

# ICM

**The Interdisciplinary Contest in Modeling:  
Culturing Interdisciplinary Problem Solving**

Edited by

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# A Short History of the Interdisciplinary Contest in Modeling

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## Roots of the ICM

Back in the pre-STEM era of the late 1980s, the Division of Undergraduate Education of the National Science Foundation (NSF) took an interest in developing more interdisciplinarity in undergraduate education. This was an insightful initiative that in some ways anticipated the needs of the more complex, virtually-connected, information-driven society that emerged at the dawning of the 21st century.

The NSF developed a special initiative called Mathematical Sciences and Their Application Throughout the Curriculum (MATC), which was designed to increase student understanding of and ability to use the mathematical sciences to solve real problems in society. This was NSF's realization that mathematics could be the linking discipline to make undergraduate education more holistic, more interdisciplinary, and therefore more valuable to society.

In 1995, the NSF funded several large-scale curriculum and pedagogy projects, along with smaller adapt-and-adopt projects to transfer these innovative interdisciplinary programs to colleges and universities around the country. Schools awarded large projects included an Oklahoma State / University of Nebraska partnership, and individual awards to the University of Pennsylvania, Indiana University, Rensselaer Polytechnic Institute, SUNY (Stony Brook), Dartmouth College, and the United States Military Academy (USMA). The MATC initiative provided an opportunity for institutions to work together to reform education by developing the skills to use mathematics and other disciplinary knowledge in broader, more realistic, interdisciplinary problem solving.

The USMA West Point MATC initiative was called Project INTERMATH. INTERMATH built a consortium of 15 schools that sought to integrate their curricula in at least two ways:

- integrate the mathematics topics under the 7 into 4 template (7 basic traditional mathematics courses integrated into 4 semesters), and
- integrate interdisciplinary problems and perspectives into all mathematics and associated disciplines under the Interdisciplinary Lively Application Projects (ILAP) concept.

ILAPs were interdisciplinary projects written by at least two faculty members from different disciplines (often mathematics and a partner discipline) for use in courses in at least two disciplines. INTERMATH curricula and ILAPs worked well at several consortium schools, including USMA, Harvey Mudd College, and Carroll College, for a number of years. COMAP and the Mathematical Association of America (MAA) published ILAPs, the MAA in Arney [1997], and COMAP in *The UMAP Journal*.

The MAA's Committee on Undergraduate Programs in Mathematics also established a subcommittee chaired by Frank Giordano, long-time Director of COMAP's Mathematical Contest in Modeling (MCM), to study, guide, and promote progress in this area. The MAA hosted many meetings with representatives from partner disciplines to develop connections between the concepts, applications, courses and programs of these various disciplines. This is the interdisciplinary-appreciative environment that spawned the Interdisciplinary Contest in Modeling (ICM) in 1999.

## The First ICM

Germane to this volume, INTERMATH, through NSF's MATC program, supported the ICM in 1999 as a sister contest to the MCM, which was already a highly-successful large international contest managed by COMAP since 1985. The ICM was INTERMATH's outreach, beyond its consortium and ILAP products, to impact the national and—as it quickly became apparent—the international undergraduate education community.

The first ICM was a contest held over the same four-day time window as the MCM for three-student teams who registered specifically for ICM. The intent and hope was that advisors would note the announced topic for the ICM problem and recruit and prepare a team for that topic. Much like the ILAP requirement of authors from two disciplines, two advisors (ideally from different disciplines) would explicitly coordinate across disciplines to enroll their appropriately-formed and -trained team in the ICM prior to the contest and implicitly connect disciplines and integrate the educational culture at their schools.

The hope was that the contest would encourage both students and faculty members to reach out to other departments and disciplines and form interdisciplinary teams. The goals were to build a more integrated educational culture and increase interdisciplinary learning and problem solving on campuses.

