Measures of Risk and Performance for a Mutual Fund: Beta, Alpha, and Sharpe Ratio (Preliminary Version)

Floyd Vest, September, 2011

Statistics for Mutual Fund X: A balanced fund, 2/3 stocks, 1/3 bonds, five year average rate of return 6.39%, a Five Star fund, Expense ratio .34%, Standard deviation 14.53, Beta 1.02, Alpha .04, Sharpe ratio .42. Risk measurements are over the most recent three years back from March 2011 (www.morningstar.com).

Mutual fund investors evaluate funds by several statistics which are indicated above. We will discuss beta, alpha, and the Sharpe ratio. (For standard deviation, see the article in this course entitled “Standard Deviation as a Measure of Risk for a Mutual Fund.”)

Beta and Alpha. According to the Capital Asset Pricing Model, a mathematical theory of economics, a Security Market Line (SML) represents “appropriate” expected returns for mutual funds with different betas (β). By definition, the SML runs through the points (0, arf) and (1, arm) where arf is the mean (arithmetic average) risk free rate of return, usually Three Month Treasury Bills. For a stock fund, arm is the arithmetic average (mean) rate of return for the stock market, often represented by the S&P 500 Index of Stocks (Alexander and Sharpe, p. 181, 391-401). See Figure 1. (See “Note about averages” in the Side Bar Notes.)
Example 1. In this example, $ar_M$ is a simulated average quarterly rate of return for the S&P 500 index through 1982, 1983, 1984, and 1985. The $ar_p$ is a simulated average (arithmetic) quarterly return for Mutual Fund A. The $ar_f$ is a simulated average quarterly rate of return for the Three Month Treasury Bills, over the same period. By definition, the beta for the market, S&P 500, is one. See Figure 1.

In Figure 1, on the graph $ar_M = 4.7\%$. Using $\beta = 1$, the slope of the SML is

$$
\frac{ar_M - ar_f}{1-0} = ar_M - ar_f = 4.7 - 2.13 = 2.57,
$$

and the vertical intercept is $ar_f = 2.13\%$. Thus the equation of the line is $ar_M = ar_f + (ar_M - ar_f)\beta$ which in this case is $ar_M = 2.13 + (2.57)\beta$.

The investor is interested in the overall risk of the fund which is the standard deviation. This variability (standard deviation) can be attributed to overall market volatility (the S&P 500) and the manager’s management and choice of securities.
Accordingly, the manager is evaluated in terms of his contribution to the variability. A relevant measure of this added risk is his beta.

A fund with a higher beta (risk) than the market is expected to produce a return as high or higher than the point on the SML line based on his beta. In Figure 1, the beta for Fund A is 1.12 and the expected return \( e\alpha r_p = 5.01\% \) and the average return for the fund was \( ar_p = 3.83\% \). The difference between \( ar_p \) and \( e\alpha r_p \) is the fund's alpha. In Figure 1, \( \alpha = 3.83 - 5.01 = -1.18 \). According to the CAPM, a negative alpha represents less return than expected for the risk.

The investor can compare similar funds by their beta and alpha. Alpha is the difference between the average rate of return and the expected rate of return based on the beta. A positive alpha means the rate of return is superior for the risk. The alpha for the market is zero. A beta greater than one means the fund took on more risk than the market (S&P 500). If beta is less than one, the fund has less volatility than the market. A beta greater than one means that the standard deviation for the fund is greater than that for the market. A beta less than one requires less return to justify the lower risk.

Risk adjusted rates of return can be used. They are \( r_p - r_f \) which is the return of the fund \( (r_p) \) minus the risk free return \( (r_f) \) (Three Month Treasury Bills). For the market, it is \( r_M - r_f \) where \( (r_M) \) is the return on the market.

