Evaluation of Five Methods of Retirement Withdrawals (Preliminary Version)

Floyd Vest, March 2015

This article is based on Blanchett, David, Maciej Kowara, Pen Chen, “Optimal Withdrawal Strategy for Retirement Income Portfolios,” Ibbotson.com, Research publications. In the following, the above article is referred to as “Their article” and this article is referred to as “this article.”

Some of the variable that affect success of retirement income withdrawal strategies are: age at which retirement starts, market returns and volatility, inflation rates and volatility, type of investments, longevity, amount of withdrawals each year, and beginning amount in retirement fund.

The five withdrawal strategies are: #1 - The Constant Real Dollar strategy, #2 – The Endowment Strategy (Constant Percentage), #3 – Constant Failure Percentage Strategy, #4 – Life Expectancy Updating Withdrawal Strategy (RMD Method), #5 - Probability of Failure Mortality Updating Percentage.

The criteria for evaluation was to run out of money on the estimated final death date for a retirement duration, with a low probability of failure. The survival of withdrawal strategies were calculated with a historical Monte Carlo algorithm called the Sustainable Spending Rate (SSR). The SSR evaluation is sensitive to time duration, percentage withdrawals, market returns, inflation, and the sequencing effect. In the discussion in Their article, by “portfolio returns” they mean real returns after inflation, unless otherwise stated.

To establish the framework for the study, they introduced a new metric, the “Withdrawal Efficiency Rate” (WER), which measures the relative efficiency of various withdrawal strategies. The WER compares the withdrawals received by the retiree by following a specific strategy to what could have been obtained had the retiree had “perfect information” at the beginning of retirement. This measure allows the quantification of the relative appeal of each withdrawal approach.

#1 – Constant Real Dollar Strategy: Based on the initial retirement funds, the first withdrawal is a fixed percentage of funds. The dollar amount of succeeding yearly withdrawals is increase at the current rate of inflation. See the several articles in this course on this type of strategy. For example, see “The 4% Rule for Retirement Withdrawals.” This strategy is sensitive to inflation, sets a goal for needed retirement funds based on expected cost of living for the first year of retirement. It is not sensitive to longevity risk, and is likely to leave a large amount of money on the table at death. Any of the retirement withdrawal strategies can be adjusted over time depending on future developments. In Their article, for this strategy, the Withdrawal Efficiency Rate (WER) was higher for the initial withdrawal rate of 3.5% for a Cash investment, and 4% for the 20%, 40%, and 60% equity portfolios. The higher the average WER, the better the withdrawal strategy.

#2 – Endowment Approach. A constant percentage of current funds is used to give the withdrawal for each year. This approach is sensitive to the amount of remaining funds and thus
reflects market returns and volatility. By the WER analysis, an optimal percentage was 5.0% for 0% equity allocation and 20% equity allocation, and 5.5% for 40% and 60% allocations.

#3 – Constant Failure Percentage Strategy. For succeeding years there is a changing percentage withdrawal based on expected remaining longevity, the probability of failure can be fixed for succeeding retirement periods. For example, consider Table 1 below, for 40% equities.

Table 1. Probability of failure is 5%, and 5% probability of outliving longevity, for a Couple Age 65, for 40% equities.

<table>
<thead>
<tr>
<th>Target Longevity</th>
<th>Withdrawal Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>35 years</td>
<td>3.1%</td>
</tr>
<tr>
<td>30 years</td>
<td>3.4%</td>
</tr>
<tr>
<td>25 years</td>
<td>3.9%</td>
</tr>
<tr>
<td>20 years</td>
<td>4.7%</td>
</tr>
<tr>
<td>15 years</td>
<td>6.0%</td>
</tr>
<tr>
<td>10 years</td>
<td>8.9%</td>
</tr>
<tr>
<td>5 years</td>
<td>17.8%</td>
</tr>
</tbody>
</table>

From Their article, Table A page 10.