**Table 1.**  
Participating teams and topics in the first 16 years of the ICM.

Year	Number of teams	Topic
1999	40	Controlling the spread of ground pollution
2000	70	Controlling elephant populations
2001	83	Controlling zebra mussel populations
2002	106	Preserving the habitat of the scrub lizard
2003	146	Designing an airport screening system
2004	143	Designing information technology security for a campus
2005	164	Harvesting and managing exhaustible resources
2006	224	Modeling HIV / AIDS infections and finances
2007	273	Designing a viable kidney exchange network
2008	380	Measuring utility in health care networks
2009	374	Balancing a water-based ecosystem affected by fish farming
2010	356	Controlling ocean debris
2011	735	Measuring the impact of electric vehicles
2012	1,329	Identifying criminals in a conspiracy network
2013	957	Planet Earth's health
2014	1,028	Using networks to measure influence and impact

Originally, the MCM and ICM were two simultaneous but separate contests, with slightly different rules. Under this separation, the ICM remained small compared to the MCM. In 2001 (ICM's third year), the ICM had only 17% of the total participants (83 ICM teams vs. 496 MCM teams). However, over the years, the ICM grew to 25–30% of the MCM contestants, peaking in 2008 with 33% (380 ICM teams to 1,162 MCM teams).

One disappointment was that there was only sparse anecdotal evidence that schools and advisors were explicitly reaching out across disciplines to prepare teams or produce stronger interdisciplinary connections. However, it was obvious from the hundreds (and eventually thousands) of high-quality team reports that progress was being made in interdisciplinary problem solving.

Since 2011, the ICM has been more integrated with the MCM and does not require special ICM registration. Any team that registers for MCM can choose the ICM problem, once they open the Website and see the problems at the appointed start time. Traditionally, the ICM problem has been designated as Problem C in the combined MCM/ICM contest; the two MCM problems are labeled Problem A and Problem B.

See **Table 1** for the specific problem topics and numbers of competing teams in each of the first 16 years of the ICM.

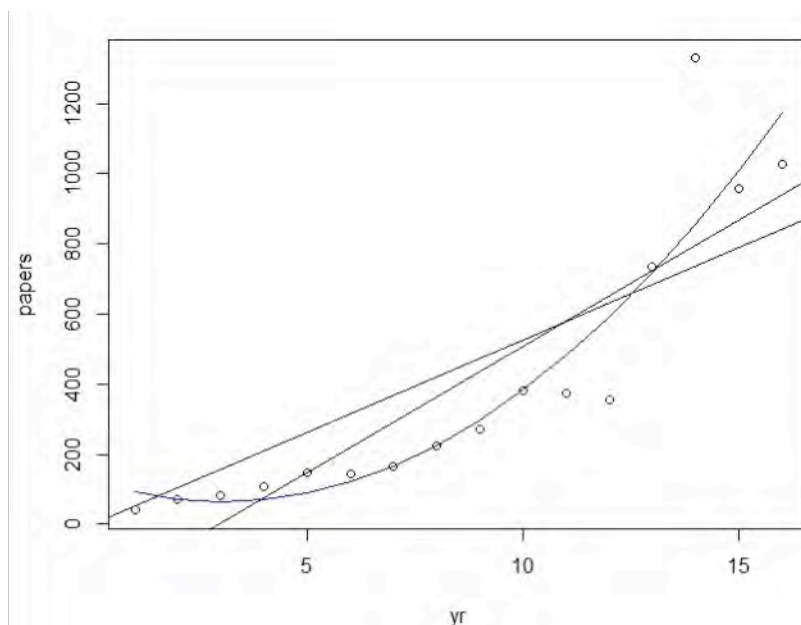
The results for every year of the contest have appeared in *The UMAP Journal*. For the first two years (1999 and 2000), the ICM results were part of the report on the MCM. For 2001–2012, the ICM had its own issue of the *Journal* to convey the results and show its Outstanding papers; in 2013 and 2014, both contests have appeared together in a double issue. *The UMAP Journal* publishes the lists of Outstanding teams, a judges' com-

mentary, often a problem author's commentary, sometimes a practitioner's commentary, and at least one of the Outstanding papers. A listing of the rankings for all the teams is published online on the COMAP Website.

It may seem a modest beginning by today's standards, but with 40 teams competing in 1999, the ICM was off to an exciting start on its crusade to make undergraduate education more interdisciplinary. The first topic theme was environmental science. Over the following years, the interdisciplinary themes and problems have involved elements from chemistry, physics, biology, engineering, information science, medicine, business, and network science. The problems also show a balance of public (government) and private (business) issues.

## Example of ICM Modeling

One skill that all teams bring to the competition is simple regression-curve fitting to data sets. As an example, if an ICM team had the data for ICM team numbers by year from 1999 to 2014 (**Table 1**), simple curve-fitting could produce several models (curves). Three possible functions are shown in **Figure 1**.