In Figure 1, with the point P(1.12, 5.01) representing P(\( \beta, e\alpha r_p \)) on the SML line, Eq. 5
\[
ear_p = ar_f + (ar_M - ar_f)\beta.
\]
Eq. 1
\[
\alpha = ar_p - [ar_f + (ar_M - ar_f)\beta] \text{ which gives the Fund Characteristic Line:}
\]
\[
ar_p - ar_f = \alpha + \beta(ar_M - ar_f), \text{ or}
\]
Eq. 2
\[
r_p - r_f = \alpha + \beta(pM - r_f) \text{ with } r_M - r_f \text{ on the horizontal axis and } r_p - r_f \text{ on the vertical axis, with vertical intercept of } \alpha_p \text{ and slope } \beta_p. \text{ We assume } r_M > r_f, \text{ see Alexander and Sharpe for other cases. The } r \text{ values are for a sequence of periodic returns. For the above example,}
\]
Eq. 3
\[
r_p - r_f = -1.18 + 1.12(r_M - r_f).
\]

Example 2. Linear Regression in the TI84. The simplest method for us to determine alpha and beta for a fund is by simple linear regression. Consider linear regression on the TI84 on List 1 (Which we mounted on the TI84) containing sixteen quarterly average excess market returns \( r_M - r_f \) and in List 2 the paired sixteen quarterly excess returns \( r_p - r_f \) for Fund A. (In Example 1.) (See Exercise 6.) We may need to turn off previous plots: Code: Y= \( \wedge \) and turn off plots. Then turn on scatter plots: Code: \( 2^{nd} \) [StatPlot] 1 Enter \( \vee \) Select the first box, the scatter plot. Enter \( \vee \) \( 2^{nd} \) L1 Enter for X \( \vee \) \( 2^{nd} \) L2 for Y Enter > Enter for + as Mark \( 2^{nd} \) Quit Then we ran LinReg(b+ax) L1, L2, Y. Code and commentary: STAT
You see LinReg(a+bx) on the home screen. Then 2nd L1 , 2nd L2 , Then we selected a Y to contain the equation of the regression line by VARS > Enter Enter Enter. We saw on the home screen y = a + bx, a = -1.1769 which is equal to \( \alpha \), and b = 1.1215 which is equal to \( \beta \). This gave

\[
(4) \quad r_p - r_f = -1.18 + 1.12(r_M - r_f), \quad \text{with } Y = r_p - r_f \text{ on the vertical axis and } X = r_M - r_f \text{ on the horizontal axis. To see the scatter plot of the points } (r_M - r_f , r_p - r_f) \text{ and the regression line: We did ZOOM 9. We could check the plotted points with TRACE. We saw the first pair of coordinates from the lists of L1: } X = r_M - r_f = -8.93, \text{ L2: } Y = r_p - r_f = -11.79, \text{ and at the top P1:L1,L2. We used WINDOW to change the scale on the axes.}
\]

To get \( r \) (correlation coefficient) and \( r^2 \), we used 2nd Catalog \( \checkmark \checkmark \checkmark \ldots \checkmark \) to DiagnosticOn Enter. Then reran LinReg and saw additionally \( r^2 = .8444 \) and \( r = .9189 \). The \( r \) is the correlation coefficient between the excess returns of the market and those of Fund A, and is close to one as it should be, indicating a high degree of correlation between the returns of the market and Fund A. A representative of the market (S&P 500, a benchmark) should be highly correlated with the type of fund and fund values for meaningful beta and alpha.

**Summary.** Thus with a \( \beta = 1.12 \), the manager of the fund took on more risk than the market. With an \( \alpha = -1.18 \), his performance was 1.18 percentage points below the expected return of \( e\text{ar}_p = 5.01\% \), for his level of risk. (See Figure 1.) So when comparing funds with the same objectives and type of investments, consider their average rate of return, their standard deviation, their beta and alpha.

**Discussion of beta and alpha.** (From [www.morningstar.com](http://www.morningstar.com), investing classroom.) Morningstar applies both alpha and beta to annual returns. A risk free investment has a beta of zero. A fund with a beta of zero has its returns changing independently of the benchmark (market). The beta represents the funds statistical variance which cannot be removed by diversification represented by the benchmark. In fund management, beta and alpha together can be thought of as separating a manager’s skill from his willingness to take risk.