Their Table A says that for a 65 year old couple, with a 30 year horizon, age 95, they should withdraw 3.4% the first year, increasing percentage withdrawals each year to 3.9% at age 70, and so on with larger percentage withdrawals. If there has been changes such as health issues, they can change their schedule. For retirement planning, for $60,000 a year retirement income for the first year, for needed assets $P$, $60,000 = 3.4\%$ of $1,764,706$ needed retirement assets. If they have Social Security of $25,000$ a year with a COLA, they can subtract $60,000 - 25,000 = $35,000 and 35,000 is 3.4\% of $1,029,412$. Other assets such as a level retirement program could be estimated and subtracted.

The withdrawal percentages in Table 1 are calculated by a historical Monte Carlo algorithm called Sustainable Spending Rate (SSR). The SSR is designed so that the portfolio is depleted to zero at the target ending date. This Constant Failure Percentage Strategy and the SSR evaluation is sensitive to longevity, market returns, inflation, and the sequencing effect.

#4 – Life Expectancy Withdrawal Approach (RMD method). The withdrawal percentage is $1/(\text{Remaining Life Expectancy})$. Table C and D on pages 12 and 13 and Table A page 10 in Their article have different equity allocations: 0\%, 20\%, 40\%, 60\% and are for an Age 65 couple. If they choose a 5\% chance of failure of funds, and a life expectancy for which there is a 5\% chance of outlasting the life expectancy (a 5\% longevity), then for any equity allocation, the first year withdrawal rate is 2.7\% which gives a 5\% life expectancy of $1/(2.7) = 37$ years, from age 65 to age 102. For each remaining year, the withdrawal percentage is $(1/\text{Remaining 5\% life expectancy})$. Consider the example in Table 2.
Table 2, for a Couple Age 65, 5% life expectancy, and 5% failure of funds.

<table>
<thead>
<tr>
<th>Age</th>
<th>Withdrawal rate</th>
<th>5% Life Expectancy</th>
</tr>
</thead>
<tbody>
<tr>
<td>65</td>
<td>2.7%</td>
<td>37</td>
</tr>
<tr>
<td>70</td>
<td>3.0%</td>
<td>33</td>
</tr>
<tr>
<td>75</td>
<td>3.6%</td>
<td>27.7</td>
</tr>
<tr>
<td>80</td>
<td>4.5%</td>
<td>22.2</td>
</tr>
<tr>
<td>85</td>
<td>5.9%</td>
<td>16.9</td>
</tr>
<tr>
<td>90</td>
<td>9.0%</td>
<td>11</td>
</tr>
<tr>
<td>95</td>
<td>18.3%</td>
<td>5.5</td>
</tr>
</tbody>
</table>

For in-between values, one can look up a 5% life expectancy table or interpolate by curve fitting. In this Table 2, as a simulation, remaining years have been calculated from withdrawal percentages for a life expectancy for which there is a 5% chance of outliving, and we have used figures from their Table A page 10 for Cash since these figures give smaller percentage withdrawal numbers (longer life expectancy).

#5 – Mortality Updating Failure Percentage. This strategy combines the Constant Failure Percentage strategy and the RMD strategy. It is first based on the retirement years remaining. Then the percentage withdrawal is calculated based on maintaining a constant probability of failure for that period. The financial planner or retiree would choose a low probability of failure of funds, and a remaining lifetime which has a low probability of being outlived. For example for the 60% equities portfolio which is in the last section of their Table D on page 13 in Their article, one might choose a 5% failure of funds, and a 5% life expectancy. For the left side of the table, they have chosen a 5% failure of funds, and for the right side they have chosen 5% longevity and thus for an Age 65 couple, they should withdraw 2.6% of funds. For each succeeding year, the retiree would need a new Table D. This would require hiring someone who has purchased access to the tables for the Mortality Updating Failure Percentage strategy.

Comparison and evaluation of retirement withdrawal strategies. For the Monte Carlo study, the withdrawal strategies were compared by the WER quotient and thus determined the optimal income and withdrawal strategy for the retiree. A Sustainable Spending Rate (SSR) was first calculated to give the maximum income such that it depletes the portfolio to zero at the hypothesized time of death, for a given sequence of market returns, and annual inflation rates, and death scenario. The numerator of the WER quotient was a utility function “Certainty Equivalent Withdrawal” (CEW). For WER, \( WER = \frac{CEW}{SSR} \). For utility functions, see Wikipedia.org and see Their article.