**Figure 1.** Data points and three simple example models for number of submissions (papers) from year 1999 (yr = 0) until 2014 (yr = 16).

linear (intercept at origin):	$\text{papers} = 52.7 \times \text{yr}$
linear (calculated intercept):	$\text{papers} = 72.1 \times \text{yr} - 212.5$
quadratic (with intercept):	$\text{papers} = 124.7 - 40.2 \times \text{yr} + 6.6 \times \text{yr}^2$

All three models show that 2012 was an outlier year with tremendous interest in the ICM problem related to the topic “Identifying criminals in a conspiracy network.” You can read that problem on pp. 146–151.

## Judging

Each year, the judges seek to categorize the submitted papers as Outstanding, Finalist, Meritorious, Honorable Mention, and Successful Participant. The judging is accomplished in two stages:

- First, the triage judges read the papers with the intent to find papers that are competitive for a Meritorious award so that the final judges can concentrate their more intense efforts on fewer papers.
- Then, the panel of final judges reads the surviving set of papers, evaluates their attributes, debates their merits, and decides on the final rankings (**Figure 2** shows a small sample of final judges).



**Figure 2.** ICM judges Rod Sturdivant and Joe Myers at final judging.

Over the 16 years of the ICM, the contest has grown from 40 papers in 1999 to a high of 1,329 papers in 2012. As the contest has grown, the judging has become even more challenging. The final and triage judging panels have been kept busy reading the 6,408 team submissions over the course of the 16 years.

The 20-page ICM submissions contain the earnest work of four days of modeling and problem solving on a challenging open-ended problem. For many students, these four days are the most intense academic work they have performed during the college experience. Most students are very proud of their submissions, even though all realize that because of the time constraints neither their work nor their reports have the benefit of being carefully polished and refined. That fact is a challenge for the expert judges, who must calibrate themselves to look past some of the rougher edges of the reports to find the real attributes and values of the teams’ interdisciplinary modeling.

The judging process ultimately determines category boundaries and classifications. There is no ICM category for “winner,” since the ICM staff feels all participants in the contest are winning teams and winning students through their participation. While there are several important administrative rules, the intellectual rules are simple:

Teams may use any inanimate source of data or materials: computers, software, references, web sites, books, etc. ALL SOURCES USED MUST BE CREDITED. Failure to credit a source will result in a team being disqualified from the competition. Team members may not seek help from or discuss the problem with their advisor or anyone else, except other members of the same team. Input in any form from anyone other than student team members is strictly forbidden. This includes email, telephone contact, and personal conversation, communication via web chat or other question-answer systems, or any other form of communication. [COMAP’s ICM Website]

Plagiarism and over-reliance on sources are monitored by the judges, and advisors are notified when these violations occur.

Looking at the range of topics over the 16 years (provided in **Table 1**), the contest shows its interdisciplinary flavor. This breadth of topics makes it a challenge to find expert judges who can confidently evaluate the reports and also calibrate to ICM-level quality. Writing a coherent science-based paper over a four-day period is a challenge no matter how proficient a team’s writing, modeling and analysis may be. The papers are all written in English, so some teams (especially the large number of Chinese participants) are writing in a second language.

## **Sibling Contests: MCM and ICM**

Thanks to the prior 14 successful years of the MCM, and armed with support from COMAP’s director Sol Garfunkel and the NSF through MATC and Project INTERMATH, the ICM was rather easy and quite natural to establish, unlike the more challenging and revolutionary start of the MCM itself in 1985.

Ben Fusaro, founder of the MCM, wrote about his road to launching the MCM with support from COMAP’s director Sol Garfunkel [1995]. Fusaro’s motivation came from both the potential and the frustrations associated with the longstanding individual and theoretical mathematics competition called the William Lowell Putnam Exam, which began in 1938 (see Arney [1994]). Fusaro desired an applied, collaborative, Putnam-like contest; with tremendous dedication and perseverance, he created the MCM. Fortunately, Fusaro and Garfunkel obtained a three-year grant from the Fund for the Improvement of Post-Secondary Education (FIPSE) to put the MCM in action. For the ICM with the benefit of building on the MCM, the small



funding by NSF as part of Project INTERMATH was not so essential. However, that national-level support, as well as encouragement by the NSF, was a confidence boost and gave a welcomed assist to the INTERMATH organizers. COMAP's HiMCM (the high school modeling contest), initiated in 1998, also derives from the MCM.

