts, one each for each course. Teachers materials include, for Courses 1, 2, and 3, an Annotated Teacher’s Edition, a Solutions Manual, and Teacher’s Resources that includes additional teaching suggestions, background readings, reproducible handouts, assessment problems, supplemental activities, and transparencies. Course 4 has all the teacher material in the Teacher’s Resource Binder, including the Solutions Manual. Other materials include a video with segments for each unit and a CD-ROM with calculator and computer programs written specifically for Mathematics: Modeling Our World.
Dr. Sol Garfunkel received his Ph.D. in mathematics from the University of Wisconsin in 1967. He was a member of the mathematics faculty at Cornell University and the University of Connecticut at Storrs for 11 years. He has dedicated the last 20+ years to research and development efforts in mathematics education. Since its inception in 1980, he has been the Executive Director of COMAP in Bedford, Massachusetts.

Dr. Garfunkel has served as Project Director for a wide array of projects, including: the Undergraduate Mathematics and Its Applications and the High School Mathematics and Its Applications Projects, funded by the National Science Foundation. He directed three telecourse projects, including Against All Odds: Inside Statistics, and In the Simplest Terms: College Algebra for the Annenberg/CPB Project. He was the Project Director and host for the series For All Practical Purposes: Introduction to Contemporary Mathematics.

Developmental history and philosophy

The work that COMAP (Consortium for Mathematics and its Applications) has done at all educational levels has been based on the idea that we wanted to teach mathematics through contemporary applications and modeling. We remember the time when applications were always an afterthought; one taught some body of mathematical technique, algorithms and facts, and then, if we were lucky, looked at an application. The analogy I like to use is that it’s like saying to someone, “Alright, the first two years, you’re going to learn how to hammer a nail. The next three years, you’re going to learn how to saw a straight cut. If you stick with it long enough, somewhere after graduate school, there’s going to be this thing called a house that you’re going to learn how to build.”

We felt that this sort of delayed gratification works for a very, very small number of students. Our interest in math education has always been not the education of mathematicians, per se—although it’s always nice when that happens—but the mathematical education of the broadest possible population. Data that was accurate 10 years ago said that the half-life of students in the math class from 9th grade on to the Ph.D. is one year. For instance, if you look at the number of students who take 9th-grade mathematics, then the number of students who take 10th-grade mathematics is approximately half of that number, and 11th grade is half of the 10th-grade number, etc. So we lose enormous numbers of students, in part because they don’t see the usefulness of the subject.

Therefore, we felt that mathematics has increasingly useful applications to more social issues and economic things, things in people’s normal lives, as opposed to the physics- and engineering-type things that we think about. It’s easier today to show how mathematics is used in a way that students might actually care about, and to involve them in the process. We didn’t want just: “Here’s a body of mathematics, and now here’s an application.” The student knows, “Well, if I just did section 3.12 in the book and someone gives me a problem, then I’m surely going to have to use the mathematics they just taught me in section 3.12.” The real world is filled with problems that come ugly and don’t tell you what mathematics to use. So that’s been a guiding philosophy behind our curriculum development.

For years, COMAP produced supplemental materials that teachers could use to find an interesting application here or an interesting model there, picking and choosing the ones that would interest their students. That world changed dramatically when the Standards came out in ’89 and when NSF started to fund the curriculum development projects. In ‘92, when we proposed and got funded for the ARISE project, it was time to put our money where our mouths had been. It had been fun to do these neat little supplemental things, but teachers who really wanted to teach from a curriculum were saying, “What do I do on Monday? What do I do on Tuesday?” We had to think about how we would present a whole body of mathematics—in this case, four years’ worth of mathematics—through this approach.

We had to spend a lot of time thinking about what this would mean. One of the things that we decided early on was that we wanted to have the mathematics “arise” out of contexts. The “ARISE” name—Applications Reform In Secondary Education—was chosen both because it sounded good and also because of the
philosophy that the mathematics should come naturally out of the kinds of contextual problems that the students might care about and that they'd want to solve. The nice thing about that kind of an approach is the fact that it is truly integrated. In a real-world problem—for instance, if you want to figure out how you make things look like they're moving when they're not moving (in some kind of an animation, for example)—it isn't obvious what mathematics you need. You might need a piece of algebra, a piece of geometry, some trig—you may need a whole bunch of things to think about that problem. So, by doing it from a contextual point of view you make the integration of topics much more natural. Therefore, if you look at our first three books, Courses 1, 2, and 3, you see that the chapter titles are contextual titles, not mathematical titles. Of course, we do let the teacher know what math they're going to teach, but from a contextual point of view.

The modeling approach

Philosophically, it wasn't simply to present students with solutions to interesting and complicated contextual problems, but it was the modeling process that we really wanted to get across. The title of the books is Mathematics: Modeling Our World. What we wanted to do—and what I hope we achieved—is to make students understand and participate in the process of modeling. We didn't want to simply show them models; we wanted to make them modelers.

In the program, we take some time to look at a contextual problem, figure out some mathematical approach to the problem, see where that takes us, and if we're not happy with it on the first approximation, we come back and cycle through it again. Because in the real world, that's what you do. You have to think: Does mathematics make this problem accessible? Is it a problem that, in fact, you can use mathematics to analyze? And if so, what mathematics? What simplifications might you have to make of the problem in order to apply some mathematics that you actually understand to the problem? The program pushes students to look at a problem, try to make some assumptions about it, get some sort of a mathematical framework that tells you something about the problem, play with the mathematics, and then go back to the original problem. You may find out that that doesn't really tell you everything you wanted to know. So you think about what else you can do: maybe you can complicate the model. Sometimes you have to come back through it several times. The hope is that, somewhere in the middle of taking these courses, students also become problem-solvers, in the sense that they're willing to be active and take a chance at an approach, see what it tells them, and if they're not happy, come back and do it again until they can refine the process.

We care deeply about developing conceptual understanding. Once we have a mathematical model for helping to tell us something about the problem that we started with, we spend time there. We analyze, we solve equations, and we simplify. In that, there's quite a bit of skills practice. We attempted to make that balance, and set it at a point that we thought was fair and reasonable. Is it as much practice as there's been historically? Of course not. There are not 50 examples for teachers to say, "Do all the odd problems." But we worked pretty hard to have sufficient skills check and practice problems. It was a constant theme in our discussions during the writing.

I think the real strength of the program is in the modeling process and all the things that go with it: using the appropriate technology, and having kids working in small groups and working with other students. The program is set up to have group activities involved, although there's quite a bit of individual work. The people who
have chosen to use the program have, in part, chosen to use it because they feel that the applications and modeling that we’ve done are the strength of the program.

**Using appropriate technology**

If you’re going to do the modeling problems and look at real problems, there’s going to be a lot of data and it’s going to be messy. You’re going to need the technology to make these problems something that you can actually work with, and not have to oversimplify them. The technology helps you look at serious problems in several ways. In a trivial way, it helps you deal with masses of data; it helps you look at harder and more complicated problems and saves you the trouble of the computation. It also, of course, enables you to look at the data, by helping you make a picture. Therefore, we make the assumption that every student has a graphing calculator at home as well as in school. There are some schools who have worked out rental agreements, where they get the calculators from TI (Texas Instruments) or some other company, and they do a lending-library kind of an arrangement for students. We also assume that, in the building, available to the students in some kind of timeframe, there is a computer that they can put spreadsheets on and where they can look at some geometry utility programs like Geometer’s Sketchpad, to experiment with as a lab tool. We use spreadsheets on the PCs rather than on the calculator just because they’re easier to look at on the PCs, and the same for the geometry utility programs. In addition, we use CBLs (Calculator-Based Laboratories) and CBRs (Calculator-Based Rangers) hooked up either to the computer or to the graphing calculator, so that they can do motion experiments or experiments with frequency where they get real data in real time.

**Introducing the problem context**

One of the prices that you pay for the approach that we’ve taken is sort of the Mozart thing—too many notes. When you open up *Mathematics: Modeling Our World*, you see a lot of words. If you’re going to spend time in a context in a serious way, then you’ve got to spend some time really learning about the context. We tried to help with the setup in a variety of ways, but nevertheless, it does take a little more time for the student to read more about the application, about the context. We’re not unhappy about that. People talk about reading and writing across the curriculum, and we feel that you can’t do that if you have a textbook that’s simply equations and algorithms with boxes around them. But it’s not typical; people look at our program and wonder why there’s so much writing and reading involved. On the one hand, it’s a motivational thing: students will care about the mathematics because they see that house being built. On the other hand, there’s the subtext that mathematics is not separate from all other areas of human endeavor.

By the nature of our grant, we were able to create video material, so that, for every chapter in the book, for every major context, there is a 10-minute video that introduces the contextual problem.