Comparison of efficiency of the five withdrawal strategies.

1. The Mortality Updating Failure Percentage strategy had the highest efficiency for 20%, 40%, and 60% equity portfolios.
2. The Constant Failure Percentage strategy ran a close second.
3. The Constant Real Dollar strategy had the lowest efficiency.
4. The RMD strategy had a greater efficiency than the Endowment Percentage strategy and the Constant Real Dollar strategy. All of the strategies except the Mortality Updating Failure Percentage strategy were fairly easy to apply. The Constant Failure Percentage strategy might be the preferred method. One might keep a copy of Table A from Their article and for in-between years, interpolate to get withdrawal rates. See Table 1 in this article.
Side Bar Notes:

For the 4% Rule: From Blanchett, David, “Estimating the True Cost of Retirement,” 09, Dec. 2013, pages 16, 18, Ibbotson.com, Published research, Morningstar, a study designed to be based on the behavior of retirees represented by a constant real model spending curve: There was a 26.7% chance of not lasting 30 years (73.3% chance of success, withdrawals increasing at a constant real model). For the 4% rule, a 4% initial withdrawal, increasing at a constant real model spending curve, an 18.5% chance of life expectancy failure, for a 65 year old couple there was an 81.5% chance of success. For the 4% rule and different historical spending curves, the highest success rate was 91.1% for 30 years and 93.2% for joint life expectancy. For 3% initial withdrawals, for 30 years there was a 97.2% chance success, and for life expectancy a 97.5% success. This Monte Carlo simulation was based on income from a portfolio of 40% stocks, 3.0% real return on the portfolio, with 0.5% expenses, and 10% standard deviation. The return on stocks was reported at 9% and bonds 4%. Inflation was reported at 2.5%. For 30 years, and the constant real model spending curve, there was 73.3% chance of survivability, and for the historical spending curve, there was a 90.0% chance of survivability. For research on the 4% rule, see several articles in this course including “The 4% Rule for Retirement Withdrawals” which found a failure rate as low as 4%. Several other studies found similar low failure rates.

Exercise: For the above Blanchett study “Estimating the True Cost of Retirement,” check the figures on the portfolio returns, inflation rate, and real rate of return.

There is a 14% probability of living to age 100 for one or both of a 65 year old couple (Their article, page 3). Discuss the implications for retirement planning. For a couple Age 90 there is a 34% probability that one or both will live to age 100 (Their article, page 10).

A 2010 Vanguard Group paper claimed that combining an immediate inflation-adjusted annuity with a RMD approach produced stable cash flows that grew at a faster rate than those of other rules of thumb (Kiplinger’s Retirement Report, May 2013).

Get back only 85% of your money. Researchers have found that on average people who put money in immediate life annuities get back 85 cents on the dollar. They found that life annuity offers varied by as much as 20 percent (Scott Burns, Denton Record Chronicle, July 7, 2013).

Volatility of the stock market along with withdrawals can kill your retirement funds. In early 1966 the DOW topped 1000. In August 1982, it was down to 777. The apparent 22.3% loss in value over 16.5 years was actually much worse when adjusted for inflation – figure a 70% haircut, excluding dividends (Forbes, July 15, 2013).

Exercise: For the above, check the inflation rates for this period of time to see if inflation adjusted withdrawals would deplete the portfolio of DOW stocks. See USInflationcalculator.com and Dow stocks in other sources. Using the average inflation rate and a beginning 4% of $1000 withdrawal adjusting withdrawals for inflation, even with 4% dividends, the fund is broke.
A Variable annuity with a guaranteed income rider, purchased by a 55 year old male in the year 2000 would have an average expense ratio of 3.7%, a balance of $95,500 in 2010 and a guaranteed lifetime income at age 65 in 2010 of $8144 per year. For the bear market of the 2000’s, $100,000 in a mutual fund portfolio of 60% large-cap stocks and 40% intermediate term bonds with a 0.75% expense ratio would have grown to about $131,600 (Money, Aug. 2013, p. 76). An AARP lifetime immediate annuity for a male Age 65 pays 5.9% (5/19/2014).