The ICM still has goals similar to the original ones written in the FIPSE grant for the MCM:

The purpose of [MCM and ICM is] to involve students and faculty in clarifying, analyzing, and proposing solutions to open-ended problems. . . . Major features include:

- The selection of realistic open-ended problems chosen with advice of working mathematicians [and, for the ICM, scientists] in industry and government.
- The ability of participants to draw on outside resources including computers and texts [and for the ICM, use of data].
- An emphasis on clarity of exposition in determining final awards with the best papers published in professional mathematics journals [so far, in just *The UMAP Journal*].

The underlying purpose and philosophy of the ICM is also in tune with that written by Fusaro about the MCM: “[T]he contest must be primarily an educational experience, not [just] a competitive one. . . . [We want] it to be closer to the spirit of traditional English sport than to modern American sports” [1995, 4].

As a demonstration of this educational emphasis, teams can get commentary and feedback from an ICM judge on the team's report, including analysis of the team's modeling skills and constructive criticism, so that students can improve their modeling.

The ICM looks at the many thousands of hours of interdisciplinary modeling experience as the most important product of the ICM—not the publication of Outstanding papers as examples for student researchers to learn from, nor the distribution of awards, plaques, and certificates to participants. The ICM is an education tool for society to develop better problem solvers and individuals to develop their interdisciplinary modeling talents. (See later chapters in this book to see descriptions and discussions of these skills.)

The judges recognize that because of the passion and intensity of teams' efforts that some teams' rankings are lower than they expect. The ICM applauds that intensity and pride in teams' products, but still sees the students' problem-solving experience as the most important and primary element of the contest.

## Making the ICM Interdisciplinary

An important issue that the ICM continually wrestles with for the contest is the nature and scope of interdisciplinary modeling at the undergraduate level. The ICM deals with this rather daunting breadth of interdisciplinarity by announcing the next year's theme or topic in the materials (press release, UMAP summary, flyer) from the year before. Following its original intent and tradition, the announced theme narrows the huge interdisciplinary world to a topic more in tune with undergraduates' interests and abilities and allows for preparation by teams.

Over the years, the announced themes have included environmental issues, ecology, health and biology, operations research, and network science. As already announced for 2015 ICM, the ICM will have a new addition to its problem menu. Like the MCM, **there will be two ICM problems starting in 2015**. The ICM will continue the network science theme for one of the problems, and the second problem (Problem D for contestants) will focus on environmental issues. Teams preparing for the 2015 contest should consider reviewing interdisciplinary topics in the areas of network science and social network analysis (for Problem C), or human-environment interactions in the areas of environmental science, climatology, food security, and geography (for Problem D).

Unlike the quantitative mathematical modeling approach to problem solving found in the MCM problems, the ICM chooses problems that benefit from perspectives from various disciplines (mathematics is often one) and (possibly) a more qualitative modeling approach. This kind of holistic or non-reductive framework is more consistent with interdisciplinary problem solving. Some ICM problems are more multidisciplinary, where perhaps reductive methods can be used to solve the tasks of the ICM problem. (For definitions of interdisciplinary and multidisciplinary frameworks, and reductive and non-reductive problem solving, see the chapter "Developing and Understanding Interdisciplinary Problem Solving" on pp 165–176.)

The ICM problems often involve the participants with large data sets, where the data need to be produced, collected, filtered, organized, and/or mined for information or patterns. Most ICM problems have involved information-based analysis and the development of measures of properties, or else the development of algorithms and/or simulations. Students can usually expect, as a minimum use of technology, to use computer software (spreadsheets, data analysis software, or programming languages), and Internet searching and browsing.

Modeling is a creative process where, based on assumptions, decisions and choices are made (often iteratively) to construct a model or framework. That model can then be solved, used, implemented, tested, and/or validated, in an effort to solve a problem, accomplish a task, understand a phenomenon, build a system, and/or make a decision. In the ICM, the problems are written to require teams to research and incorporate science

knowledge and perspectives in their models in order to engage with a real-world (usually human-based) problem. (See the Interdisciplinary Problem Solving Chapter in Part 2 of the volume for a more thorough description of modeling.)