**Choices about mathematical topics**

We looked at various strands and threads of mathematical ideas through the
curriculum, like algebra, the discrete topics, and the statistical and data analysis topics, and tried to be sure that we covered them in a reasonable amount and in a reasonable order. We visited these ideas often enough so that students got comfortable with them and understood them.

When you look at Mathematics: Modeling Our World, you can find the basic algebraic skills, the Algebra I and Algebra II, and a fair amount of trig and data analysis. There are probably more discrete topics in our program than people normally see in a course. For the first three years, there may be a little less trigonometry than traditionally would be done. Of course, the organization is different, and the way in which it's developed is different. But there's a comfort level as far as the basic content of high school mathematics, once you're used to looking at the pedagogy and reading the words and so on.

We took a very broad view of geometry, so that it wasn't simply the first five books of Euclid. While there certainly are units that are basically Euclidean geometry, there are a number of units that deal with things that are geometric in the broad sense, but that may not be thought of by the typical 10th-grade high school teacher, like graph theory models. There are a lot of geometric ideas. We're always graphing. We're always looking at different representations of functions and so on, but you don't see that sort of standard historical development from the first principles in Euclid. Once you get away from that particular form of organization, there really are a lot of choices to be made and more degrees of freedom than you'd think there are.

Our fourth course looks like a precalculus course, although it's certainly got an enormous amount of applications. That is the one concession we made: in our Course 4, we organized the topics along mathematical topic lines, with a heavy emphasis on applications and modeling, rather than the organization being along contextual lines. Our feeling there was that we have to prepare students to go on into calculus and to read mathematical texts as they are typically written. They are not reformed at the college level, and so we have to show more of the mathematical structure and make that more apparent to students who are going to go on in mathematics and science.

**Algebra**

Though it's an integrated course, Course 1 spends a lot more time in algebra than in other topics. It won't be unfamiliar to an Algebra I teacher. We begin with the development of linear functions, which, of course, is a natural place to begin. It's the function concept and the concept of linearity that leads. Within that, we have equations for straight lines; we're solving linear equations and we look at graphical representations, as well as symbolic representations, as well as tabular representations, and so on. We start with looking at phenomena that can be modeled by linear functions, and looking at some of the data. We look at how close we may be to a linear function if we look at things in the real world, where things might be a little off, because things aren't perfect.

Then we move both to exponential functions and to quadratics. Once a student sees that not every function is linear, that functions may have funny shapes or move in different ways, and that these shapes can be described by other functions, then an algebra of those functions makes sense. That's what you need in order to solve the contextual problems that you set up. If a phenomenon is not linear and it turns out to be quadratic, you're going to need to solve that quadratic equation
if you want to understand and make predictions about that particular phenomenon. Once we’re there, of course, we solve quadratic equations. We have the quadratic formula. We complete squares. We factor. We do all of the things that people do in the development of those concepts, but it’s always for a purpose. It’s always because we’ve got a problem, for which we come up with a mathematical description, which requires us to learn some of the manipulations.

Implementation
In most cases we’ve seen, there has been a pilot phase, where some number of classes and teachers have gotten involved in the program. In some cases, it’s actually been a sort of two-stage pilot, where you’ve had some of the teachers teaching a certain number of sections, and the following year there have been additional teachers, until the entire system has adopted. A program has more success in a place where you pilot, and you bring more teachers on board each time, and the enthusiasm for the new curriculum grows—where it feels as though the curriculum is growing internally, not simply being brought on from the outside.

I think the biggest pitfall is to isolate the people implementing the program. The nature of the beast is that you get some people who are extremely enamored of the program and who want to make the change. But it may happen that these people are either isolated, or at least are the minority in their school or their district. As a consequence, the success or failure may depend on the energy of only a few people, without the requisite sort of political support of a significant percentage of people in their district or school. That can be a problem. People can really feel as though they’re doing these wonderful things, but then the success is totally dependent on their personal energy, and not widespread support. That’s a serious pitfall.

Tracking issues
In locales where kids have been tracked early, Course 1 is essentially being taught in the 8th grade. So we simply have the algebra-in-the-8th-grade phenomenon, with students taking our Course 1 in the 8th grade, and then taking a precalculus course for their 11th grade, and an AP Calculus or Statistics course in their senior year. We have seen a lot of examples of that.

I think that our experience with lower-end students who have used our materials is that it’s all about pacing. What happens is that in some of the lower-end classes, the students simply cover less material. But the point is, they stay in school, and they come to class. Even if they cover half a book instead of the whole book, they’ve learned something, because they’ve stayed with it. I think that’s a value in and of itself. So where it’s been used with tracked and lower-tracked kids, we’ve had actually quite a bit of success.

Changes for teachers’ practice
As is true for all standards-based programs, there are a variety of changes for teachers, in the content, the process, the pedagogy, the technology, the assessment. When you do a contextual problem, for example, you may very well be entering areas that, as a math teacher, you are not necessarily an expert in. In fact, the context itself may be something that students or their parents or their friends may know better, or have more background in, than you do. So once teachers start to look at the real world and the real contextual things, they open themselves up to not being the divine providers of truth. It’s the same situation with technology; you could easily have students who are more adept at using
These materials certainly are different from traditional materials. Teachers will be teaching material that may involve some subject matter that they haven’t recently taught. But we find that teachers actually enjoy that fact. So what happens, in general, is that the teacher’s role changes from simply being lecturer—“This is the way it is. Go to the board, and do the following 20 problems”—to actually being a student, as well, who tries to lead the class but who also has to learn a great deal about the content and about the context. I don’t think that our program is unique in that respect; all of the standards-based programs have this same effect. The teacher’s role is simply different. It just isn’t going to be the same anymore.

As I said, you can’t do contextual things unless you spend some time in the context. And the context may, in fact, be different and new for the teachers. So, for example, in one chapter we do LandSat images where you’re looking at the satellite imagery, trying to determine something about area. The problem we use is two pictures of the forest in Czechoslovakia, three years apart. The question that’s asked is: “How much was lost to deforestation?” Well, obviously, there’s something to be looked at about similarity. The teacher may or may not understand how a LandSat satellite works, and may not understand some of the physics involved. And the students may, in fact, have played with some of these ideas, because images like these are available on the Web, for example. So there’s a little bit of the physics, a little bit about the cameras, a little bit about satellites and how they work, that you really do need to understand. There’s material in the book, and there’s extra reading and Web sites and so on, but nevertheless, this is something where you really need to learn a little bit more about the context than you might historically in a math class. Exploring the context may take a little extra work on the teacher’s part the first time through.

Training teachers to use the program

The truth is that teachers need to go through the program in some form, maybe a week or two weeks at a time, before being able to teach it. A lot of the material is familiar and comfortable, but there is a lot that’s different. What we’ve found is that the second time through is a lot easier than the first time through; teachers need to see how the students react to these questions and these different activities, and hear what kinds of things they say, and see what kinds of directions they take.

When you open up these problems and you open up the context, you want the students to explore. You want them to come up with different ideas. On the other hand, you can only let them play and go off in different directions for so long, because you only have a finite amount of time. The whole experience is one that, unless you’ve had it, is hard to appreciate. Partly, teachers can do it by being students themselves, as they are when they work through the materials in our training programs. But I really think that the best training, aside from being a student yourself, is actually teaching it at least one time through.

There’s an enormous amount of support for teachers in the program. In addition to the student text, we have a wraparound teacher edition with a lot of information in the margins. There’s a Teacher Resource Guide, which, for every lesson, has additional material about the subject and additional handouts for the students, either for those who are going quickly or for those who are going more slowly. On our Web site, for every one of the chapters in the text, we have additional information, including hot buttons for various Web sites, where teachers and
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students can learn more. For example, for the LandSat satellite imagery, you can pull up a page for that chapter, and there are contact people for you to talk to, who are the authors of that chapter, and their e-mail is available. There's a set of Web sites that you can click on for more information about that particular contextual problem. Plus, there are chat groups and bulletin boards where teachers can talk to others who are teaching the material, and share their experiences. So we’ve tried to provide a fair amount of extra background and support for teachers. Plus, we have an 800 number that teachers can call if they have a problem.

In the most successful implementations, there is no question that there’s just simply been more staff development. As part of the COMPASS\(^1\) group, we go everywhere. So if people want staff development at their site, we will send people out to help train. We also hold a leadership conference in the Boston area every year. But where the program has been the most successful, people have had a planned staff development process, organized locally. In the Minnesota area, for example, we’ve had situations where, one summer, people worked on Course 1, and the next summer they worked on Course 2, and the next summer on Course 3. This was full-blown staff development, organized at the local level. They had two-week sessions each summer. In one week they would take a year's worth of material and go through it, and then folks from COMAP would come in, in addition to local people there. It's certainly the case that if you have more time to go through the material, I think it certainly helps.