Alpha is the difference between the fund’s expected return based on beta and it actual return. Two funds with the same returns could have different alphas if they have different betas. Further if a fund has a high beta, it’s quite possible for it to have a negative alpha. That’s because the higher the beta, the greater the expected return.

In general, you would want to have high alpha funds where alpha depends on beta. Beta can be meaningless if the fund’s correlation to the benchmark is below .75. Then both beta and alpha are not valid. Alpha fails to distinguish between underperformance caused by poor management and underperformance caused by fees. Morningstar includes fees, loads, and commissions in evaluating funds.
Calculating beta. “A misconception about beta is that it is a measure of the volatility of a security relative to the volatility of the market” (www.wikipedia.org, beta). Another misconception is that it is the correlation between the returns of the security and those of the market. Consider the following definitions, and the theorems in the exercises.

Definitions: \[ \beta_p = \frac{\text{covariance}(r_M, r_p)}{\text{Variance}(r_M)}. \]

\[ \text{Covariance}(x,y) = \frac{\sum (x - \overline{x})(y - \overline{y})}{N} = \text{mean}[ (x - \overline{x})(y - \overline{y})] \]

\[ \text{Variance}(x) = \frac{\sum (x - \overline{x})^2}{N} \]

\[ \text{Correlation}(x,y) = \frac{\text{covariance}(x,y)}{S(x)S(y)} \text{ where } \]

\[ S(x) = \sqrt{V(x)}. \] See the Exercises.

Sharpe ratio. Sharpe ratio uses a funds risk adjusted returns divided by the funds standard deviation. Both are usually annualized. The higher the fund’s Sharpe ratio, the better a funds returns have been relative to the risk it has taken on. Because it uses a fund’s standard deviation, it can be used to compare funds across all categories. The higher a fund’s standard deviation, the higher its risk adjusted returns need to be to get a higher Sharpe ratio. Conversely, funds with lower standard deviation can have a higher Sharpe ratio if they have decent returns.

Calculating Sharpe ratio. (www.wikipedia.org, Sharpe ratio) Assume a long term annual return of the S&P 500 of 10%. Assume that risk free returns have averaged 3.5% per year. The average standard deviation of the S&P 500 is about 16% per year.

Calculating, the long term Sharpe ratio for the S&P 500 gives \[ \frac{10\% - 3.5\%}{16\%} = .4. \] This number gives a meaningful Sharpe ratio to be used in evaluating stock funds. Sharpe ratio and standard deviation, as reported, are annualized.

For another example, we use

(5) \[ \text{Sharpe ratio} = \frac{ar_p - ar_f}{S_p} \text{ where } S_p \text{ is the standard deviation for the fund. Assume } \]

\[ ar_p - ar_f = 15\% \text{ and } S_p = 10\%. \] Sharpe ratio = \[ \frac{.15}{.10} = 1.5. \] This was an exceptional investment.

It is easier to compare funds of different types using the Sharpe ratio than with beta and alpha which are calculated with different benchmarks for stock funds and for bond funds. We can therefore use Sharpe ratio to compare stock funds and bond funds. When you compare one fund’s Sharpe ratio to those of similar funds, you get a feel for its risk adjusted rate of return relative to other funds.
**Exercises:** Show your work. Label numbers and variables.

1. (a) According to the CAPM economic theory, what two major components of a mutual fund’s risk are being considered?  
   (b) Give a formula for calculating $ear_p$. Explain.  
   (c) Name ten “other” kinds of risk. Some may affect the market.

2. (a) Using the values in Figure 1, for a Fund B with a beta of .9, what is the $ear_p$?  
   (b) How does this compare to $ar_m$? Draw a graph.  
   (c) How does the effect of beta = 1.12 compare to the beta of .9?  
   (d) Does the manager of Fund B have to beat the market to get a positive alpha?  
   (e) Do we know $ar_p$ and alpha? Give a general formula for $\alpha$.  
   (f) Summarize the work of the manager.

3. (a) For Equation 4, draw a graph. Label axes.  
   (b) In numerical and general terms, what is the horizontal intercept and the vertical intercept?  
   (c) Show from the graph that slope = $\beta$ = 1.12.

