Joint Life Expectancy and Sustainability of Retirement Funds (Preliminary Version)

Floyd Vest, Oct. 2013

In Blanchett’s study “Joint Life Expectancy and the Retirement Distribution Period”, he found that for the 4% Rule, and a 60/40 portfolio, the failure rate for retirement funds to last 30 Years was 4.1%. (Reference #1, p. 59 in the References). For the 4% rule, the first withdrawal is 4% of retirement assets and withdrawals increase each year at the rate of inflation. (For a review of the 4% rule, see the article in this course “The 4% Rule for Retirement Withdrawals.”) For a 65 year old couple, based on the funds lasting as long as either of a couple are alive, the probability of failure was 0.7% (%1, p. 59).

Making comparisons of this type, the author concluded that based on joint life expectancy, the withdrawal rate could be 1-2 percentage points higher than acceptable rates for the fixed 30 years (%1, p. 54).

Joint life expectancies. These results come from a Monte Carlo study of 81 calendar years from 1927 to 2007 and a 2004 table of joint life expectancies (Social Security Administration) (%2, p 70). From the life expectancy table, for a couple age 65, the probability of one or both living to

Age 70 is 99%. Age 75 is 95%. Age 80 is 85%. Age 85 is 67%. Age 90 is 39%. Age 95 is 14%. Age 100 is 3%.

Depletion rate vs. withdrawals for living expenses. The base study (%2), used historical earnings of market sectors without considering income taxes and the expense ratios of mutual funds. If the expense ratios average 1% of assets, a withdrawal rate of 4%, would deplete funds by 5% after expenses. The actual withdrawal rate for living expenses would be less than the depletion rate.

The 60/40 portfolio (%2, p. 73) included four asset categories: 1. Cash. 2. Intermediate term bonds. 3. Domestic large blend equity. and 4. International equity. Annual returns were generated by bootstrapping monthly returns from random years from which monthly inflation rates were subtracted and 12 months of data were annualized for annual real returns. Blanchett reported the average annual inflation rate was 3.51%.

We report part of his Table 2: Annual Real Returns and Annual Standard Deviations for Selected Asset Categories (%2).

<table>
<thead>
<tr>
<th>Asset Category</th>
<th>Geometric mean</th>
<th>Standard deviation</th>
<th>Arithmetic mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash</td>
<td>.72%</td>
<td>4.10%</td>
<td>.81%</td>
</tr>
<tr>
<td>Intermediate-Term Bond</td>
<td>2.34%</td>
<td>6.14%</td>
<td>2.52%</td>
</tr>
<tr>
<td>Domestic Large Blend</td>
<td>7.36%</td>
<td>20.99%</td>
<td>9.48%</td>
</tr>
<tr>
<td>Equity</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>International Equity</td>
<td>5.16%</td>
<td>21.59%</td>
<td>7.25%</td>
</tr>
</tbody>
</table>

For the 60% equity portfolio, the ratio between Domestic large blend equity and International equity was held constant at 66.67% and 33.33% respectively. For the 40% fixed income/cash portfolio, the ratio was 50% each (%2, p. 80).

We report part of Blanchett’s Table 3 in #1 giving the probability of portfolio failure while either is alive: Table 3: Probability of Portfolio Failure While Either or Both are Alive.
<table>
<thead>
<tr>
<th>Initial Depletion Rate</th>
<th>Initial Joint Age of Couple (Years)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>65</td>
</tr>
<tr>
<td>3%</td>
<td>0.1%</td>
</tr>
<tr>
<td>4%</td>
<td>0.7%</td>
</tr>
<tr>
<td>5%</td>
<td>4.8%</td>
</tr>
<tr>
<td>6%</td>
<td>15.8%</td>
</tr>
</tbody>
</table>

Starting at age 65, for a 3% depletion the first year, and yearly withdrawals increasing each year at the rate of inflation, the probability of funds failing to last as long as either shall live was 0.1%. For a 4% first year depletion, the probability is 0.7%.

Starting at age 70, for a 4% first year depletion, the probability of failure is 0.3%. For 5%, the probability is 2.3%.

