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Reviewing a mathematics textbook

COMAP's Mathematics: Modeling Our World Course 1

1998. 762 pages. ISBN of the student's edition: 0-538-68210-8. Developed and copyrighted by COMAP (Consortium for Mathematics and Its Applications), of Lexington, Massachusetts. Sold by W.H. Freeman and Company, 41 Madison Avenue, New York City, New York 10010.

Editor's Introduction -- The text that is reviewed here is the first in a three-book series. The series is intended to provide a three-year high-school curriculum in math, replacing elementary algebra, plane geometry and intermediate algebra.

My Answer Is "Yes"

Tom VanCourt

During the past two years or so, I have done reviews of three successive versions of Glencoe's fuzzy and frenetic schoolbook *Glencoe Pre-Algebra*, a mathematics book that, in all of its incarnations, has made me wonder how anyone might learn any math from its pages [[see note 1, below](#)]. Now it is my agreeable task to tell you about a book -- *Course 1* in the three-book series *COMAP's Mathematics: Modeling Our World* -- that is refreshingly different from Glencoe's product.

The visual features of *Course 1* signal immediately that it is a serious book. The COMAP designers have used simple typography, they have used illustrations only in situations where illustrations enhance the book's text, and they have used color only for helping the student to grasp meaningful information. As a result, *Course 1* bears no similarity to Glencoe's riot of jumbled fonts, meaningless pictures, and splashes of color that signify nothing. While Glencoe's book continuously batters the student's gaze with glitz, *Course 1* invites the student's eyes to move smoothly across its pages.

In their introduction, on page iii, the COMAP writers say:

We have attempted in this text to demonstrate mathematical concepts in the context of how they actually are used day to day. The word "modeling" is the key. Real problems do not come at the end of chapters in a math book. Real problems don't look like math problems. Real problems ask questions such as: How do we create computer animations? Where should we locate a fire station? How do we effectively control an animal population? Real problems are messy.

By acknowledging that "Real problems are messy," at the very start of their book, the COMAP writers earned my respect -- and my respect for them grew as I read further. *Course 1* is a thoughtful textbook that treats the student as a capable, intelligent person, encourages critical and self-critical reasoning, and assumes that the student can hold onto an idea for more than a few minutes.

While trendy math books like *Glencoe Pre-Algebra* offer a mad whirl of topics and unrelated techniques, *Course 1* takes a single subject, develops it at length, and uses it as a context for helping the student to acquire analytic skills and to adapt them to a succession of new situations. Indeed, adaptation is one of the skills that the student acquires.

The body of the book consists of eight units. Unit 1, "Pick a Winner," deals with elections and systems of voting, and the student soon sees that applications of math in the real world can be messy indeed. The writers show that any plurality election [\[note 2\]](#) is subject to this failing: If the election involves more than two candidates, the winner may not be the candidate whom most of the voters would want to see in office. The writers then tell how "preference diagrams" can be used for representing voters' attitudes in a three-candidate election, and they invite the student to develop an alternative voting system that might avoid the flaw which is inherent in the plurality system. Next they ask the student to test his alternative system quantitatively, and they remind him that one successful test doesn't mean that the new system always will work well:

If you feel you have a method that never produces the plurality method's flaw, congratulations! However, it's possible that your method has other problems. Think about difficulties your method might encounter. Write them down. Finally, tell whether you think these problems are more serious or less serious than the plurality method's flaw. [page 12]

The writers thus invite the student to engage in subtle, rigorous adult reasoning.

Next, in three dozen pages or so, the COMAP writers introduce the use of directed graphs (page 18) and matrices (page 25) for analyzing elections, and they discuss runoff systems, point systems, pairwise voting, approval voting, and more -- and they do all of this without even using linear equations.

Unit 2, "Secret Codes and the Power of Algebra," introduces linear relations, graphs, functions, multi-step operations, symbolic equations, subscripted variables, and other topics. These are presented in measured, competent ways which reflect the assumption that the student possesses intelligence. When the writers introduce calculator graphing, on page 75, they assume that the student already knows (or can find out) how to use a graphing calculator, and they focus on teaching principles -- not on forcing the student to execute rote sequences of key-strokes. Their material isn't specific to, and doesn't require the student to purchase, any particular model of calculator. This is good, because calculators become obsolete and disappear from the market even faster than textbooks do.

