**Health Savings Accounts** (Preliminary Version)

Floyd Vest, Jan. 2014

Families with certain high-deductible health insurance can save up to $6500 pretax in a health savings account (HSA). After age 65, funds not used for medical expenses can be withdrawn tax and penalty free, when posted against documented out of pocket medical expenses. (Money magazine, Jan 2014, p. 28, and p. 54). We will compare taxed savings with HSA savings.

**Taxed Savings.** Let \(x\) = before tax dollars. \(t\) = marginal income tax rate. \(x - tx\) = after tax money invested. Taxes at the rate \(t\) are paid on each year on the earnings. After \(n\) years,

\[
S_t = (x - tx)[1 + i(1 - t)]^n
\]

is the accumulated after tax savings, where \(i\) is the interest rate compounded annually.

**Example 1.** Let \(t = \) a marginal income tax rate at 25\%, and after tax dollars = \(x - tx = \$1000\). Solving \(x - .25(x) = \$1000\) gives \(x = \) before tax dollars of \$1333.33 equivalent to \$1000 after tax. Let \(i = 5\%\) and \(n = 35\) years from age 30 to age 65.

\[
S_t = 1000 \left[1 + .05(1 - .25)\right]^{35} = \$3627.30
\]

**HSA Savings.** Let \(x = \) pretax dollars invested in an HSA Savings at the interest rate \(i\) compounded annually for \(n\) years.

\[
S_H = x(1 + i)^n
\]

**Example 2.** Let \(x = \) pretax dollars of \$1333.33 tax equivalent to the \$1000 after tax above. Let \(i = 5\%\) and \(n = 35\) years.

\[
S_H = 1333.33(1 + .05)^{35} = \$7354.67
\]

which can be withdrawn tax and penalty free after age 65. You note that the HSA Savings is more than twice the taxed savings.

Most likely, annual savings deposits for 35 years comes to mind.

**Future Value (FV) of annual deposits in taxed savings.** Let the first deposit at the end of the first year accumulate to \((x - tx)[1 + i(1 - t)]^{n-1}\). The saving at the end of year \(n\) is \(x-tx\). For the ordinary annuity, the

\[
FV = (x - tx) \left[ \frac{(1 + i(1 - t))^n - 1}{i(1 - t)} \right].
\]

(See the Exercises for the derivation.)

**Example 3.** For taxed deposits and savings for 35 years from above,

\[
FV = 1000 \left[ \frac{(1 + .05(1 - .25))^{35} - 1}{.05(1 - .25)} \right] = \$70,061.38.
\]

**Example 4.** FV of an ordinary annuity for HSA savings is
\[ FV_H = 1333.33 \left[ \frac{(1+.05)^{35} - 1}{.05} \right] = $120,426.78 . \]

**Example 5.** What interest rate for the taxable investment is required to match to HSA savings rate of 5% in Example 4? We set up

\[ 120,426.78 = 1000 \left[ \frac{(1+i(1-.25))^35 - 1}{i(1-.25)} \right] \]

and let \( y = i(1-.25) \) to get

\[ 120,426.78 = 1000 \left[ \frac{(1+y)^{35} - 1}{y} \right] \]

and solve for \( y = 6.363 \). This gives \( i = 8.484\% \) as the required interest rate for the taxable investment to match the 5% for HSA savings.

The reader may have thought of comparing HSA savings with savings in a traditional IRA, a Roth IRA, or a tax deferred annuity. This is left as exercises. (See the References in this course.)

With the HSA savings, you get

(a) By investing pretax dollars you get a deduction off your income tax for the year of \( tx \) since you don’t have to pay taxes on \( x \). For the above example, \( tx = .25(1333.33) = $333.33 \).

(b) Your money compounds tax free. Some HSA accounts have accumulated to $100,000.

(c) The money is withdrawn penalty and tax free after age 65. Before age 65, money can be withdrawn tax free to pay acceptable medical expenses.

Although for most people, the HSA money is used for health insurance deductibles and other medical costs, the expenses may be paid out of pocket. People with an adequate family emergency fund can manage this.

**Side Bar Notes** (Forbes, Jan. 20, 2014, p.54).

**HSA rules.** See at irs.gov, IRS Publication 969, Health Savings Accounts and Other Tax-Favored Health Plans (12 pages of HSA rules) and Form 8889, Health Savings Accounts.