## Problem Authors

The ICM has benefitted from its connections with expert interdisciplinary problem solvers and researchers who have shared their challenging, data-rich problems with the ICM participants. The list of the ICM problem authors and their primary discipline is found in **Table 2**. Beginning in 2011, the ICM Director and associated staff have authored the problems.

**Table 2.**  
ICM problem authors, with their disciplines and affiliations.

Year	Author	Department/Discipline	Affiliation
1999	Yves Nievergelt	Mathematics	Eastern Washington Univ.
2000	Anthony Starfield	Biology	Univ. of Minnesota
2001	Sandra Nierzwicki-Bauer	Biology	Rensselaer Polytechnic Institute
2002	Grant Hokit	Biology	Carroll College, Montana
2003	Sheldon Jacobson	Computer Science	Univ. of Illinois
	John Kobza	Operations Research	Univ. of Tennessee
2004	Ronald Dodge	Computer Science	United States Military Academy
	Daniel Ragsdale	Computer Science	United States Military Academy
2005	Paul J. Campbell	Mathematics/ Computer Science	Beloit College
2006	Heidi Williams	Economics	MIT
2007	Paul J. Campbell	Mathematics/ Computer Science	Beloit College
2008	Kathleen Crowley	Psychology	College of Saint Rose
2009	Melissa Garren	Marine Biology	Scripps Institute
2010	Miriam Goldstein	Marine Biology	Scripps Institute

## Chinese Participation

In several ways, Chinese teams have dominated the ICM from its beginning. In the first ICM in 1999, a Chinese team from Zhejiang University was one of two Outstanding teams in the Ground Pollution problem, and that year 5 of the 9 Meritorious teams were from China.

Chinese teams have a substantially higher propensity to compete in the ICM. The Chinese have had a higher percentage of their teams participating in the ICM compared to other countries (including the U.S.). Since 2005, there have been more Chinese ICM entries than U.S. entries.

Jinxing Xie of Tsinghua University reports, in regard to a similar Chinese contest (CUMCM), that “more than 80% of the participants are engineering,

economics, management, and even humanities majors, other than mathematics majors one might expect" [2013, 437]. Perhaps this involvement and interest in modeling by a wider segment of undergraduate students in China is one reason for the ICM's popularity with Chinese teams. This broader disciplinary profile is quite different from that of U.S. participants, who in both the MCM and the ICM are mostly mathematics majors. Xie's data show that over the years the Chinese participants are about 18% more likely to work on the ICM problem than their U.S. counterparts. For instance, in 2006, Chinese teams made up 62% of the MCM participants and 87% of the ICM participants. In 2014, Chinese teams were 98% of the ICM participation.

The CUMCM (China/Contemporary Undergraduate Mathematical Contest in Modeling) has been held annually in China since 1992. It is "a Chinese copy of MCM/ICM" [Xie 2013, 437] but with many more teams entering than in the U.S. contests: more than 23,000 in the Fall 2013 contest compared to 7,783 total for MCM/ICM in February 2014.

## Results

Over the 16 years of the ICM, there have been 59 Outstanding teams representing 33 different schools. Among those, 29 teams were from Chinese schools and 30 teams from U.S. schools. Thus far, no team among the few entries from other participating countries has achieved an Outstanding award. **Table 3** provides the honor roll listing of the Outstanding schools and the year(s) when the school received the Outstanding award. Harvey Mudd College, with seven Outstanding rankings, has produced excellent team performance in the ICM.

## People of the ICM

Two people were instrumental in getting the ICM idea into the INTER-MATH proposal and included in the COMAP contest framework: COMAP Director Sol Garfunkel and MCM Director and COMAP coordinator Frank Giordano. Once they had their plan, the first contest went smoothly. Jack Grubbs and Chris Arney were the head judges and contest directors for the first two years, and Arney has directed the contest ever since. Paul Campbell, editor of *The UMAP Journal*, has expertly assembled and edited all 16 reports of the contest. One of Campbell's duties is to pick the one Outstanding paper that will appear in print. The other Outstanding papers appear in electronic form on an annual CD-ROM from COMAP.

Early on, Gary Krahn, John Kobza, Richard Cassidy, and Kelly Black did considerable special work as judges. Some continuity in the judging panel is necessary to calibrate the panel to the ICM goals and standards.

**Table 3.**

The honor roll of schools with teams who earned Outstanding rankings on the ICM (1999–2014).  
There have been 59 Outstanding rankings over the first 16 years of the Contest.