Engaging students in the mathematics

We've spent an enormous amount of time trying to get students involved in mathematics, to get them thinking about problems. In fact, when we have done various evaluations, one thing that comes out—the thing that probably I’m proudest of—is that teachers will say, “The students are not afraid to attack a new problem.” Whatever their level of expertise, whatever they've learned or not learned, students who have come through this program seem to have a willingness to try—or at least not a fear of—problems that just come new, where they don’t know particularly where to start.

Parents

I have this wonderful piece of videotape, in which the mother of a student in a rather upscale private girl’s high school says, “Look, I went to high school 25 years ago. The world’s changed a lot in 25 years. If my daughter were taking the same curriculum that I took 25 years ago, then the schools would be doing her a disservice.” That's a lovely sentiment, which I, of course, agree with.

Some parents whose kids historically have done well with the old curriculum look at the new curriculum and wonder, “What's going to happen here?” We haven’t gotten any specifically negative responses; it's been more questions, like, “Are you experimenting with my child?” or, “You’re doing something different. There was this game, and it was being played by a set of rules, and my children were winning with that set of rules. Why should I think that they'll win with the new rules?”

Another quote that I’ve got on tape is from a father of a student. He said, “You

\(^1\)COMPASS is a national, NSF-funded high-school implementation center based at Ithaca College. For more information, see page ii of this publication.
know, I never could help my daughter with her homework. Sure, I could ask her questions and see if she got them right or wrong—I could be a flash card. But now, for the first time, these questions are about things that affect my life. I actually have life experiences that I can really bring to bear. I can talk about problems in the real world. I can actually help her in a way that I could never have helped her before.”

We've had, actually, very positive responses to the content. Mostly, we've seen sort of generic fear of change, and the kind of question that basically says: “Is my child going to get as good SAT scores or ACT scores as they would have before? Will they get into as good a college as they would have before?”

**Results on standardized tests**

What we have so far is SAT, ACT, PSAT, and state test data, and we are obviously collecting more all of the time. Where we’ve looked at SAT and PSAT data, we've found 3–4% increases for the students in our program. And where we've looked at the state tests—these are state tests that are more in the spirit of standards-based state tests—the students in our program score 10–14% over a comparable group of students in the same school not in our program, or over students from previous years, in the case that there is no comparable group.

In terms of the SAT/PSAT data, 3–4% is probably not statistically significant data. I don't think anyone should expect, at this stage, to see statistically significant changes on big exams like that. On the state exams, you're dealing with tests that are more aligned to standards-based programs than others. To try to do the comparisons now is just chilling—maybe we should wait until four or five years from now, when these things are being used by many more people, in a much bigger setting, before we make claims about test scores.

Yes, our students do better. They do significantly better in some cases, and just better in other cases. I think that's true with every program, and all the data, and I think it's useless. The mathematician in me rises up and says, “Don't talk about this yet. Wait.”
Kay Francis is a mathematics teacher and mathematics division leader at Gresham Union High School in Gresham, Oregon. Kay has been teaching mathematics at the secondary level for 28 years.

Gresham Union High School has been using Mathematics: Modeling Our World for five years, since the early field testing. Last year’s graduating class had students who had been through all four years of the program. Gresham Union High School is one of two high schools in a suburban district outside of Portland; each has 1500–1600 students. While approximately 85% of the student body is Caucasian, the remaining population is widely diverse, including African-American, American Indian, Hispanic, Asian, and Pacific Islands students.

Using Mathematics: Modeling Our World

We’re using the entire Mathematics: Modeling Our World (MMOW) curriculum with a certain group of kids. We basically divide our kids into about three groups mathematically; MMOW is used with our upper-level kids, which is probably about half of our math students. They’re kids who think they’re going to a four-year college or at least a two-year college. The next level down uses an integrated program by McDougal Littell called Integrated Math. The third group, our IEP students, use Middle Grades Math from Prentice Hall.

I describe MMOW as a program where the students learn by a lot of discovery, using technology, and working with other people. Students will come upon most of the traditional math concepts that they would learn in a traditional program, but it won’t be through a traditional approach—it will be more from a discovery approach instead of a repetition approach. It’s integrated in two ways: the math concepts are integrated—you might have some geometry concepts and some algebra concepts within the same lesson—and it’s integrated with the other areas in your school. It might be a math and science integration one day; some days it’s integrated with physical education; some days it’s integrated with social studies.

Before I used this program, I don’t think the students saw the connection between what they were doing in math class and what they did after they left my classroom. It was just like we did math for that little class period and that was it. They would ask me, “When am I ever going to be able to use this?” and I would have to drum up some good examples. One of the things I like about using this program is that now nobody asks that. A whole unit is based on some real-world context, and then the math arises from that. Students see that what they’re learning is used in real life, and they see a lot of connection in their physics and science classes.

Why Mathematics: Modeling Our World?

I used to buy materials through COMAP (Consortium for Mathematics and its Applications) for my Calculus class, and then they were advertising for people to check out MMOW. I think they sent out cards to some schools. We knew that for the Oregon standards, we needed to go to an integrated program, because they are trying to get our 10th graders to reach a thing called a “Certificate of Initial Mastery.” In order to do this, there are five strands that 10th graders need to be in pretty good shape in. If we didn’t go to an integrated program, the only two strands most 10th graders would have would be algebra and geometry. So the other three strands we have now are measurement, statistics, and probability/estimation. In a traditional text, those strands are either not even in the 9th and 10th grade or maybe in the back of the book, so we felt like we needed to go to an integrated approach. Here was an offer to pilot this material, and if we did—we didn’t know it at the time, but it was sponsored by NSF—we’d get some breaks on calculators. It made a way for us to get some technology into the building.

We didn’t have anybody else asking us to try something new, so we went with MMOW. We didn’t look that hard at other programs; our division leader sent two of us, Gayle Meier and me, off to Boston one summer and said, “See what you think of the COMAP program.” And we came back and started it. We just kind of jumped in with both feet.
Implementation strategy
The first year, we had seven sections of MMOW Year 1, and the two of us taught all of them. The next year, we had at least five or six sections of Year 2. The original two of us taught those and we brought on new teachers to teach Year 1, although one of us taught one class of that at the same time with them. The third year, the two of us moved on and taught Year 3 and we taught some of them how to teach Year 2. So that's what we've done each year, except for Year 4. The Year 4 kids, who are basically the precalculus kids, have been with just Gayle and me. We've got enough people involved now to teach the Years 1, 2, and 3, and then we teach the Year 4, because by the time the classes get to Year 4, there are only five sections. So the two of us can teach those and then still have some room in our schedules to teach other things.

We've now got six teachers out of 12 in our department who are teaching the program at the different levels. So half the people are teaching this and the other half are teaching the more traditional-looking integrated program.

Instructional approach
Over the course of using this program, I've changed my mind about strategies that work best for my students. I used to do pretty much direct instruction, giving examples, and then expecting the kids to repeat what they had seen me do in a variety of types of problems. Now I use more of a discovery approach or cooperative groups, where students aren't necessarily working on problems that they've seen before. Instead, they're starting by trying to make a model for a particular situation. Students have to take a lot more responsibility for their own learning. They have to read a lot. That's the way the world is headed; when you go out and buy a VCR, you have to read the directions to learn how to use it. If you decide to put lighting in your house, you read the directions on what to do with the light fixture. The program forces students to read and reason and problem-solve and not just repeat a whole bunch of the same type of thing. Sometimes they would much prefer for me to just stand up there and show them how to do it. But I think they're learning more, and they can definitely do more when they're put out on their own than they could back when I felt like I had to tell them everything. Also, I think the fact that there's a lot of use of technology in the program— with graphing calculators and in the computer lab— helps the teacher do less lecturing and the students do more work on their own.

Mathematics
Some of the really strong mathematics in the program is regression and any kind of data analysis. At the beginning, that was one of the areas where I didn't have a clue about what was going on. I thought, "I've got a degree in mathematics, and I have done none of that." Things like where you have to take a set of inputs and outputs and see whether they should be linear or quadratic, or do any type of data analysis. We had not done that in traditional classes before, but in the first year of this program, it comes up two or three times, and that's consistent through the program. Any one of our fourth-year MMOW students just automatically knows how to put all those things in the calculator and get the prediction equation, and see what the residuals are. They're very good at anything where they have to make up their own model.

Traditionally when you had a set of exercises, some would be easy, some harder, and then, at the end, they'd be hard. Well, the way MMOW is set up, there might
only be 15 exercises, but every problem is different. They're not building up in
difficulty—some of the hard ones might be right off the bat. I think that's good;
that's the way things go in real life. Also in real life, just because you can't do one
skill doesn't mean you're not going to be able to do the next thing. This approach
helps students to realize that we don't just quit because we couldn't do the first one.
There might be one later on that we actually could do. I think students are a little
more willing than they were before to try things that they don't know how to do.