For the high-deductible policy, the deductible has to be $1250 for an individual, and $2500 for a family. The premiums for the high-deductible policy can be claimed against the HSA.

**Setting up a HSA.** You need to work with a qualified HSA trustee such as a bank. You may need to take a tax deduction for your HSA contribution. You must file Form 8853 in any year where there is activity in your HSA account. The HSA may be set up by you, or your employer and contributed to by both. Ten million Americans have Health Savings Accounts. They have been around for about ten years. For 2014, the maximum contribution for a family is $6550 and for single is $3300.

**Retroactive claims.** Save the medical bills paid out of pocket, and you can make retroactive claims. From a HSA Bank, you may get a debit card to draw on the HSA. An HSA administrator can charge a fee.
Pretax contribution to a HSA is not counted as income on your income tax and may reduce your marginal rate and AGI.

After age 65, HSA money withdrawn and not matched against medical costs is taxed as ordinary income, as in a 401k. This says to keep records of all out of pocket medical costs. For HSA withdrawals, you can’t claim medical expenses you have used as an itemized medical deduction on your tax return. But for most people, medical expenses don’t exceed the required 10% of AGI. Once on Medicare, you can no longer contribute to a HSA but you can claim against the account for premiums such as for a Medicare Supplement. Turbo-Tax will ask you if you have a HSA account.

Who pays income taxes? The bottom 40% in earnings get more in tax credits than they would otherwise owe. They make a profit on income taxes. About 47% pay no income tax. The top 10% in earnings pay about 73% of the taxes.

Real household income has fallen about ten percent since Oct. 2011, the start of the last recession (New York Times, Sept. 6, 2013). Groups with a low income, such as those who did not attend college, and parents under age 25 has a steeper decline. Median income for households headed by people ages 65 to 74 increased by 5.1% to $43,000. For those nearing retirement, ages 55 to 64, there has been a 10% decline since June 2009.

How are their investments doing? For those nearing retirement and beginning retirement, their portfolios were down 25% (Kiplinger’s Personal Finance, p. 48, 10/2013). Rather than bailing out of stocks, most stayed the course. Stocks began to rebound after March 2009. Check finance.yahoo.com for the S&P 500 history, or the article in this course “Annual Total Return Table for the S&P 500 Index of Stocks.”

Stock and bond investments are in danger (Money magazine, Jan./Feb. 2014). After a five year rally that has more than doubled the S&P 500, stock prices are among the frothiest in history. P/E ratios are above 25. Robert Arnott, Chairman of Research Affiliates, says “But that’s a game I choose not to play.” Bonds values are certain to decline as the Fed raises interest rates. The long term government bond has already lost 12%. What’s a person to do? But, this is a good problem to have.

Where are HSA accounts invested? HSAAdministrator.info says that it is an affiliate of Vanguard Funds and offers 22 Vanguard no load funds. Annual administrative fee is $45. Custodial fee is 80 cents per $1000. Mutual fund charges are low at Vanguard, as low as .10%.

Many Stock Dividend Yields are higher than CD rates (3%, 4%, and up). Since 1930, stock dividends have accounted for nearly 40% of returns. High dividend stocks are less volatile. There are high dividend stock mutual funds. The Vanguard ETF Vanguard Dividend Appreciation (VIG) has outperformed the S&P 500 and had a monthly beta of .83.
Floating rate bond funds are offered in some HSA accounts. They currently yield from 3% to 4.5% and have some protection from losses due to rising interest rates, since when interest rates rise, they raise the interest rates they charge on their loans. Historically, they have taken a dip, but recovered.

More information. See morningstar.com and register as a free user. Do a search for health-savings-account and read some articles. One contributor was happy with his provider saying that “I invested my HSA with Saturna Capital, home to Amana and Sextant funds. No fees, also superior service, and excellent funds.” They pay medical expenses out of pocket.

Check for load, expense ratio, performance, and selection.

Starting Social Security. LaVoice says that the average age for starting is 63.8 Years. For women 49% start at age 62. Only 0.6% of men wait until age 70. (Scott Burns, “A good strategy for increasing future retirement income,” Denton Record Chronicle, Jan. 26, 2013)

Exercises. Show all your work. Label numbers, variables, and answers. Provide information to your class by giving them written reports and sources. Some exercises require a TVM Solver. See the article in this course “The Mathematics of Financial and Social Responsibility” for the annuities with rents increasing at a constant rate used in several of the problems.