For comparison, we will report part of Blanchett’s Table 2 in #1 for fixed 30 and 35 year periods:

Table 2: Probability of 60/40 Portfolio Failure for Various Real Withdrawal Rates and Distribution Periods.

<table>
<thead>
<tr>
<th>Depletion Rate</th>
<th>Distribution Period</th>
<th>35</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>3%</td>
<td></td>
<td>.8%</td>
<td>.3%</td>
</tr>
<tr>
<td>4%</td>
<td></td>
<td>7.5%</td>
<td>4.1%</td>
</tr>
<tr>
<td>5%</td>
<td></td>
<td>24.2%</td>
<td>16.4%</td>
</tr>
<tr>
<td>6%</td>
<td></td>
<td>48.8%</td>
<td>39.0%</td>
</tr>
</tbody>
</table>

For a person age 65, they might calculate for 30 years to age 95, or 35 years to age 100. For a person retiring at age 70, they might calculate for 30 years to age 100, or for 35 years to age 105.

As pointed out earlier, for 4% depletion, the probability of funds failing in 30 years is 4.1%, for example from age 65 to age 95. For age 65, the probability of funds not lasting as long as either shall live is 0.7%.

Another figure is for 4% depletion and 35 years, the probability of failure is 7.5%. For age 65 the probability of funds not lasting as long as either shall live is again 0.7%. Examination of a 5% rule would be of interest since it translates into an approximately 4% withdrawal after mutual fund expenses.

For 5% and 30 years, the probability of failure is 16.4% while for age 65, the probability of funds not lasting as long as either shall live is 4.8%, which is 29% of the 16.4%. Perhaps this risk could be managed by revisiting the calculations of fund’s survival each year and making appropriate adjustments (#3 and #4).

Changing life expectancies. Since a 2004 longevity table was used, and most readers are aware that longevity has been increasing, readers would be concerned with updating these figures. Blanchett reports that studies have found that since 1950, for a 60-year-old white male, the average life expectancy has increased by one month a year, 0.7 month a year for age 70, and 0.4 month a year for age 80. Individuals with higher lifetime earnings and more education tend to live longer.
The real rate of return for the 60/40 portfolio. From Blanchett’s Table 2 (#2), we estimate the real return for the fixed income/cash portfolio at 

\[ .50(0.72) + .50(2.34) = 1.53\% \].

The real rate of return for the equity portfolio is estimated as 

\[ .667(7.36) + .333(5.16) = 6.627\% \].

For the 60/40 portfolio we get 

\[ .60(6.627) + .40(1.53) = 4.59\% \].

Funds last forever. We note that if the average real rate of return of 4.59% was applied to each year, with the first withdrawal of $1,

\[ P = \left( \frac{1}{1.0459} \right) = \$21.786492 \] (approximately) or more would last forever. For the 4% rule, $25 would last forever. (See the article in the course, “The Mathematics of Financial and Social Responsibility.”) (See the Exercises.) This points out what is called sequencing risk, risk associated with the sequence of gains and losses in earnings, and inflation, during retirement withdrawals.

The sequencing affect. To illustrate further this sequencing affect, we will reproduce part of a table published by Fidelity Funds (https://guidance.guidance.fidelity.com/viewpoints). Look for Safeguard Your Retirement Income. Portfolios A and B each started with $100,000 and had an average return of 4.06% for 12 years and experienced the same annual returns but in different orders. Withdrawals of $7000 occurred at the beginning of each year. For Fund A, the sequence of positive returns was applied first and after 12 years the balance was $53,633.11. For Fund B, the sequence of negative returns was applied first and after eight years, the fund could not provide the $7000 withdrawal. The returns were 16.21%, 20.72%, 19.52%, 8.95%, 14.59%, 14.30%, 6.79%, -18.47%, -15.39%, -4.59%, -19.14%, and -18.39%.

Something similar to this happened to the S&P 500 Index from Jan. 2000 to Jan. 2012. For 2000, the S&P was 1425.59, for 2001 it was 1335.64, 2002 was 1140.21, 2003 was 895.84, 2004 was 1132.52. By 2009, it was 865.58, and by 2012, it hadn’t recovered its former high of 1325.58. (See the Exercises and see the article “Annual Total Return Table for the S&P 500 Index of Stocks” in this course.)