There are some basic skills that I think aren't in the program; I don't know if the
authors feel like students don't need to know them. For instance, in a traditional
program's Algebra I year, you factor. I used to factor and solve equations using
factoring for about six weeks. The NCTM Standards say to play that down. W ell, we
went right along with what the program had, which was no factoring. But then we
came up to piloting the fourth-year material, and it basically assumed kids knew
how to factor. So we decided we'd quickly teach the students in fourth year how to
factor. The amazing part was, because they've done work with patterns and different
things, they could catch on to the factoring. I think they probably were as good in
about three days of factoring as my freshmen had been in six weeks of it. So we just
taught it at a different time. But occasionally in the fourth year things will come up
that assume the kids know how to do something, but it hasn't ever really been
taught with more than one or two problems. So we've had to add more practice.

Skills practice

I believe that students need traditional skills if they're going to jump the hoops
and get into an engineering program. If you want to have students be good at
something, you have to do it more than one time. MMOW doesn't do a lot of
repetition, so we supplement the program with some drill and practice. I know that
the authors don't want it to be skill and drill and kill, but there are certain topics
that I think you need to do that on because they come up repeatedly throughout
students' careers in math. If they don't have those skills, they're just going to lose
out. For example, we just did the oscillation unit this fall. In order for a student to
get it in their head that sine is the relationship opposite over hypotenuse, they
need to do more than two problems. So we supplement a worksheet of problems for
those kinds of things. It's a lot easier to create a worksheet on drill and practice
than it is to create a worksheet for an application.

I want students to be solid in their fundamentals, like the language of algebra
and those kinds of things. But problem-solving skills and being able to converse
with mathematics and use it comfortably to do things like make change or
measure to know how much paint to buy for a house are sometimes more
important. We're not going to make engineers out of every student that comes
through. So if they can feel comfortable with using mathematics in everyday life,
that's probably the bigger picture.

Assessing students

I used to spend less of my time planning and more of my time grading. Now it's the
other way around. I spend a lot of time preparing for class each day and then I
don't collect as many papers. I go by how well kids are working in groups, and I
look to see what they did on their homework. I used to take a paper from every
student every day and then tried to look at a few specific problems to see what they
were doing wrong. Now I'm more apt to have them take a few minutes while I take
attendance to look at their homework in groups and see which ones we need to go
over as a class.
Certain things now do take a long time to grade. I used to be able to get through a set of tests in a couple of hours. Now, if I give a big project, it might take me a lot of hours to grade a whole class set; I might spend 15 minutes reading one student's project. We assign maybe one big project per unit. I don't test as often as I used to; I used to give lots of little individual quizzes and then a test at the end of a chapter. Now we give a quiz maybe in the middle somewhere and then a test at the end of a unit. If I feel like I need to do a little quiz, I might just give one problem situation in class.

If you're patient with it and can watch the kids that come through the entire program, they're way ahead of the traditional approach as far as problem-solving. They're not ahead on bunches of little math facts, but for problem-solving and using it in real life, they are ahead.

**Changes for teachers**

It's scary for teachers to change to a new curriculum. When we started this program, it was not only a new approach, but also new technology and new material and new pedagogy. All of those things at the same time make a large chunk for somebody to bite off. You have to have some faith that what you're doing might actually be better than what you've done before, but you also have to be willing to make a large time investment to learn to do things differently. When you see the results of what kids are able to produce, I think it's well worth the investment. So, if somebody is thinking about changing to a new program, they need to be patient with it and be patient with themselves and give it time to work. We just kind of went in the first year on blind faith and there were many times that I questioned my sanity.

I used to do what most traditional math teachers did in the classroom: I spent the first third of the period going over questions from the homework the night before, the next third going over the new material, showing a bunch of examples, and then a third where students started working on that. Now, each day is its own thing. You go in one day and start out by reading the context to get an idea of what real-world situation you're working with and what math you might need in order to solve this problem. Then there might be an activity set up where students work in groups to try to do this. They might pick somebody from the group to do a presentation. And sometimes I'll have each group do a different problem and present it so that we can cover a wide variety of problems in one period. The funny part is, now when I have to do days where I start out with a lecture, I feel like it's boring. If we go a few days with a lot of new material that I have to do by the lecture approach before I let the kids go to work on it, the kids just look bored to death. I don't blame them—I even feel bored doing it that way.

I don't think classroom management is any tougher with this new program. I've learned to accept more noise because there is more noise. When I'm lecturing, I don't accept anybody talking; it's my turn. But when they're doing group work, I expect there to be some discussion between them, so I'm more tolerant of noise. I do have to get around and make sure they are on task. When I was lecturing, somebody could look like they were paying attention but they could be zoned out. In group work, if somebody's slacking, the other kids get on him or her about it. The only tricky thing about the management part is when you need to call them back out of small groups to one large group to talk about the lesson. That takes a little bit of time sometimes.
Manipulatives and technology

MMOW activities involve different materials. You might need a variety of things to do an activity; for instance, you might be doing something for which you have to have clay or balloons or whatever. So you always have to be looking ahead, knowing what materials you need and gathering up all that stuff. That’s real different for a math teacher. This program has caused us to have to have measuring tapes and meter sticks and all of these things for each group of students.

Learning to use all the technology has also been a challenge. In the first year, when we were learning how to use a new calculator and then CBL (Calculator-Based Laboratory) units and all those things, that was a lot to learn. With certain units, you use the CBL and then you might not use it again for months. So you kind of forget how it works. For the first year, I panicked when I had to program something into the graphing calculator and I couldn’t remember what keys to use. I spent more time looking up in the manual how to do these things. But part of the technology support is built right into the program; they tell you what keystrokes to use. They have worksheets that tell the kids what to do. You can just follow right through on their directions. When you’re using the technology for the first time, you don’t want to look stupid in front of the kids. So it is a challenge. I really enjoy it, myself, but I understand why people are reluctant.

Training and support for teachers

Our first year, when the material was in the packet form, just two of us were piloting. We had all the 9th graders that year. At the end of the year, we gave an in-service, showed other people how to do it, and they just followed our lead the next year. We had the same packets, and whatever we had done, the new teachers did, so even though it was new to them, it was all laid out. After we got through piloting a couple of years, along came the book, so the two of us started back with the book again. We went through and figured out what we thought were the important things in that book that needed to be taught to go with our standards and what things we could leave out or what things we needed to add. The book looked a lot different at times from the piloted materials.

Nobody has piloted the second-year book for the people teaching it this year, and they are really having a fit about it. The two of us that were the original field-test teachers did not go through and do all that preliminary work for them and say, “In this unit, I would do these lessons and I would do these activities.” When you’re on your own with the book for the first time and you have to do that, it’s a lot more time-consuming. People complain that the book isn’t very user-friendly because it’s just not set up like a traditional textbook. There’s a notebook that goes with it that has teacher suggestions. If people would actually read the teacher suggestions, they might not feel that way. Also, the solution key is a whole separate manual. That’s the way calculus books have always been, but that’s not the way Algebra I or geometry books have always been, so it means a change of habit.

As far as professional development, mostly Gayle and I learned the first year material between us and then we taught other people, so it has just kind of trickled down. We taught others in an in-service week. We only had three or four days to work with them and wanted to make sure that they felt confident about the technology, so we spent at least one day down at the computer lab. Without that training, I think people would have been a lot more reluctant to use the technology.
One helpful strategy was when our department head could fix it so that people who taught the same class had the same prep period. Gayle and I have been teaching the same class now for five years, and one year we had a common prep period. That was wonderful, because then we could sit down and plan together and pick each other's brains. We could look activities over and ask each other “How would you do this?”

Another thing we did was have one person, either Gayle or me, on a duty period each semester to be a resource for anybody in the district who was teaching MMOW. So, for instance, if a teacher was having problems doing the animation unit, I would take off that period and go over there and help. We tried to do the things that would support the people that were new to it.

**Transitions into and out of the program**

As the new division leader this year, I’ve done some work with the 8th-grade teachers and learned that they’re trying to emphasize the same strands that we do. Their book is maybe a little more traditional-looking than ours is, but it’s definitely a problem-solving approach. I think it’s a decent match; kids can either go to the Integrated book or to MMOW and be successful. To be successful in this Modeling book, students need to be good readers or at least willing to try.

If MMOW students go on to a calculus class or a precalculus class, I think they’re prepared for college mathematics. But if they take a traditional college entrance exam and don’t do well and are put back into college algebra, they’re not prepared for that. We haven’t taught them just traditional algebra concepts. That type of test, mostly would look like what students probably would have learned if they had taken Algebra I/Geometry/Algebra II. Knowing this, I make sure that during senior year, Year 4 of the program, I go over some of those traditional things. As we’re piloting the material for Year 4 and we come upon a situation where one of those traditional topics comes up, we just expand a little bit so that the kids will have seen some of those things that they might see on those tests.