#1. Do examples similar to Example 4 with deposits beginning at the beginning of each year, and one example with an additional deposit at the end of the last year.

#2. Do examples for Examples 4 and 5 for (a) a traditional IRA, (b) a Roth IRA, and (c) for a tax deferred annuity.

#3. For Examples 4 and 5, what rate of return is required to accumulate $100,000?

#4. Derive Formula 3.

#5. If HSA contributions can increase at 1.5% a year, for Example 4, what is the FV? Do this for beginning with $6500.

#6. For paying medical expenses and health insurance deductibles, what is the advantage of withdrawing from a HSA? Make an example and discuss. What is the percentage savings in expenses?

#7. Assuming deposits of $6500 per year in the HSA, 35 years, 5% interest rate, $3500 per year in medical and deductible expenses paid out of pocket and not from the HSA. How does this compare with paying the $3500 per year from the HSA? Do the calculations, summarize, and discuss. There are several ways to interpret this question.

#8. Other things being equal, how does the after tax return from a Traditional IRA compare to a Roth IRA? Is there a basic financial advantage of changing a Traditional IRA to a Roth IRA?
#9. (a) For $6500 deposits at the end of each year to year 35 and 5% interest rate, graph 
\( FV_n \) for \( n \) from 1 to 35. Graph taxed savings \( FV \) on the same graph. Discuss the graph.

(b) What rate of return is required to reach $1 million? (c) Give an estimate for the annual cost of living after 35 years. What inflation rate did you use? How long should retirement funds and resources last?

#10. Dallas Morning News, Feb. 4, 1993: “Dear Scott Burns, I am 17, and have a part time job. I can save $250 a month. My dad says that if I put $2000 into an IRA for 10 years, it will be worth $1 million by the time I retire at age 65.” Try different inputs, calculations, and assumptions to justify this claim. Explain your results. You can use a SOLVER to solve for \( i \).

#11. With a tax rate of \( t \), give a formula for before tax dollars in terms of after tax dollars.

#12. Scott Burns told one of his readers that the reader was in a low tax bracket. Look up and describe the different IRS tax brackets and how taxes are calculated. Do an example for someone you know. How do they qualify for the 0% tax bracket. See the Side Bar Notes.

#13. For the Side Bar Note above “Where are HSA accounts invested?”, for a Vanguard fund paying 6% on $10,000 for ten years, estimate the \( FV \) after expense ratios of .10% and .08% and an annual custodial fee of $45 (paid with outside dollars). Estimate the after expense rate of return. Estimate the total expense ratio. When your account reaches $500,000, what would be the average total expense ratio for ten years? Look up HSAAdministrator.info and summarize some the particulars. Do you get a debit card to pay medical expenses?

#14. For a single investment over ten years averaging 6%, but with rates varying yearly, does the order of return make a difference in the final sum? Why? What limitation would you put on the returns?

#15. Go to irs.gov and summarize key features of HSAs in IRS Publication 969.

#16. Go to hsaadministrators.info and click on 22 Vanguard funds. An HSA can use four funds at a time. Check the features of the funds and select a portfolio. Justify your choice. Most of these funds hold stocks and/or bonds. For a study of stocks, see in this course “Stock Pricing Models” and its Side Bar Notes. Give information on a run up in stock prices and a following decline. For bonds, see “The Effect of Interest Rate Increase in a Bond Investment in a Low Interest Rate Environment,” Jan. 2013. Gives expected declines in NAVs as interest rates rise from their recent lows (2014, eg: 2% to 3%). See other articles in this course on stocks, bonds, and mutual funds.

#17. Suppose you are afraid Share Prices (NAVs) of a mutual fund will decline by 36% in the next year and you use dollar cost averaging for the year by investing quarterly starting at the beginning of the year when NAV = $30. In the year, the NAVs drop from $30 to 30(1-.36) = $19.20. Investing $6500 at the beginning of the year, the end of year balance would be

\[
\left( \frac{6500}{30} \right) (19.20) = 4160.
\]

(a) Instead you invest 6500/4 = $1625 at the beginning of each quarter with a 9% decline in NAV each quarter. How many shares do you own at the end of the year and what is the end of year balance? (b) It could be that monthly or annual deposits are the only options. For monthly deposits, how many shares do you own at the end of the year and
what is the fund balance? In order to do the monthly calculations, develop and apply a general formula.