In 1929, the S&P was 24.86 and later was down to 7.09 (a 71.5% drop) and later to 8.93 and didn’t recover its former high until 1953.

Side Bar Notes:

Using the TVM Solver to verify Formula 1. We will investigate a claim that $21.8 will last forever.

TVM code and comments: \( I\% = 4.59 \), \( PMT = 1 \), PMT: END, \( PV = -21.6 \). For \( N \), Alpha Solve gives 106 years. Using \( PV = -21.7 \), for \( N \), Alpha Solve gives 123 years. Using \( PV = -21.8 \), for \( N \), Alpha Solve gives Error which means that \( N \) is infinite. (See the Exercises.)

The real rate of return is usually the return after the effect of inflation. It is often calculated as 

\[ y = \frac{r - I}{1 + I} \]

where \( r \) is the nominal rate of return and \( I \) is the inflation rate. (See the article in this course “Living and Investing with Inflation, Fisher’s Effect.” See Exercise #1.)

Savers are seen as a better dating match, from a study by the University of Michigan. Spenders can signal a lack of control (Money, Nov. 2013, p. 18).

Working during retirement. Three-quarters of workers believe they will have a job in retirement, a third of those will work for the enjoyment. But reality doesn’t match expectation. EBRI found that only 25% worked for pay during retirement (Money, Nov. 2013).
Average number of years people expect retirement to last is 21 years (Money, Nov. 2013, p. 12). Evaluate their expectations.

Retirement portfolios became extinct. Many of the people who retired after the 1990s saw their portfolios become extinct over the next ten years – or they drastically reduced their standard of living (Scott Burns, “Your portfolio could be an endangered species,” Denton Record Chronicle, Sept. 13, 2013). Study the history of retirement fund earnings during that period – earnings on CDs, stocks, fixed annuities, short term bonds, real estate funds, and so on. See the article in this course, “Annual Total Return Table for the S&P 500 Index of Stocks.”

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Buying high and selling low is common among individual investors. Study after study has shown that when the stock market goes up, people pour money into equity mutual funds, and when the market goes down, they pull money out. During bear markets they pull even more out. For example in one study, this average behavior over a period of time resulted in a 2.1% return, less than the rate of inflation, when the S&P earned 7.8% and bonds earned 6.5%. (See DALBAR on the internet.)

High returns for domestic small-cap equities occurred over the test period (1927 – 2006) with a 10.46% geometric annualized real rate of return (13.89% nominal). Such high returns seemed to Blanchett as too aggressive for forward looking research (#2, p. 73).

Skewed left. Financial markets exhibit non-normal characteristics, such as kurtosis and skewness (#2, p. 79). (See the article in this course, “Investment Portfolio Design to Optimize Performance and Minimize Risk.”)

Experts forecast, (#4, p. 74) in 13 studies, future real returns by equities for the next approximately 20 years, (periods varied) ranged from 2% to 7% averaging 5.2% which is below the 7.3% for Domestic large blend in Table 2 (#2) and small-cap real returns reported above of 10.46%, for the 81 years from 1927 to 2007. It is interesting how often such forecasts are wrong.

Glide paths. Target date mutual funds have a glide path where exposure to equities (stocks) is reduced as the years pass and investment in bonds/cash is increased through the years such as starting at year 40, 30, 20, 10, 0. In Blanchett’s study #2, concave methodology proved the most optimal of certain methodology. The concave methodology has equity reduction that is fastest at the beginning and decreases at a faster and faster rate. Blanchett’s formula is (Next years equity allocation) = (Previous years equity allocation) – (Distribution Year)^2 × (.00002) starting at the year with the largest number and down to zero. See an example in the Exercises.