4. For Fund X in the introduction, all data is annualized.  
   (a) Estimate the risk-free rate of return $ar_f$ by using Sharpe Ratio.  
   (b) Do a graph as in Figure 1. Calculate $ar_m$. What are its $\beta$ and $\alpha$ on the graph?  
   (c) Interpret its alpha.  
   (d) How would you design a benchmark for this fund?  
   (e) What is $ear_p$? Interpret.  
   (f) Apply the standard deviation to gauging an expected range of returns for Fund X.  
   (See the article in this course on standard deviation.)

5. (a) Derive Formula 2 from Formula 1.  
   (b) Explain differences, if any?

6. (a) On the TI84, enter the following sixteen pairs of simulated quarterly excess returns $r_M - r_f$ in L1 as X, and $r_p - r_f$ in L2 as Y.

<table>
<thead>
<tr>
<th>L1, $r_M - r_f$</th>
<th>L2, $r_p - r_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-8.93</td>
<td>-11.79</td>
</tr>
<tr>
<td>-6.1</td>
<td>-9.14</td>
</tr>
<tr>
<td>10.82</td>
<td>11.24</td>
</tr>
<tr>
<td>12.84</td>
<td>23.03</td>
</tr>
<tr>
<td>9.91</td>
<td>1.76</td>
</tr>
<tr>
<td>9.45</td>
<td>8.6</td>
</tr>
<tr>
<td>-2.9</td>
<td>-2.49</td>
</tr>
<tr>
<td>-2.19</td>
<td>-1.89</td>
</tr>
<tr>
<td>-4.78</td>
<td>-5.29</td>
</tr>
<tr>
<td>-4.29</td>
<td>-9.11</td>
</tr>
<tr>
<td>6.19</td>
<td>6.09</td>
</tr>
<tr>
<td>-.87</td>
<td>-1.3</td>
</tr>
</tbody>
</table>
Then run LinReg as reported in Example 2. To mount data in a List, Code and commentary: STAT Enter You see columns headed L1 and L2 into which your enter data. Data is entered on the bottom line with Enter. When you have L1 and L2, each with sixteen items, do 2nd Quit to go to the home screen and continue as in Example 2. Report your results. (b) Then go back and turn on DiagnosticOn and rerun. Give r and \( r^2 \). Explain r. (c) Give \( \alpha \) and \( \beta \) and the equation of the characteristic line.

Summarize the success of the manager.

7. (a) If Fund C has a beta of .9 and the S&P loses 10%, what is the expected return for Fund C? (b) If a stock index fund had an average risk adjusted annual rate of return in the last five years before June 30, 2011 of 2.54% and a standard deviation of 14% per year, what was its Sharpe Ratio? (c) For Fund X (in the introduction) what was \( ar_f \) and its average risk adjusted rate of return? Compare Fund C with Fund X. (c) Discuss the use of the Sharpe Ratio.

8. (a) Given that mean of \( x = \bar{x} = \frac{\sum_{i=1}^{N} x_i}{N} \), prove that the mean of \( (x + y) = \text{mean of (x)} + \text{mean of (y)} \). (b) Prove that for a constant \( c \), \( \text{mean of (x + c)} = \text{mean of (x)} + c \). (c) Given that the variance \( V = \frac{\sum_{i=1}^{N} (x - \bar{x})^2}{N} \) and the standard deviation \( S = \sqrt{V} \), prove that \( \text{V} = \frac{\sum_{i=1}^{N} x^2}{N} - \bar{x}^2 \) and \( S = \sqrt{\frac{\sum_{i=1}^{N} x^2}{N} - \bar{x}^2} \). (d) Prove that for a constant \( b \), the standard deviation of \( (x + b) = \text{standard deviation of (x)} \).

9. (a) Use the TI84 to calculate the mean and standard deviation of L1 and L2. Explain their meanings. See the Side Bar Notes for code. (b) Use the theorems in Exercise 8 and this data to verify \( ar_M \) and \( ar_p \) in Figure 1. (c) Use the theorems to estimate the standard deviations of \( r_M \) and \( r_p \). Compare and explain. (d) Use the TI84 to calculate the geometric mean of L1 (the average annual total return, sometimes called the geometric mean. See the answer to Exercise 9 for an example).