School	Years designated Outstanding
Harvey Mudd College	2000, 2001, 2003, 2004, 2004, 2006, 2008
Duke Univ.	2006, 2006, 2007
Humboldt State Univ.	2001, 2003, 2011
North Carolina School of Science & Mathematics	2000, 2000, 2011
Northwestern Polytechnic Univ.	2011, 2012, 2013
United States Military Academy	2004, 2006, 2009
Zhejiang Univ.	1999, 2011, 2013
Beijing Univ. of Ports & Telecommunications	2008, 2013
Carroll College, Montana	2003, 2010
Maggie Walker Governors School	2002, 2005
National Univ. of Defense Technology	2008, 2014
Olin College of Engineering	2002, 2005
Southeast Univ.	2011, 2014
Tsinghua Univ.	2014, 2014
Univ. of Electronic Science & Technology	2004, 2012
Beijing Jiaotong Univ.	2010
Central Univ. of Finance & Economics	2014
China Univ. of Mining and Technology	2009
Cornell Univ.	2012
East China Univ. of Science	2005
Earlham College	1999
Hangzhou Dianzi Univ.	2010
Huazhong Univ. of Science and Technology	2012
Lawrence Univ.	2010
Lewis and Clark College	2001
Nanjing Univ. of Information Science & Technology	2012
Peking Univ.	2013
Princeton Univ.	2007
Rensselaer Polytechnic Institute	2013
Shanghai Jiaotong Univ.	2012
South China Univ. of Technology	2011
Xidian Univ.	2014
Zhuhai College of Jinan Univ.	2012

Later on, Joseph Myers, Rod Sturdivant, Kathi Snook, and Tina Hartley filled the continuity roles as regular triage and final judges and assistant directors.

The problem authors are an integral part of the endeavor. **Table 2** provides the names and affiliations of these important contributors.

This contest could not continue without the administrative support provided by COMAP, and primary in that role have been John Tomicek, Roland Cheyney, and Anne Sterling.

There have been hundreds of triage judges, mostly from the Mathematics Department of the United States Military Academy, and scores of final

judges from many disciplines and organizations. They are vital contributors to the success of the contests.



**Figure 3.** John Tomicek at final judging at COMAP.

More recently, Jed Wang has written books in Chinese about the methods and problems in the MCM and ICM; these books reinforce the educational focus of the contest.

However, foremost in terms of people who enable the success of the ICM and the education of the contestants are the active educators at the schools who serve as team advisors of the 6,408 teams that have entered the ICM so far and the approximately 19,000 student team members who have worked on the problems and written the reports. The advisors are charged with organizing the teams and supervising the conduct of the contest, ensuring that the rules are followed and that their teams benefit through participation.

It is amazing how thorough a job these undergraduate (and even some high school) students do in both their modeling and their presentation of such considerably challenging problem-solving work. Four days is very little time for an undergraduate student even to understand complex problems such as those in the ICM, much less solve and write up a solution (usually 20 pages). Yet, year after year, these committed students do just that and should be proud of their accomplishments and their development as interdisciplinary problem solvers.

## References

- Arney, David C. 1994. Army beats Harvard in football and mathematics. *Math Horizons* 2 (September 1994): 14–17.
- \_\_\_\_\_ (ed.). 1997. *Interdisciplinary Lively Applications Projects (ILAPs)*. Washington, DC: Mathematical Association of America.
- Fusaro, Bernard (Ben) A. 1995. Background and history of the MCM. In *Tools for Teaching 1994: MCM the First Ten Years*, edited by Paul J. Campbell, 1–5. Lexington, MA: COMAP. 2005. Reprinted in *The MCM at 21:*

COMAP's *Mathematical Contest in Modeling*, edited by Paul J. Campbell, 1–5. Lexington, MA: COMAP.

Xie, Jinxing. 2013. An introduction to CUMCM: China/Contemporary Undergraduate Mathematics Contest in Modeling. In *Educational Interfaces between Mathematics and Industry*, edited by A. Damlamian, J.F. Rodrigues, and R. Straber, 435–443. [http://faculty.math.tsinghua.edu.cn/~jxie/papers/EIMI2011\\_xie.pdf](http://faculty.math.tsinghua.edu.cn/~jxie/papers/EIMI2011_xie.pdf).