The triple tax break. For Health Savings Accounts: Your contributions are tax deductible (paid with pre tax dollars if made through an employer’s plan); the money grows tax deferred; and it can be used tax free for out of pocket medical expenses in any year. To get these breaks, you must have a health insurance policy with a deductible of at least $1250 for individual coverage and $2500 for families in 2014. You can contribute up to $3300 for individual coverage and up to $6500 for families for the year. See Publication 969, Health Savings Accounts and Other Tax Favored Health Plans at IRS.gov for more details. With Obama Care, things may change.
(Kiplinger’s Personal Fianace, 12/2013) Do some comparisons and long term estimates and calculations. This information would make some nice math problems. For example, pay copays and deductibles with other money and leave the HSA to grow tax deferred. Look up what people (seniors) pay out of pocket in medical expenses. Check with EBRI. Scott Burns by checking bluezones.com learned that his life expectancy is 96.2 years (the average) and that his “healthy” life expectancy is 88.7 years (7.5 years unhealthy) (“Getting ready for the longevity haul,” Denton Record Chronicle, Nov. 10, 2013).

**Exercises:** Show your work. Label answers, variables, and numbers. Give formulas and code when appropriate. Discuss when appropriate. Name your device.

#1. Use the formula for the real rate of return

\[ y = \frac{r - I}{1 + I} \]

and \( I = 3.51\% \) and the real returns given in Table 2 from #2, to estimated the average before inflation nominal rate of return \( r \) for each category of assets. Prove that for standard deviations, \( S_y = (1 + I)S_y \). Build a table for Table2, #2 giving for each market sector the arithmetic and geometric mean of the nominal rate and the standard deviation. If monthly real returns were calculated \( r - I \) and annualized, then the standard deviation could be different.

#2. Do the calculations to verify the claims under The sequencing effect. Show your demonstration or code. Name the device.

#3. Calculate the minimum average real rate of return applied yearly for the 4% rule to last 35 years.

#4. To get a nice retirement curve, graph \( Y2 = 1 \left(1 + .0351\right)^X \times \left[ \frac{1 - (1 + .0498)^{-(60-X)}}{.0498} \right] \) for Window: X: 0, 60, 5 and Y: 0, 100, 5. How much would it cost to retire at Year 30? Interpret. Would you say that up to a point, the older you get the more it costs to retire. Use Trace to record and discuss key points on the graph. Discuss what they tell about this particular retirement curve. Interpret the numbers in the formula. At year 30, calculate for \( R = $40,000 \) as the first withdrawal and interpret.

#5. For the 4% rule, solve \( 25 = 1 \left[ \frac{1 - (1 + .0498)^{-X}}{.0498} \right] \) for \( X \). What does this tell you? Try $20 for PV.

#6. Calculate the Total Return for the S&P 500 from Jan. 2000 to Jan. 2012. See measuringworth.com or some other source. Do the same from Jan. 1929 to Jan. 1952. See the article in this course “Annual Total Return Table for the S&P 500 Index of Stocks.”

#7. From Money, Nov. 2013, p. 26: “I’m 32, I have $125,000. With 8% growth, that’ll be $1.8 million at age 67. Am I set?“ Do the calculations to answer the question.

#8. According to Money, Nov. 2013, p. 71, if a person wants to retire at age 65 and are starting saving at age 35 – when baby boomers typically start, they need to save 15% to 19% a year. Considering inflation of salary, and saving 15% of inflating salary a year, what rate of return do they need, not considering taxes, to meet their goal. A long term average inflation rate has been 3.24%. In “America’s Retirement Challenge” in this course, to supplement Social Security, Fidelity recommended that they should save eight times their last year’s salary. You may need to derive a formula for future value and
apply a TVM Solver. What rate of return is needed if they save 19%? What would you advise them to do? You could use the 4% Rule with first year’s withdrawal a certain percentage of the last year’s salary, with or without considering Social Security.

#9. According to Money, p. 76, Nov. 2013, the S&P 500 has turned in a 16% annual gain over the past three years propelling the typical 401(k) balance to a record high this year, Fidelity reports. For workers 55 or older, their average high of $255,000, was nearly doubled the amount in March 2009, the depths of the bear market. Notice that \(0.5(255,000)(1+1.16)^3 < 255,000\). If they made up the difference by saving the same amount each year at 16%, how much did they save each year? Is this number reasonable? If they chose to increase the yearly saving by 3.24% per year starting at the end of the first year at \(R(1+I)\), how much did they save at the end of the first year, the second, and the third?