Exercise. For the above, make up and solve some problems. Don’t forget the $8144 should be discounted back to age 55. Did the mutual fund keep up with inflation?

An inflation adjusted lifetime annuity for an Age 65 male, female couple for $100,000 pays for the first year $3489 and later payments are adjusted for inflation. (Americks, John and Nathan Zahm, “Estimating internal rates of return of retirement income annuities,” Research by Vanguard, March 2012, vanguard.com). Considering the Constant Real Dollar strategy, the first payment is 3.5% of the $100,000 . A fixed lifetime annuity pays $5660 per year (the sum of the monthly payments). The author gives the median life expectancy of 24.76 years. He says that for the median longevity and the fixed annuity, the IRR on the $100,000 is 3.18%. The real IRR for the inflation adjusted annuity is -0.07% . For a 75th percentile couple, the real IRR = .99%. The author didn’t say where he got his inflation figures.

Exercise: For the above, calculate the median rate of return on the fixed annuity with annual payments. Do the calculations with monthly payments of $471.67 . Do it for payments at END and BEGIN. The fixed annuity would be quoted as paying what percent? Discuss why an insurance company would pay a lower IRR on an inflation adjusted annuity. Calculate the real rate of return on the inflation adjusted $100,000 annuity for a couple with the median longevity. Does your number come close to Americk’s? (Note: you may be using a constant average inflation rate.) How did the author get his number? Using average inflation \( I = .032 = 3.2\% \), what is the nominal rate of return? How does this compare to the nominal rate of return for the fixed annuity? (You may need to derive a formula for the present value of an annuity with withdrawals increasing at a constant rate.)

Long term financial planning and inflation. (From “Don’t let inflation crack your nest egg,” 5-24-2014, Marketwatch.com.) According to Senior Citizens League, since 2000, seniors have lost about one-third of their buying power. SLC found that 27 of 33 typical costs have increased faster than the average Social Security COLA of 2.7%. Calculate the rates of increase: Since 2000, gasoline has increased from $1.31 to $3.41, homeowner’s insurance has increased from $508 to $1,135, Medicare Part B has increased from $45.50 to $104.90. Do the math for the rate of decline of purchasing power. Summarize in a table.

Do you believe this? In 2014, Current age 40, Current salary $100,000, Annual savings 10%, Total retirement assets $0. They calculate, you cannot retire until age 67. Do the calculations. Estimate Inflation, Increase in income, Social Security, Rate of return on investments, Formula for Retirement funds needed, Longevity, Cost of living for the first year of retirement, and so on. (Marketwatch.com, Personal Finance, 5-25-2014)

What do you think? Boomerang families (Their adult children have moved back home.) are near the bottom of the heap. While just 4.9% of boomerang families say they are financially secure, 6.2% of single parent families say they are. Just when parents think they can finally
make up on savings for retirement, they find themselves financially insecure. Many boomer women say they spend over $10,000 per year on adult children. (Marketwatch.com “Your kids will never let you retire,” 5-23-2014).

Ninety percent of working age households are not saving enough for retirement. For ages 45 to 54, about $60,000 is saved, and for ages 55 to 64, $100,000 (National Institute on Retirement Security.) What do you think of someone Age 64 with less than $100,000?

Can they live on Social Security? How much Social Security do they have? Is their home paid for? Perhaps they enjoy working. If the $100,000 was invested, how much a year could they draw from it? What does Blanchett’s study say?

Financial mathematics and the mortgage crisis. Read about it on the internet. Citigroup has been fined $7 Billion for deception in handling subprime mortgages by minimizing the risk to investors (Associated Press, 7-15-2014). According to SIAM (Society of Industrial and Applied Mathematics), those responsible were not people who have been quantitatively trained. Far too often they were raised to their position on other criteria, such as deal chasing ability, and sales. “The Gaussian copula formulas assumes the independence of individual components mortgages” (Wikipedia.org). “It Assumes constant correlations where correlations for financial products are fluid.” The author has a quote from a financial mathematician who said he saved his bank from the mortgage crisis. See “Financial Mathematics ’08; Mathematics and the Financial Crisis, Jan. 10, 2009, SIAM.org.