I find, though, that Unit 2 is deficient in two ways. First, the writers haven't taken account of a common problem that arises in the use of graphing calculators: It is hard to get a display in which the two axes have the same scale. As a result, a line whose true slope is 1 may be shown as a line that forms an angle of 30 degrees, 45 degrees, or 75 degrees with the abscissa. This makes it difficult for a novice to develop intuitive connections between algebraic coefficients and geometric angles.

The second failing of Unit 2 lies in its treatment of "secret codes," because the material is obsolete and creates inaccurate impressions. For example, the writers convey the idea that encryption and decryption are inverse processes that necessarily proceed forward and backward along the same path, but this is no longer true. A modern code may be a kind of cycle: Encryption pushes the message partway through the cycle, then decryption pushes it the rest of the way -- following a different path -- to regenerate the readable message.

Unit 3, "Landsat," is given chiefly to geometry. Here is how it begins:

How much land has been cleared of forests in a particular region of the world? How can police agencies identify fugitives who have tried to change the way they look? What do these two problems have in common? One common thread is the geometry of size and shape.

That opening reference to the clearing of forests put me on my guard, because I remembered Marianne Jennings's account of a fluffy math book that was full of "save the trees" messages and other pop confections [[note 3](#)].

Course 1, though, omits political slogans and presents an exercise in which the student, using an image transmitted by the Landsat 5 satellite, assesses how much deforestation took place in a specified part of the Czech Republic during a specified period of time. In performing this exercise, the student must carry out two tasks: finding the areas of forested regions shown in the Landsat photo, and listing additional information that is needed for completing the analysis.

The second is the more important of these tasks, and the additional information that is required for computing the extent of deforestation won't appear until later in the unit. Does this mean that the COMAP writers are taunting the student with a problem that he can't possibly solve when it first is presented? No. In my view, it means they are teaching that, in the real world, the selection of techniques and information is as important as competent calculation. Real problems are messy, and they rarely come with all of their parts neatly packaged.

Later in Unit 3, the student uses a map (apparently a U.S. Geological Survey map) to find the area of Crater Lake. This exercise, too, does dual duty: The writers teach a technique for estimating the area of an irregular shape, and they also teach the concept that an estimate has upper and lower bounds. Then, as a bonus, they explain how an area can be estimated by using Monte Carlo integration -- a logical extension of the process of approximating an area by counting grid squares.

At a few places in Unit 3 the student is encouraged to use a computer for solving a problem (e.g., problem 2 on page 188, problem 8(b) on page 198, and problem 8 on page 249). However, most of the problems can also be solved with pencil and paper, so the student is seldom *required* to use a computer, and he never is required to have access to the Internet.

Unit 4, titled "Prediction," concentrates on methods for devising predictions founded upon empirical correlations -- for example, the correlation between a 14-year-old person's height and the length of the person's head, or the

correlation between an adult's height and the length of his tibia, or the correlation between an adult's height and the sum of the lengths of his tibia and his femur. While solving problems that invoke these correlations and many others, the student learns about scatter plots, about the graphing of linear equations, about ways to assess the reliability of some simple mathematical models, and (eventually) about the least-squares technique for fitting a line to a set of data.

Early in Unit 4, the student finds a brief sidebar about Mildred Trotter, the physical anthropologist who showed that the sizes and proportions of human bones were correlated with age, sex and other variables, and that such correlations could be utilized in identifying individuals from their skeletal remains. The sidebar is a sober little sketch of Trotter's science, not an ostentatious attempt to attain political correctness by ballyhooing a female. *Course 1* is admirably free of politically correct digressions, and the COMAP writers tell about individuals only when there are good reasons for doing so. They never resort to stunts like the ones that I saw in the Glencoe book, such as Glencoe's attempt to convince students that the first appearance of a black woman on the cover of a fashion magazine was a benchmark in the history of mathematics.

Unit 5, called "Animation," deals with the rudiments of computer animation, starting with simple ways to describe the movements and transformations of geometric lines and figures within a reference system. The writers then discuss how linear equations can be used to describe how the position of a point changes with time, and they relate this to the problem of causing a dot to travel horizontally through the pixels of a computer monitor. Next they show how, by the use of parametric equations, the dot can be made to move diagonally too. Eventually the student learns to use a graphing calculator to create an animated display of two dots chasing one another, and joins other students in generating a display of a star-burst firework. The writers thus construct vivid ways to help the student to understand parametric equations, intersecting lines, and collisions.