#10. According to Money, Nov. 2013, p. 120, $2533 per month is the maximum Social Security retirement benefit. What will it be when you retire? Discuss.

#11. How would you combine the standard deviations of the 50% Cash (4.10%) and 50% Intermediate-Term Bond (6.14%) to get the standard deviation of the combined portfolio of cash and bonds? See the articles in this course on standard deviation and correlation.

#12. WSJ.com, 10/14/2013 says that if you had retired Jan 1, 2000, with an initial 4% withdrawal and a portfolio of 55% stocks and 45% bonds, with the withdrawals increased each year at 3%, your portfolio would have fallen by a third through 2010. You would have less than a 29% chance of making it through another twenty years. If the retiree continues the 4%/3% program, what rate of return on the investment would be required for funds to survive the 20 years? If the couple had retired at age 65, what is the probability that they will live another 20 years, another 25 years? What should they do?

#13. If a 65 year old couple wants the probability of failure of funds to be less than 1%, for the 4% rule, what initial withdrawal rate should they choose and what is the probability of failure? In the joint survivability table, what initial withdrawal rate could they choose and what is the probability of failure?


#15. Use the concave methodology formula under the Side Bar Note Glide paths:

\[(\text{Next years equity allocation}) = (\text{Previous years equity allocation}) - (\text{Distribution year})^2 \times (.00002)\]

starting with year 40, then 39, and so on. Use \(0.40 = 40\%\) as year 40’s (the first year) equity allocation and show that the yearly allocation approached zero in the neighborhood of 19 years. Show that the slope of the curve decreases as the number of the year decreases, and it approaches zero as the number of the year decreases from 25 down. Some students could do a graph. Put years on the horizontal axis. (Starting the first year at 100% equities, referred to 100/0 concave, was best (for non-constant) for survivability in 53.93% of all scenarios. The 40/60 concave didn’t rank high. If you can do the math, calculate with the 100/0 concave. To get a formula for \(A_k\), allocation at year k, you could try using two pieces of

\[1^2 + 2^2 + 3^2 + \ldots + k^2 = \frac{k(k+1)(2k+1)}{6}.\]

#16. In the Side Bar Note: “Using the TVM Solver to verify Formula 1”, it was demonstrated that with the real rate of return of \(y = 4.59\%\) per year applied each year, and the first withdrawal of \(R = $1\) at the beginning of the first year, and increasing each year at the rate of inflation \(I = 3.51\%\), that \(P = $25\) would last forever. Use \(I\) and the real rate of return \(y\) to calculate the rate of return \(T\) on the investment. Write a program for 100 years to illustrate Formula 1. We did this and got a positive final balance S for
$25, and a negative final balance for $20. Your final sums may be different from those of other people because of time line differences and rounding. You could do this on a spreadsheet. Try P = $1M and R = $40,000.

#17. Blanchett says that under the 4% rule, for a 65 year old couple, the probability of failure of funds lasting as long as either shall live is 0.7%. Describe the details of how this probability could have been arrived at.

References: To access Blanchett’s articles used in this paper, do an internet search for David M. Blanchett and click on research-www, and click on the articles marked View or Link. These four articles are marked View.


In this course:

“Funding Retirement During a Worst Market Scenario,” based on an article at Vanguard Funds.

“The 4% Rule for Retirement Withdrawals,” based on the Trinity Study and follow ups.

“America’s Retirement Challenge,” based on a speech by a Fidelity Funds official.


Also see “Two New Retirement Income Withdrawal Strategies,” which examines the 4% rule, a "valuation strategy" based on stock P/Es at the time of initial withdrawal, and a "dynamic strategy" based on portfolio losses, with a (FIWR) feasible initial withdrawal rate for which funds are sustained for 90% of 30 year rolling periods. Go to troweprice.com. Search: T. Rowe Price Report, Fall 2013 and download the issue.

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