#18. Obama’s new budget proposes a $3 million cap on IRA and other tax preferred accounts (accountingweb.com, Jan. 1014). For someone age 20 who plans to retire at age 70 with beginning retirement income of $100,000 in today’s dollars (a) with 3% inflation, what is the needed income at age 70? (b) With investments earning 7%, what retirement funds are needed for age 70 to age 100? How far short is Obama’s recommendation? What do you think of Obama’s proposal? Do you think they know mathematics? How dumb do they think the American people are? Let us know what you think.

#19. Kiplinger’s Personal Finance, 1/2013 says 401(k) fees are a major issue. If you pay an extra 100 basis points (one percent) over a 40 year career, it reduces your pile at retirement by about 20%. Make up an example to verify this. Give your time-line, investment program, assumptions, and conclusions. Label variables, input, and answers. Did you consider a level savings program or one with savings increasing at the rate of inflation? What rate of inflation and return on investments did you assume? How did you apply the extra expense ratio?

#20. (From Scott Burns, “The conspiracy for failure in 401(k) plans,” Denton Record Chronicle, Aug. 25, 2013) A reader is in a 401(k) which has a 1.8% expense ratio. Scott says it is easy to manage a 401(k) for 0.7% by using index funds. He says that 70% of managed funds underperform the earnings of the “index fund.” Not only do they charge more but don’t earn as much. See Standard and Poor’s SPIVA report, for S&P Indices Versus Active Funds Scorecard. Can managers earn the extra 1.8%, or would they do as well or worse, without the charge, than the index? What is the probability that a managed fund would outperform the index fund? What is meant by “the index fund”?

The following problems are taken from Scott Burns, “A good strategy for increasing future retirement income,” Denton Record Chronicle, Jan. 26, 2014:

#21. Scott says “Social Security is even more valuable in a low interest rate environment.” Do examples for Social Security with a 2.5% COLA which demonstrate this. A common low interest rate for the environment up to 2014 is 2%. The student can pick a figure for a high interest rate environment since this less well defined. You might start Social Security at a age 62 at 12($1035) = $12,420 per year. Age might run from 62 to 95.

#22. Scott quotes Richard LaVoice, an executive vice president at the Symetra Life Insurance Company as saying that: A couple age 65, has an 88 percent chance that at least one will live to age 80, a 73 percent chance that at least one to age 85, and a 49 percent chance to age 90. What does “will live to” mean? Choose a meaning and stay with it. (a) For the 100% at age 65, what percentage are both dead before age 80, before age 85, before age 90? What percent will live beyond age 90? What percentage fade (results in both are dead) from age 80 to 85, from age 85 to age 90? (c) What interest rate is earned if the money lasts to age 80, to age 85, to age 90, to age 95, to age 100.

#23. Scott says that to replace Social Security from age 62 to age 70, LaVoice gives an example of an eight year term monthly annuity starting at $1035 a month at age 62 with the equivalent of a 2.5% annual COLA which costs $100,081. Estimate the rate of interest paid by this annuity. In what year does the owner get their money back? Will they pay income taxes on
the Social Security, on the annuity? (For the annuity, LaVoice estimates that 92% of every dollar will be tax free. For Social Security, up to 85% is taxable. See the article in this course “Income Taxes on Social Security.”)

#24. Scott says that if the above person waited to age 70 to withdrew Social Security, they could get $2523 per month with a COLA for life, more than double the $1035. Such an annuity (a joint and survivor annuity) would cost about $320,000 (See ImmediateAnnuities.com).

(a) What interest rate does the annuity pay if the money last to age 80, to age 85, to age 90, to age 95, to age 100? (b) He says the $101,181 premium has grown to $320,000 growing at 15.7%. Check his figure and discuss. (For the mathematics of postponing Social Security, see the article in this course “How Much Longer Will Many of Have to Work? – To Secure Retirement Financially.”)

#25. Look up the cost of an annuity beginning at age 70 that pays as long as either of a couple is alive, starting at $2353 per month indexed to 2.5% inflation (See ImmediateAnnities.com). Estimate the interest rate it pays if money lasts to age 80, to age 85, to age 90, to 95, to 100. Estimate the probability of the age 80 rate, the age 85 rate, the age 90 rate, the age over 90 rate? See the above problems. What assumptions are you making?

References:
See other articles in this course on stocks, bonds, retirement, mutual funds, and so on.

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