Exercises: Show your work. Label answers, numbers, and variables. Conduct discussions in complete sentences.

#1. Examine the below references for the percentage of failure for the 4% rule for thirty years survival. Report.

#2. Look up Utility Functions on Wikipidea.org and report. See articles in this course on Utility Functions.

#3. Show that if you start with one dollar and withdraw a small \( s \) (for example $0.03 at the beginning of each year for \( N \) years, and the real rate of return applied to year 1 is \( r_1 \) , and for year 2 it is \( r_2 \) , and so on to year \( N-1 \) where the real rate is \( r_{N-1} \) and you spend all of the funds by the beginning of year \( N \), then for \( N-1 = 3 \), 
\[
\left[ (1-s)(1+r_1)-s \right] \left(1+r_2\right)-s \right] (1+r_3)-s = 0
\]
See of you can prove this formula for \( s = \text{SSR} \) from Their article on pages 4 and 18. Why is \( s \) a constant and not increasing yearly at the rate of inflation? See the article in this course “Varying Annuities and the Real Rate of Return.”

#4. See the article in this course “The Mathematics of Financial and Social Responsibility,” 1999, including the Examples. Why would a person need these complex calculations if there is a simple 4% rule?

#5. From Scott Burns, Denton Record Chronicle, May 18, 2014: For an index fund with expense ratio of 0.06 percent, invest $1000 per year for 40 years, with 8% return before expenses, you would earn $254,999. For a similar managed fund with an average expense ratio
of 1.22%, how much would you accumulate? What is the difference? You could manage your money in a low cost index ETF for 20 years before you would spend as much as it will cost to manage the same money in the managed version for any one year. What assumptions does this make? Check Scott’s numbers.

#6. Discuss the methodology advantages and disadvantages of each of the five strategies studies in Their article. Name and describe a plan you might prefer.

#7. Look up Monte Carlo simulation on some source such as Wikipedia.org. There should be articles on random numbers and Monte Carlo simulation at COMAP.com. Give a brief report.

#8. What would be a weakness of randomly selecting 30 consecutive year periods over a periods of time (for example 100 years)? What is rolling 30 year periods?

#9. For $50,000 for the first year of retirement, calculate the retirement funds needed for 30 years by the 3% rule, the 4% rule, and the 5% rule. Using numbers from Blanchett “Estimating the True Cost of Retirement” of 40% stocks, 3.0% real rate of return on the portfolio, 0.5% expenses, 10% standard deviation, return on stocks 9%, return on bonds 4%, and 2.5% inflation, calculate with a formula the needed retirement assets for a married couple Age 65 for 30 years, 35 years, and 40 years. Why 40 years? You will probably use constant averages. What is wrong with constant averages? If you used a long term inflation rate of 3.24%, what would be the numbers? How much money is requires to last forever?

#10. Copy the Uniform Lifetime Table III from IRS Publication 590 for RMDs and build a table comparing the 50% longevity figures in Their article. From your available sources what longevity figures would you prefer for retirement planning?

#11. In his above study of retirement funds, Americks says that IRR is the rate that discounts the cash flow to $100,000 (IE). State and derive this formula. What does mirr mean? How is it calculated?

#12. May 2014, for a male Age 65 an immediate lifetime annuity for $10,000 pays $7044 per year ($587 per month). Build a table using the yearly figures to calculate the interest rate earned if he lives to Age \(x = 70, 75, \ldots, 100\), and the probability of death at Age \(x\), and for Age \(x – 5\), the probability of death at Age \(x\). See the mortality table in Their article page 19. What does this suggest about: if he lives to Age \(x – 5\), the greater probability of living to Age \(x\), and the probability of earning the higher rate to Age \(x\)? Estimate the median death age for the 65 year old male. What rate of return does the average annuitant make? At what age do they get their money back? Since the insurance company makes money on those who die by Age 79.2, how does this affect the rate they can offer annuitants? What expenses do insurance companies have? Sales commissions for annuities range from 2% to 10%.

References in this course:

“Funding Retirement During a Worst Market Scenario.”


See “The Utility of Insurance” under Additional Articles in this course for examples of utility functions.

References for other sources:

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