**Parents**

If I had it to do over again, I would do it differently. Our parents were not aware that we were going to change programs, and that made the whole process more difficult. Some of the kids had had traditional algebra in 8th grade and hadn’t done well enough to go on to a geometry class as 9th graders, but they felt like they would just come in and waltz through a traditional algebra class. When they got into a class that they actually had to work at, they were pretty fired up. Their parents didn’t feel like they could help them because the materials looked so different. In hindsight, I wish we had been a little more straightforward about educating the public. But we didn’t know ourselves what we were getting into until we were right in it.

For others trying to implement this, I would recommend doing either a parent night or something like that where you don’t open it up for argument, but you do a presentation to parents about why you’re going to a different program, what you think the strengths are of your new program, and how you plan to monitor what you’re doing. Show that you’re not just changing for the sake of change; you’ve got a reason. That will help get a few parents in your corner; anytime you change something, you’re going to have some parents who are not in your corner.

I think that the biggest issue with parents has been that they don’t feel like they can help their kids. As a teacher, I don’t think that’s bad. I think kids need to do
their own work and take responsibility for their own work. A parent can sit down
and help their child with this program if they’re willing to sit down and read and
have the student tell them what they’ve been doing in class. They just can’t jump
in, like they have in the past. It doesn’t look like a traditional book where you read
Example 1 and then there are some exercises that look exactly like that example.

Since that first year, we’ve done a lot more with PR, and it’s gotten easier each
year. I’m still battling some parents at the 8th-grade level over “Why don’t you
also run a traditional program so that my child can just take a traditional math
class?” because their worry is how their students are going to do on the SAT. The
SAT and college placement exams are still a big worry. Some colleges are still
testing on very traditional topics and are not doing a lot of problem solving
because those types of exams are not easy to grade. I tell parents that we haven’t
seen that our SAT scores have gone down, and if students stay in the program for
four years, they will have covered the material that they need for college
placement exams. I think eventually the colleges will catch up.

Support within the school

Our department bought in to change. When we said we were going to the
integrated approach, half of us went with the Modeling, and the teachers who were
a lot more traditional, who weren’t willing to go out on a limb quite so much, went
into this other integrated program.

The neighboring school had just a couple of classes of MMOW, but they ran a
traditional algebra class right alongside, so as soon as students got to the point
where they didn’t like the Modeling class or they were looking for something
easier, they had something to bail out into. This is now their fourth year and they
have one section of 15 kids left in MMOW. Kids are human enough that they’re
going to look for the easy way out. There were also lots of teachers in that school
who weren’t convinced that it was a better approach. Their department has fought
about it from day one.

Our math teachers were fine about the change, but other teachers in the building
didn’t know much about the program. I felt very supported by our administration,
and very supported by the department head. When parents would complain to the
administration, Gayle and I or the division leader would be called in. We were
financially supported, too. If we needed calculators, we were able to get some of
the technology that we needed. The district supported our in-services during the
year, as well as our week-long in-services in the summer, with Eisenhower funds
or other district funding.

For kids of different abilities

I think it must be possible to use MMOW with all students, but we would have had
a hard time trying it with all of our students at Gresham, because it is so different.
For us I think it worked better having two programs, because I don’t think every
single student has the same capabilities. But I do know that this type of program
can work with lower-level students, too. One of the inner-city schools in Portland
came to an in-service and saw what we were doing, and they started using the pilot
materials with every single 9th grader. They had to take it slower; they felt like
they had to do a lot more reading to the students because the kids weren’t very
good readers, but as far as actually doing the work, it could be done by kids who
weren’t just the fast kids. ■
JERRY LEGE  ▶  MATHEMATICS TEACHER

Goals for students
For way too long, we've been on this fast track of making sure that kids get every little thing, or at least the chance to see every little thing, because they “need it for the next year.” I used to see that every day with my really high-level Calculus kids. They didn’t have any idea what they were doing or why—they just knew that that’s what they had to do. And so I’ve loved the ARISE curriculum (now published as Mathematics: Modeling Our World), for at least taking the approach that maybe less is better, and for basically forcing kids to understand, and to try to construct, what they’re doing. If kids can construct what they’re doing, then they truly understand, and they’ll never forget that or get confused about how to use it.

I want kids to do math. Kids today are passive and not real confident about what they’re doing. If they’re just watching instruction, they’re really not getting anything out of it; they need to do things. They need to build spreadsheets. They need to get that calculator in their hand and start punching those buttons like crazy, with some idea in mind of what they’re doing. I encourage them to recognize that playing with a problem, building tables and exploring, and doing the things that mathematicians actually do, is really okay.

I stress with students that there is no right way, that maybe you need to do a problem more than one time to verify your work, that you want to think about things in terms of their connections to the context, and connections to other areas of mathematics we’ve been studying. I try to frame for kids that math’s not just a bunch of recipes to be memorized, but that it’s a language and a process for approaching the world. The ARISE approach really strongly develops that, because the kids are studying the math in context. They have to always be aware of their ability to step in and out of the reality of the situation.

Selecting the program
Before we got involved in ARISE at Hogan, we piloted some curriculum programs that focused on writing in mathematics and the use of manipulatives. Then we started thinking about how that translated to a college prep curriculum. About that time, I saw Sol Garfunkel speak in San Francisco about this three-year project. So when Sol said, “We’re looking for pilot sites,” I applied, and we got selected. My good friend and I started ARISE and College Prep Math (CPM) at that school at the same time.

We had been using a hodgepodge of stuff, and had problems with textbooks not being available because there hadn’t been enough money for things. Also, there had pretty much been a lack of cohesion in the district, and the message that they sent to people like me was, “Stay in step. Don’t get ahead of the rest.” So we had to knock down some doors. There was one district level administrator who was pretty good about giving me some freedom to try some things out.

Basic approach of MMOW
The program uses a modeling approach; it is application-based, and thematic, and all the math is taught in context. After that, it’s open for teachers to take it and
Once you finish doing all that simulation and all that setup for the problem, then the data analysis becomes kind of like, “Gosh, I want to know the answer.” So the modeling is a framework for kids to do problem-solving.

Mathematics

The three-year sequence of ARISE (the fourth year is under development)\(^1\) does a really outstanding job of preparing kids for precalculus, for AP Stats, for discrete math, or for basically any type of technological vocational training program.

Linearity is a concept that I think is extremely well developed in the program in a way that makes a lot of sense to kids. They start out by looking at patterns in coding processes that are basically going up by the same number each time, and they see that there’s a line function that’s being created, and they see the numbers that are describing the line functions. Then they get into trying to describe data that’s maybe not perfectly linear, but kind of linear. They do regression, and they see the other ways of expressing linear equations. They get into parametric form for linear equations as a way of describing animation routines. They get into additive models for population growth, with migration being the context. By the end of the first year, they’ve done a lot on linear behavior. The distinction between, “Okay, this is linear and this is nonlinear,” is pretty clear for the kids, as are the concepts: “When do you know it’s linear?” and “How you describe it when it’s linear?” and “Why would this be important for us to know?” It’s a whole lot better than just a chapter of a book, where you study it and then you walk away from it wondering, “What was that all about?”

Another thing that is really well developed is data analysis. First of all, kids get freaked out by it, but then they start to just dig at it. They say, “Well, I can do the calculator stuff,” and eventually, they start to feel really empowered about when to do it, and how to do it, and what you get when you do it. By the end of the first

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\(^1\) Since the time of Jerry’s interview, Course 4 has been completed and published.
year, they're actually discriminating between various types of models based on the patterns that they're looking at in the residuals. That's tremendous growth for the kids, and it's due to the underlying structure of this curriculum, since it's got a lot of data and it's trying to teach kids about modeling. The kids have to be able to select a model based upon some criteria. The kids get that pretty early on, and then they get to practice it a whole lot.

Another good thing for kids is that it's functions-driven, in the sense that, especially the first year, they have a lot of exposure to linearity, but they also see exponential functions, rational functions, and quadratic functions. There's not the expectation of total mastery the first time, but there is this concrete understanding of, “Okay, what's this all about? What's the thing that makes exponential functions be the way they are? What is it about quadratics that distinguishes them from exponentials? What are the various forms for these equations?” There's an awful lot of stuff that they process by comparison.

Program materials

The materials are pretty comprehensive. There's probably about five years' worth of material in the curriculum; I've never finished an entire book. I tend to use the activities in class, and the homework questions outside of class. That's how the book is structured. You'll see Activity 1, and then Individual Work 1. The activities are what I hit on in class, and we have discussions about what happened. If there's any time left, maybe I’ll go to a question or two from the homework, and we work out that stuff in class. But then the Individual Work is what I tend to have them work on at home, unless there's something that requires a calculator. I’ve never had a classroom of kids that had access to graphing calculators outside of the class, so I have to kind of dance around which assignments require it and which ones don’t.

