

2009 MCM Judges' Commentary—Problem B

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General Remarks

As in past years, the diverse backgrounds of the undergraduate participants yielded many interesting modeling approaches to the stated problem. The judges assessed the papers on the breadth and depth of analysis for all major issues raised, on the validity of proposed models, and on the overall clarity and presentation of solutions.

The Executive Summary is often still below par; the Summary should motivate the reader to read the paper. It must not merely restate the problem, but indicate how it was modeled and what conclusions were reached, without being unduly technical.

Assumptions must be clearly stated and justified where appropriate. Some teams overlook important factors and/or make unrealistic assumptions with no rationale. It should be made clear in the model precisely where those assumptions are used.

Graphs need labels and/or legends. They should provide information about what is referred to in the paper. Tables and graphs that are taken from other sources need to have specific references. Failure to use reliable resources and to properly document those resources kept some papers from rising to the top. The best papers not only list trustworthy resources but also document their use throughout the paper.

The Problem and Selected Modeling Approaches

Problem B involved the “energy” consequences of the cell phone revolution and five Requirements were delineated. To receive an Outstanding or Meritorious designation, teams had to address issues raised in each of these Requirements. Additionally, outstanding papers considered both wireless and wired landlines and the infrastructure to support cell phones and landlines.

Requirement 1:

Teams were first asked to estimate the number of US households in the past that were served by landlines and also to estimate the average size of those households. They were then to consider the energy consequences in terms of electricity utilization of a complete transition from landline phones to cell phones, with the understanding that each member of each household would get a cell phone.

In order to address this problem, the energy used by current landlines had to be considered. Whereas corded landline phones use relatively little electricity, the same cannot be assumed

about cordless landline phones. The top papers researched this issue and arrived at documented estimates of the number of corded versus cordless landline phones and the average number of each per household. This led to a more realistic appraisal of the energy used by the landline phone system.

With regard to cell phones, teams that rose to the top considered the infrastructure necessary, for example, the building of numerous additional communication towers if cell phones are to completely replace landline phones. This was of special importance in the analysis of the transitional phase. Also, the varying amount of electricity required by different types of cell phones was a consideration in the transitional and steady state phases.

Interesting models were constructed for the transitional phase of the cell phone “takeover.” Some teams considered the spread of cell phones as the spread of a disease and used the Verhulst model for logistic growth, using the population of the United States as the carrying capacity and estimating the rate of growth of cell phones from published reports on the growth of cell phone use in the United States. Other teams generalized this to an SIR model or used the Lotka Volterra predator-prey model, with cell phones as the predators and landline phones as the prey. A few used the competing species model. The judges looked very favorably upon models for which sufficient rationale was given as to why that model might be appropriate in this circumstance. Interpretation of the parameters and solutions as they applied to the problem at hand was essential.

Many papers ignored the transition phase and only considered the energy comparison for the steady state in order to determine the energy consequence. Some teams merely talked their way through the issues and did not construct a mathematical model. After estimating energy costs associated with landline phones and cell phones, many teams used linear equations to model the total costs associated with the numbers of phones.

Requirement 2:

Teams were asked to consider a “Pseudo US”—a country similar to the current US in population and economic status, but with neither landlines or cell phones. They were to determine the optimal way to provide phone service to this country from an energy perspective. The teams were also to take into account the social advantages that cell phones offer and the broad consequences of having only landlines or only cell phones.

Once again, consideration of the infrastructure for phones was important. In addition to landline and cell phones, many teams considered the VoIP (Voice over Internet Protocol) communication option. Not every team that considered VoIP took into account the costs for laying the cables. Some assumed that such cables were already in place, a questionable

assumption, but failure to consider the VoIP method of phone service may have kept a Meritorious paper from becoming an Outstanding paper. If one were to assume that households would already have one or more computers with internet access, the energy costs associated with VoIP would be quite small.

In terms of finding the optimal way to provide phone service from an energy perspective, some teams used linear programming, utilizing the costs determined in the first requirement and quantifying in various ways the social advantages of cell phones, as well as the preference for landlines in certain circumstances. Other teams used AHP (Analog Hierarchy Process), which worked well to get parameters used in the optimization routine, but did not work as an optimization tool in itself. Teams that considered the advantages and disadvantages of various phone types not just for individuals, but for businesses also, demonstrated a thoroughness that was commendable. Another factor that some teams considered was the number of children under 5 who would have no need for cell phones.

Requirement 3:

This was an extension of Requirement 2, asking teams to consider the electricity wasted when cell phones are plugged in that do not need charging and when chargers are left plugged in after the phone is removed. Teams were to continue to assume that they were in the Pseudo US and were to interpret energy wasted in terms of barrels of oil used.

To determine the amount of energy wasted, teams had to first estimate the number of hours that a “typical” cell phone charger is in the state of charging a phone, left plugged into a phone not in need of charging, and left plugged in when the phone is not connected to it. Some teams did their own informal surveys, but better estimates were arrived at from publications and surveys. In some papers, probability distributions were used to describe this behavior, but use of such distributions was not always justified.

Teams that were more comprehensive took into account the fact that some cell phones and chargers use less power than to do others, based on brands, age, and capabilities of the phones and chargers. Assuming that all electrical energy is generated by oil, translating kilowatts of energy into barrels of oil used was a straightforward transformation.

Requirement 4:

This extended the concepts in Requirement 3 and asked teams to estimate the amount of energy wasted by all electronic chargers. Since this question was very open-ended, contest papers showed a wide variety of estimates for the energy wasted in terms of barrels of oil. The top teams estimated the average hours that appliances are left plugged in charging and not charging and also the number of hours chargers are plugged in without the appliance.

More comprehensive papers considered many electronics and, by comparison, showed that the amount of energy wasted by cell phones is relatively small compared to many other electronic devices.

Requirement 5:

For this part, students were to consider the population and economic growth of the Pseudo US for the next 50 years and predict energy needs for providing phone service based on their analysis in the first three Requirements. Predictions were to be interpreted in terms of barrels of oil used.

Papers needed to consider both economic growth and population growth in order to estimate energy needs in the future. Various types of regression fits were applied to historical population data and economic data such as GDP. Using earlier estimates of energy requirements, coupled with the regression equations from historical data, predictions were made for the amount of energy used every decade for the next 50 years. Some teams predicted greater efficiency in future phones and the development of chargers that would use less electricity.

Papers showed estimates for the number of barrels of oil used on a per day basis or perhaps on a per year basis. There was no one right answer and answers given depended on assumptions made at the start. Some papers contained graphs displaying future values but did not give tables. A table with a graph is a better way to display information.

Concluding Remarks

Mathematical modeling is an art that requires considerable skill and practice in order to develop proficiency. The big problems we face now and in the future will be solved in large part by those with the talent, the insight, and the will to model these real world problems and continuously refine those models.

The judges are very proud of all participants in this Mathematical Contest in Modeling, and we commend you for your hard work and dedication.

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