Near the end of their unit, the writers direct the student to compose a new animation that can run on a graphing calculator, and they add this: "Your program must be unique and show your creativity." I am very pleased to see applied math described as a creative undertaking.

Unit 6, "Wildlife," starts out with a tale about moose -- not fuzzy, fictional moose but real moose that inhabit the Adirondack State Park in upstate New York:

In 1988, it was estimated that between 15 and 20 moose were in the park. In 1993, new estimates put the number at 25 to 30 moose. The New York State Environmental Conservation Department (ECD) conducted a survey at that time to determine what policies the public favored. A majority of the people surveyed favored a "gradual increase in the moose population as the animals migrate from nearby New England states and Canada and an expansion of their numbers through natural reproduction." Conservationists suggested moving 100 moose into the park over a three-year period. The ECD determined that such a plan would cost \$1.3 million.

Put yourself in the position of commissioner of the ECD, and suppose that you must make a recommendation to the governor about this situation.

In the context of that assignment, the student now undertakes to build a mathematical model of projected changes in the moose population, beginning with the selection of factors that must be reflected in the model and must be used as inputs. These include migration rates, sex ratios, reproductive rates, and rates of importation, which fold into a recursive formula. As the unit continues, the student learns about exponential functions (page 556), about

graphs of exponential equations, about the multiplication of terms containing exponents (page 571) and about probabilistic models (page 588). He tests his moose model to learn how sensitive it is to changes in the values of the inputs, and he examines the model to see whether it works -- i.e., whether it gives reasonable results. The COMAP writers are honest enough to say that when a model doesn't work, the only practical remedy may be to tinker with some of the parameters until the output seems right. Engineers know this dirty little secret, and now the student knows it too.

In Unit 7, called "Imperfect Testing," the theme is probability and the context is the job of determining whether athletes have been using performance-enhancing drugs. The unit starts with basic ideas about sampling, followed by discussions of probability and joint distributions. The writers' next step, though, is unusual: While considering the use of surveys to sample people's attitudes and habits, they point out that some survey-questions -- such as "Do you use outlawed drugs?" -- are likely to elicit some replies that are lies. This leads to discussions of conditional probabilities and to exercises in the analysis of data. Next, the writers tell about laboratory tests that may yield false positives or false negatives, and they go on to describe how, by building models based on first-order rational functions, an analyst can estimate the number of true positives in a data-set that contains false positives too.

In Unit 8 -- "Testing 1, 2, 3" -- the writers continue their exposition of probability. They discuss the expected value as an analytic tool, they formally introduce quadratics, they apply quadratics to probabilistic modeling and to regression, and (in the unit's final lesson) they teach how to solve a quadratic by completing the square.

Course 1 sometimes resembles a novel because of the writers' practice of providing foreshadowings of things to come. For example: Recurrence relations appear in Unit 5, although they will not be treated extensively until Unit 6; and quadratics appear several times (in Unit 2, Unit 3 and Unit 5) before they are formally introduced and developed in Unit 8. The COMAP writers recognize the value of exposing math students to a concept before making them responsible for knowing it.

In some places, the writers even seem to look ahead to other courses that the student will take. Hence they briefly introduce delta notation (on page 171), although it doesn't play any important role in the rest of the book. They also introduce matrices, gently, and suggest how matrices can be employed in computer animation. Next year or the year after that, the student will need to use matrices for vector algebra or for solving large systems of equations.

After reading *Course 1* I've asked myself: If I were to learn, at some time in the future, that a bridge had been designed by engineers whose knowledge of math was founded on *Course 1* and books of comparable quality, would I be willing to drive on that bridge? My answer is "Yes."

Notes

1. See ["Glencoe's Manual of Fuzz"](#) in *TTL* for May-June 1999; ["This Schoolbook Is Stuck in Time"](#) in *TTL*, September-October 1999; and ["Less Is Better -- but It Still Isn't Good"](#) in *TTL*, September-October 2000. [\[return to text\]](#)
2. In a plurality election, the winner is the person who gets a plurality of the votes cast, even if this plurality isn't a majority. [\[return to text\]](#)

3. See the review "[Rain-Forest Algebra and MTV Geometry,](#)" by Marianne M. Jennings, in *TTL* for November-December 1996. [[return to text](#)]

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