Using modeling as a motivational tool has translated into a lot of words on the page in the student book. There's so much reading. I know that the authors deliberately put a lot of reading in because kids need to learn how to read math. There are pros and cons to that, but my kids don't read well, so I end up reading a lot of things to them, and we discuss it and paraphrase it. I give a lot of oral instructions, rather than just throwing them a book and saying, “Here, read this page, and get to work.” If I did that, my kids would read some of those activities and say, “I don’t have a clue what you're asking me to do here.”

Assessment

In my current school, we've done almost none of the given assessments. The school was built with the predication that it would be involved in competencies, and that the testing—in fact, all the instruction and the homeworks and things like that—would be about preparing kids for these competencies that we wrote. I used the assessments a lot more at Hogan. I found that kids started out being pretty poor at them, but eventually I saw a transition. First off, they’d say, “I don’t get it.” And then eventually they'd read the problem and try to play around with it and then say, “I don’t get it.” And then they’d read the problem again and play around with it and kind of come up with some reasonable answers, but they wouldn’t do a lot of the extra stuff like exploring it again with a different approach or seeing if they could verify their work. So the concrete questions would be answered, but not a lot of the higher-reasoning questions. Eventually, you see these kids starting to move towards high-level thinking. The assessment problems
They fight me tooth and nail about learning math in the first place, but when we start talking about wildlife population growth, and looking at animation routines and things like that, then all of a sudden they’re curious about the mathematics, in a way that I couldn’t have generated otherwise.

were constructed totally to test for that, which I think is an interesting way of testing. Rather than testing what they have done, it’s testing for what they can do: “What else can they do besides that? Where can we take these guys beyond that?”

Implementation
Hogan High School was a three-year high school, so when the students came, the top students had already started the college prep sequence as 8th and 9th graders at the junior highs. The ones who were starting their college prep sequence when they came to high school in 10th grade got to choose between two curriculums: ARISE and College Prep Math. We basically had 10th, 11th, and 12th graders in ARISE Year 1, and 11th and 12th graders in Year 2, and only 12th graders in Year 3. But we played around with the placement a lot.

The new high school, which I’ve been at this year, chose to go school-wide with this curriculum. We are on an intensive block, with kids in totally heterogeneously grouped classrooms, in 90-day courses of 90-minute blocks. So all 9th graders were in ARISE 1 this year, with no exceptions, no matter what background they had. About half of the 10th graders were in ARISE 1. And then there were three sections of ARISE 2, and two sections of ARISE 3, for 10th graders. That will change next year. The district has mandated that kids who start their algebra sequence in 8th grade be given the opportunity to continue that same program in 9th grade with the second year of that course. We’ve given kids the option of testing out of the first year of ARISE if they could demonstrate that they knew the material.

This year we also had some kids who had finished Algebra and Geometry in the 8th grade and the 9th grade, and who were transferring to this brand new high school as 10th graders. Our superintendent insisted that they be placed in the third year of a curriculum, so I actually had kids starting with ARISE 3. There were some of the units in that third-year book that I thought maybe they didn’t need as much as they needed some background stuff. So I did two units from Year 1 and one unit from Year 2 with them, as kind of a transition. That gave them a pretty strong emphasis on data analysis and function development, so that they’d be ready for Precalculus next year. It worked pretty well.

We did the wildlife unit first, and the fact that we took three weeks to solve a problem really bothered them. It was hard for them to see what they had done and what they had learned. I would ask them to write a summary of the modeling process as they applied it to this problem, and they wrote all superficial level stuff, because it was so new to them. Our high-level kids couldn’t believe that this was something from the first year. These were kids who were supposed to be pretty sophisticated, and all of a sudden they were being very humbled. It was good for them.

Impact on students
The connection to the applications has been a tremendous motivation for my students. They fight me tooth and nail about learning math in the first place, but when we start talking about wildlife population growth, and looking at animation routines and things like that, then all of a sudden they’re curious about the mathematics, in a way that I couldn’t have generated otherwise. For most kids, trying to get them to do abstract math to prepare them for a math degree at the college level is really a waste of energy. There are so many other things that they need—not just business math, or voc-tech math prep. Kids need to know that
they're going to do a lot of math in their life, no matter who they are, and they need to know how to do that math.

There are some specific algebra skills that may not be developed in the curriculum, but I've seen firsthand how the thinking is developed and how kids can transfer the mathematics from one context to another. Ironically, one of the contexts that students are very able to transfer the mathematics to is traditional mathematics. They will go into something like an SAT question, having never been taught the specific content, and will be able to apply what they have been taught to figure out how to approach questions that they've never been taught. I think they're being armed for life and for college and for testing.

Use with kids of different backgrounds and abilities

I think that MMOW makes some assumptions about the abilities, preparation, and motivation of the kids who will use the program, and my students don't match those assumptions. That's not a criticism of the program, it's just a reality for me. Kids who can't read are going to have a hard time with this book. Kids who can't compute are going to have a hard time with the mathematics involved, even things as simple as percent calculations. I also think that the program makes an assumption that kids can construct their own learning, by going through one example of how to do a calculation and then generalizing that to a formula. I've had students who can do that. But I've had a lot of students who can't do that when you just give one example—maybe they're not at that level of abstraction yet. I'm not advocating for doing it 20 times over and over again, but I think that we need to temper what we want with what our real world is. I believe the curriculum does meet their needs, but we, as teachers, have to be aware of their needs and make some adjustments to make it accessible. The curriculum is just a book.

I have kids in the same room who have such diverse backgrounds and abilities and motivations that, for some, I can just literally throw them a book and say, “Why don’t you play around with this idea a little while and see what you come up with?” And then I have kids whose attention span is five seconds, who don’t understand the question that’s being asked, and so I have to reword the question and coach them through it. So, as an instructional practice for class, I tend towards providing some structure for the kids who need it. I give them what they need to understand what I’m asking them to do, and to give them the confidence to be able to get started at doing it. Then, I take a step back and see what happens.

My challenge is how to make the program work for my kids who aren’t ready to work at this level. It's easier for me to structure and make the mathematics a little bit more visible for them, than to try to create the situations. There are a lot of notes to teachers with suggestions; most of the time, I look at my kids and I look at what I want to accomplish and I look at the book, and I say, “Okay, this is what I’m going to do.”

I have my kids work in groups almost 100% of the time. I want the kids in a proximity where they have some people whom they can work with and lean on and exchange information with, and not feel like they’re isolated. I spend a lot of time thinking about how I should organize the classroom to optimize that. I try to give each group somebody who's a resource, and somebody who's going to be a spokesperson, and somebody who's comfortable enough with calculators to be able to be a resource for that group.

In my experience in the classroom, there are some contexts that guys find more
interesting, and girls tend to prefer the verbal work, explaining their thinking and reasoning, constructing an understanding of what's going on. There's enough of a balance between applications and verbal construction in the curriculum that I've never really noticed any difference between the genders' responsiveness and ability with the program.

Adapting the curriculum
Whatever's going to happen in my classroom is going to happen, and the curriculum is my tool for making it happen. If I need to, I can go away from the curriculum materials and supplement in some way. I've done that. We didn't teach the LandSat unit in the first-year pilot, because we just didn't have the time, and we didn't have the computers to do the satellite imaging photos. So instead, when we got to the second year with those students, we took a little bit longer on the “Right Stuff” unit, and gave them opportunities to learn about finding the area of irregular-shaped regions and similarity relationships, which they would have learned in LandSat.

Technology
I think that the technology stuff is certainly all there for the kids. They learn a lot of skills really quickly, and get pretty proficient by the end of the first year. It's like the modern-day slide rule. You give them that power, and then they're able to go. I try to impress them with some of the things that they're doing early on, with the help of technology, like learning matrices in the first unit. And I tell them, “In a traditional curriculum, you wouldn’t see that until the middle of the third year.” They don't think it's a big deal that they're studying exponential functions in the first year, because they haven't seen the flip side, the traditional approach. They get to exponential functions as an intuitive way of exploring population growth, instead of just looking at a new function.

The uses of technology built into the curriculum help with the focus on thinking and getting some exposure to the real world and mathematics. It's really kind of weird to be talking about deforestation in Czechoslovakia to a bunch of kids who have never been out of Vallejo. But then they see the satellite images, and all of a sudden there's this whole new venue for them, the Internet. It just becomes a tool for them. The same thing is true with the ARISE videos. I've had kids who, for instance, see the animation video, and they see these people talking about their work and showing their work, and all of a sudden they start thinking, “Jeez, maybe I should become an animator.” There's this careers infusion in the curriculum that I wouldn't even have the slightest idea how to begin developing without the curriculum.

