

2017 MCM

Problem B: Merge After Toll

Multi-lane divided limited-access toll highways use “ramp tolls” and “barrier tolls” to collect tolls from motorists. A ramp toll is a collection mechanism at an entrance or exit ramp to the highway and these do not concern us here. A barrier toll is a row of tollbooths placed across the highway, perpendicular to the direction of traffic flow. There are usually (always) more tollbooths than there are incoming lanes of traffic (see former [2005 MCM Problem B](#)). So when exiting the tollbooths in a barrier toll, vehicles must “fan in” from the larger number of tollbooth egress lanes to the smaller number of regular travel lanes. A toll plaza is the area of the highway needed to facilitate the barrier toll, consisting of the fan-out area before the barrier toll, the toll barrier itself, and the fan-in area after the toll barrier. For example, a three-lane highway (one direction) may use 8 tollbooths in a barrier toll. After paying toll, the vehicles continue on their journey on a highway having the same number of lanes as had entered the toll plaza (three, in this example).

Consider a toll highway having L lanes of travel in each direction and a barrier toll containing B tollbooths ($B > L$) in each direction. Determine the shape, size, and merging pattern of the area following the toll barrier in which vehicles fan in from B tollbooth egress lanes down to L lanes of traffic. Important considerations to incorporate in your model include accident prevention, throughput (number of vehicles per hour passing the point where the end of the plaza joins the L outgoing traffic lanes), and cost (land and road construction are expensive). In particular, this problem does not ask for merely a performance analysis of any particular toll plaza design that may already be implemented. The point is to determine if there are better solutions (shape, size, and merging pattern) than any in common use.

Determine the performance of your solution in light and heavy traffic. How does your solution change as more autonomous (self-driving) vehicles are added to the traffic mix? How is your solution affected by the proportions of conventional (human-staffed) tollbooths, exact-change (automated) tollbooths, and electronic toll collection booths (such as electronic toll collection via a transponder in the vehicle)?

Your MCM submission should consist of a 1 page Summary Sheet, a 1-2 page letter to the New Jersey Turnpike Authority, and your solution (not to exceed 20 pages) for a maximum of 23 pages. Note: The appendix and references do not count toward the 23 page limit.