Professional development and training
It's very teacher-intensive; people need to examine that pretty carefully before they decide that they want to go this route. It's not something that is going to be easily incorporated into every school district in the country. Any teacher who wants to adopt this curriculum should expect a five-year transition period. I went through a growth process that's unparalleled in my professional career, and I've taught in three subject areas. Even five years into this program, I still find things that I haven't thought about.

I think that people should get training in the specific facets of the program, like technology and cooperative learning and authentic assessment, and some of the content background, and some of the context background that would be different from what you've taught before. All that stuff needs to be in place before you ever
walk in the classroom. Planning for the first year should be a high priority, because you only get one chance to do it right the first time. You need to have a pretty good idea about what you want to accomplish. Otherwise, you’re going to have a pretty bad time of things.

I got my training from COMAP while at Hogan. At the new high school, we actually ran our own training; I think that we did a pretty decent job. We had six teachers, and three of them were new to the program. All of them, I think, got what they needed to teach the materials.

Community support

If you were a farmer, you would prepare your soil before you plant your crops. I think you need to do that with the community. The implementation of ARISE in my old high school and my new high school were two totally different experiences. In the old high school, we were offering students a choice, and the strategy was to build some awareness for people about the differences between the two programs, and let them choose. We did a lot of orientation, like parent meetings and things where you try to show them what you’re trying to accomplish, and the differences in the programs. Parents basically are in the decision-making position, and that keeps them happy.

For the new school, we did a series of parent workshops last summer. Unfortunately, they were not very well attended, because the district didn’t advertise it like they should have. But the people who came were greatly appreciative that we would take the time to explain to them what the program was about, what a sample lesson was, how we were going to be evaluating the kids, what this program meant in terms of the greater goals of math teaching. We went through a whole week of just giving them things like the district framework, and the NCTM Standards. We looked at some of the documents that have come up saying that, “Yes, we need to change math education.” It’s okay to do that with parents, because they need to hear it. What gets publicized most nowadays is all the reaction to that, which is “Hey, don’t change.”

I don’t hear much from the parents in my town. Most people are busy trying to eke out an existence, and they don’t really have a whole lot to say. I think that both teachers and administrators take advantage of that—I’ve had freedom to do a lot of things that, at a high school with more active parents, might have met with tremendous resistance. But less parent involvement also gives administrators the same freedom to make decisions in favor of vocal parents.

Everybody in the school system needs to be in agreement that it’s a good thing to do a program like this. We’re having a problem right now in our district with a couple of district administrators who, for whatever reason, have decided that this is not a good direction for the school. I know my principal wants this program. He talks a lot to parents about what the program is doing for kids and about the school philosophy. But people who aren’t happy don’t go to the principal or to the teacher, they go to the superintendent and the assistant superintendent.
BERNIE KRAWCZYCK > K–12 MATHEMATICS SUPERVISOR

The selection process

We started the selection process three years ago. We identified our problem to be that our students were not meeting the state goals at the 10th-grade level on the CAPT (Connecticut Academic Performance Test). We did research on the students and realized that we had tried to do a lot of band-aid kinds of things to change, such as little practice booklets and practice problems and practice tests. But as we were going through the process, we knew that what we needed was to look at why our kids had a low level of performance and how we were going to change that level of performance on our state tests. We had a mandate and we had research on the students’ scores and their backgrounds. We realized that we had tried all these extra things to add onto our existing program, but we weren’t changing the most important piece—instruction. We weren’t changing the role of the teacher. All we were doing was giving the students more things to do.

So as we were looking for a program to help us change the role of the teacher and change the instruction, we narrowed it down to looking at the reform-based programs. We were reading in the literature and hearing from the state department of education that we were going to have to look at these reform-based programs if we were going to really make a change. We developed a selection committee with a total of nine members from two high schools. We had a teacher representative for each of the different mathematics courses at the high school—people who taught Calculus, people who taught Algebra I, people who taught General Mathematics. On the committee, we also had 30-year veterans and brand new teachers, and a mix of male and female.

The committee met on a regular basis and had a plan for what they were doing and where they were going. They constantly kept the entire department informed. They gave monthly reports of department meetings and shared actions. That was important. This wasn’t going to be something dumped on the teachers or a mandate from the top down.

As we looked at the reform programs, we decided to get preview copies of four programs: MATH Connections, Core-Plus, IMP, and Mathematics: Modeling Our World. But just viewing materials wasn’t getting us an answer. What we really needed to do was to visit teachers teaching. We wanted to know what it looked like in the classroom.

One thing that was extremely helpful was that the state department of education showcased those four programs, so we were able to see a full-day overview of them. That assured us that we were going in the right direction in our process, and that our mission statement was going to be met through one of these programs. Every member of our entire committee made visits to classrooms where these programs were being taught. It was very interesting.

The observers of the classroom visit for Mathematics: Modeling Our World saw a lesson from Course 3, an Algebra II lesson that was taught non-traditionally. What really sold them, when they looked at the role of the teacher and the role of the student, was that the teacher was helping the students, leading the students to discover the algebra. She was giving them clues. She was helping them with the activities that they were doing. She was evaluating her students as they were going through that process. The students were active and were seeking out evidence to
solve their problem. Our teachers felt that this was higher-level thinking, that this
was a process they wanted students to do, even though this was going to be hard
for students and teachers. They knew that this was going to lead to the open-ended
problem-solving process needed for the CAPT and to move student learning.

**Why Mathematics: Modeling Our World?**

What finally worked out sounds sort of like the Goldilocks and the three bears
approach: one program was too hard, one program was too easy, and the third
program was just right. This COMAP program suited us and met our needs. The
teachers could see the integrity of the program and could understand where it was
going. In the observation, they immediately identified with the teacher who was
teaching the program. They could see their students sitting in that classroom and
doing that work. The other two programs that we visited just didn’t do that, at least
not for us on that day and in that visit.

The committee made the decision to go with MMOW. As a supervisor, I use a
constructivist approach with the teachers as well. I believe that they have to go
through the process of making decisions. I do not make the decisions for them. So
they went through a little disequilibrium and discomfort; they were going to have to
make these decisions and know that they had made the right decision in the end.
That was so hard for these teachers because they were making decisions that would
change instruction and change where we wanted the students to be. It was also
tough because they were representing their colleagues. It would have been much
easier, and they would have liked for me to have made the decision and said, “Oh,
why don’t you go with COMAP.¹ I mean, stop all this fussing.” But I wouldn’t do it.
I just would not do it.

I got some very strong support from the deputy superintendent, who assured the
teachers that what they were doing was important. Now I see the smiles on their
faces. They keep saying to me, “We’re doing it! This is great stuff. I believe in this
stuff!” It fits with our district initiatives to move towards problem-based learning
for all students.

Our district had a formal process for passing our decision by the school board. We
had a subcommittee from the board, called the Curriculum Committee. That
Curriculum Committee met with the teachers, who did a presentation and told
them why they had decided that this was the right program. The committee asked
questions about the change. As with any board, the bottom line was cost, so we ran
through the cost factor with them. And then the materials were put on display for
one month for any of the board members or anyone that the board would invite to
come and look at them.

Only hindsight can tell you what you would have done differently. I think we
probably would have done a little bit more development of a timeline for where we
were going and what we were doing. We knew where we had to be and when we
had to be there, but we hadn’t set up a true timeline for the process. When do you
order your materials to review? When are you going for your visits? Who are you
going to be talking with? What are your questions? You can’t do that until after you
have the formal discussion about what you want the program to look like, who are
your clients—all those pieces that you want to go through before you start. It’s not
change for change’s sake; somebody once said, “if you do what you’ve always
done, you’re going to get what you’ve always gotten.”

¹ Consortium for Mathematics and its Applications, the developers of MMOW.
We currently use Mathematics: Modeling Our World in the 9th and 10th grades for college-bound students. It’s also being used at our middle school with precocious students who are taking Algebra in 7th grade. Our teachers are using MMOW as a full curriculum, nearly exclusively. The only materials that they’re using in addition are some practice materials and materials the department has developed to help get students ready for our statewide testing.

**Goals for mathematics instruction in the district**

At our middle school we also are using a reform-based program, one of the NSF programs, Connected Mathematics. We’ve been using that for three years and the students are used to and know the investigation process. They are familiar with working in groups and they’re used to thinking without the teacher giving them answers. So, they’re used to a non-traditional approach. The problem comes with the expectations at the high school, which has always been a traditional expectation, for being able to sit in straight rows and do answer recall. Some students are very good at that. They’re very good at the structured kinds of lessons, and have balked all the way through middle school at not having this kind of a structure.

To capsulize it, we still want to maintain in our program the integrity of abstract mathematics such as algebra and geometry, trig, calculus. But we want it to be more the process of problem-solving, leaning more towards problem-based learning and giving the students an inquiry-based, constructivist approach. So we wanted to change not necessarily the content or the integrity of high school mathematics, but we wanted to change the role of the teacher and the role of the student.

**Presentation of the mathematics**

At first, the students were saying, “This isn’t math.” There are still a few who say that. But I was in a class where they were working on the second unit in the book. They were looking at variables and defining variables by working on codes. I walked over to some students and I said, “What are you doing?” And they said, “Oh, we’re developing a code for the other group.” And I said, “Oh, really?” I said, “I thought this was an algebra class.” And they said, “It is.” And I said, “What does this have to do with algebra?” And they looked at me and they said, “Miss, don’t you know? Don’t you understand that algebra is made up of letters? Didn’t you ever hear of x and y in an algebra equation?” And I said, “Yes. I don’t see any x’s and y’s.” And they said, “It represents something. Don’t you understand that x or something in an equation represents something? And so you’re going to see us doing equations like you’ve never seen us do equations before.” They were so defensive because they thought this was the best thing that they had ever done in their lives.

Students buy into the program because it’s based on real-life problems. Every unit starts with a video presenting a real-life problem. The coding unit video started with World War II and the need for cracking codes. There was going to be a bomb dropped someplace in the South Pacific, but they didn’t know where; they had to crack a code in order to know. The kids sat there watching that video, mesmerized. They were totally glued to the idea.

The unit that they worked on next is a forensic unit. They look at bones that have been discovered archaeologically and how bones relate. Students buy into it because it’s based on real life and because they’re learning actively, not passively.
They get more and more used to that. In the past, they liked to be given worksheets: “Give us a worksheet, tell us what to do, give us the answers and we'll go home.” Now they're finding that it's hard work to think, so I don't think we've won that battle yet. I think that they need to continue with MMOW in a four year sequence.

**Teacher materials**

One of the things that the teachers say is the best part of the program is a wonderful notebook full of teacher background material. They get a lot out of that in terms of the mathematics and in terms of coming on board with the lessons and the preparation.

There were two teachers from another district who were looking at COMAP who came for a site visit to us, which was a nice turnaround. Our teachers insisted that these teachers who are looking at the program borrow our instructional notebooks. They needed to see them. The traditional teacher's manual with the surround materials on the page was good, but it didn't give them this kind of background, transparencies, or an idea of what the whole lesson looked like and what all the pieces were in it.

**Implementation**

Although we're in the process of reviewing tracking and grouping, we're currently tracked and leveled. Mathematics: Modeling Our World is being used right now with our Honors and Level 1 students who are college-bound. We were originally looking for a book that would be appropriate for Level 2 students or for non-college-bound students. However, when we came to the conclusion that this book fit a very important need for our district, we decided to push it into our Level 1 so that the problem-based learning wasn't labeled as a lower-level, less challenging program.

We also have the algebra program that begins in 8th grade— and with some 7th graders as well— and they're still in the traditional program. To get people off of the tracking concept is going to take a lot of discussion; this change is all about having conversations with teachers so that we come to the understanding of where students should be and where we want them to be. If we had a mixture of Level 1 and Level 2, then we could bring the Level 2 kids up to the Level 1 abilities. We have a diversified population, with students with a lot of different needs and a high number of special ed students.

**Seeing results**

Now that we're implementing, teachers are seeing students change from wanting to just get answers and give back answers, to being thinkers. These students are learning the mathematics more deeply than they've ever learned it before. Even though in the past they've had exposure, for instance, to equations, students are now learning what an equation really is, what a variable is, how you use them, and how you apply them to real-world situations.

I think the teachers were very scared to start, and were going through a lot of disequilibrium in the beginning. After doing the first unit, I think probably four of the six teachers were ready to just throw in the towel. We had a lot of help and a lot of support from the COMAP people, who came down and got them through and over their questions. All the first month, the teachers kept saying to me, “Are you sure this is going to meet our CAPT needs? Are you sure this is going to meet the SAT? I don't see the algebra for the SAT. Are you sure that this is going to help
these students learn mathematics?” By the second unit, they were beginning to think, “This might work.” And now that they’re heading into their fourth unit, they’re saying, “This is really working. This is doing it for our students.”

**Professional development**

The only formal training our teachers have had with *Mathematics: Modeling Our World* was an overview session, done by COMAP last summer, called a Leadership Training Institute. Since then, we’ve tried to do a few additional things. One of the authors and one of the people working with the implementation came down from COMAP in October to do a question-and-answer session. Another district that has adopted MMOW came, and our teachers made it into a planning day. The interaction with the visiting teachers was a big plus, too. Otherwise, these teachers have gotten together on their own. They have had several structured release days, but they don’t want to be out of their classes. We also have some district-wide conflicts in terms of release time for teachers.

Our MMOW teachers did get together to develop a district-wide midterm, so that they were all giving the same midterm, and final exam, as well. They went to another training session this past summer with the COMAP people, especially those preparing to teach the second course.

We don’t have other formal plans for professional development, although teachers here always have the opportunity for professional development. Our state has the PIMMS program, which is the Project to Improve the Mastery of Mathematics and Science in Connecticut. Teachers can go for one-week trainings in the summer. Otherwise, we have a district-wide technology grant and so we are looking at the possibility of teachers taking online courses.

The first year, our MMOW teachers presented to our entire staff about what they’ve been doing with the program. We had a two-hour professional development for the entire staff. The first year, our MMOW teachers presented to our entire staff about what they’ve been doing with the program. We had a two-hour professional development for the entire staff. This is teachers teaching teachers. They spent 45 minutes with one of the units and 45 minutes with another unit, giving the staff a taste. What they were doing was showing their peers where the algebra is; they showed them a dynamic piece of algebra that they could use in their own classrooms as long as it was taught as a discovery for the students. So I think that was important; they were willing to work with their own peers. That’s such a big step.

**Challenges for teachers**

The instructional change to teaching *Mathematics: Modeling Our World* has been very difficult because five of these six teachers are veteran teachers. There were two brand new teachers at the middle school, so they had no experience and started tabula rasa with this instructional approach. They’re doing fantastically well. I think the thing that they struggle with most is the preparation for the classes. They take a lot of time to prepare for these classes, trying to stay one day ahead of the students, and trying to understand the deep mathematics that they’re teaching. I had one staff member say to me that she had to first understand the mathematics before she could teach it. She thought she knew what the mathematics was, except that this program requires you to understand where the mathematics is coming from.

Being prepared with the graphing calculator and knowing what to do and having the materials ready is another big piece of it. Of course, some things haven’t changed. Teachers are still doing a lot of copying, and they still have to do lesson plans like they always did before. But they’re spending an inordinate amount of
time planning now because these are the first years through the program. They already know what they're going to do differently next year. They keep a log of changes they need to make.

I don't think instructional change can be a top-down decision. It has to come from within. These teachers have to know what they're doing and where they're going, and that they have to make decisions. If you want them to teach with a constructivist approach, you have to treat them with a constructivist approach and let them make the decisions. I don't believe that you can do it any other way and have success.

**Evaluating teachers**

Teacher evaluation has become tricky because other teachers are not yet teaching a non-traditional approach. I don't want to play favorites. That's really hard in my supervisory position, because I am in the position to evaluate and hire and fire; I work very closely with the principals. We have an evaluation process, where we set goals and strategies, and the evaluation is based on where teachers want to grow and what they want to do with their individual goals and strategies. The MMOW teachers have a goal to grow and develop with the COMAP program. Other teachers may have technology or parent communication or some other goals.

For teachers who are teaching with direct instruction, it's not necessarily a bad process; it's just not where these MMOW teachers are. I don't know that I've totally figured out how to treat that yet. I've had some very hurt staff at one school come to me and say, "You know, I want to change, too. I wouldn't mind doing this new program." Some of the staff who want to do it may not be ready to do it or don't understand what they're getting themselves into, or have to stay behind for the other students who aren't on board. For those teachers who haven't changed their instructional approach, I think their reluctance is fear that they wouldn't be in control. Philosophically, I think they agree with the program. So that's not a bad place to be.

**Community Support**

We haven't had very much questioning about the program. A handful of parents have been concerned about where the algebra is in the program. They also were curious about how this book was decided on; that was easily answered because we had had a process. Another parent, whose student had received a C in this class and who had never received a C before, raised the question of whether it was the program or the kid.

Our middle school people are applauding the high school people for coming on board because they've been doing this for three years. And the high school principals are ecstatic because they can say to the NEASC (New England Association of Schools and Colleges) that they have a piece of problem-based learning coming on board. We're right on target with NCTM, and I think we're right on target with our state goals and our state framework. So those are the pieces that are behind us, knowing that this was the right decision. I just hope that it works for two years and three years and four years and five years. Then we'll see where it evolves after that.