10. (a) Graph the average excess returns from Exercise 9 against \( \beta \). (b) Estimate \( ea(r_p - r_f) \).
11. The performance of Stock Fund D, and a stock index fund (holding the stocks of a major stock index) over the last five years is as follows:

<table>
<thead>
<tr>
<th></th>
<th>Fund D</th>
<th>Index Fund</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average quarterly excess returns</td>
<td>0.5%</td>
<td>0.4%</td>
</tr>
<tr>
<td>Standard deviation of quarterly</td>
<td>9.2%</td>
<td>6.4%</td>
</tr>
<tr>
<td>of quarterly excess returns</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Beta</td>
<td>1.05</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Which fund has the best historical performance and management of risk? Show the use of several measures. What are some other considerations? Which would you recommend based on all considerations you have available?

12. Do a study project on evaluating mutual funds. Consider wikipedia.org, morningstar.com, investopedia.com, websites of mutual fund families, and other sources. Do some mathematics and submit an article to COMAP.com.

13. Do a project comparing several different mutual funds of the same type in terms of performance and risk. Build a table reporting on as many factors as available. You can find candidates on TV advertisements, in magazines, on the internet – fund screeners are available. There are at least ten performance and risk factors. See the articles on mutual funds in this course and the internet for factors to consider. Write a summary including definitions of measures. Begin by stating your personal investment objectives. If you get important insights, share them with your class including a written report.

14. Prove Covariance(x,y) = mean( x \times y ) - \overline{x} \overline{y}.

15. Prove Covariance (x,y) = \frac{1}{N^2} \left[ N \sum x y - \sum x \sum y \right].

16. Prove Beta(x,y) = \frac{N \sum x y - \sum x \sum y}{N \sum x^2 - (\sum x)^2}.

17. Prove Correlation(x,y) = \frac{N \sum x y - \sum x \sum y}{\sqrt{\left[N \sum x^2 - (\sum x)^2\right] \times \left[N \sum y^2 - (\sum y)^2\right]}}^{1/2}.

18. Prove Covariance (x,x) = \text{Var}(x).

19. Prove beta for the market is one.

20. Prove that if (x_1, y_1) and (x_2, y_2) are on y = x, then the correlation (x,y) = 1, and that the covariance (x,y) = \frac{(x_1 - x_2)^2}{4}. Prove that if they are on y = -x, then correlation (x,y) = -1.
Answers to Exercises.

1. (b) $ea_r = ar_f + (ar_m - ar_f)\beta$. $ea_r$ is the expected average return for Fund A based on its beta.

2. (a) $ear_r = 2.13 + (4.7 - 2.13)(.9) = 4.44\%$ (b) $ear_r = 4.44 < ar_m = 4.7$ (d) No. (e) No. $\alpha = ar_p - ear_r$ (f) The manager took less risk than the market. They don’t have to beat the market to get a positive alpha. $\alpha = 4.44 - 3.83 = .61$

3. (a) From Formula 4, $r_p - r_f = -1.18 + 1.12(r_m - r_f)$. (b) The horizontal intercept on the $r_m - r_f$ axis is $\frac{-\alpha}{\beta} = 1.054$. The vertical intercept on the $r_p - r_f$ axis is $\alpha = -1.18$. (c) slope $= \frac{rise}{run} = \frac{0 - (-1.18)}{1.054} = 1.12 = \beta$.

4. (a) Sharpe ratio = .42 $= \frac{ar_p - ar_f}{S_p} = \frac{6.39 - ar_f}{14.53}$. $ar_f = .287\%$, $ar_p = 6.39\%$, $\alpha = .04$ (b) By Formula 2, $ar_p - ar_f = \alpha + \beta(ar_m - ar_f)$. $6.39 - .287 = .04 + 1.02(ar_m - .287)$, $ar_m = 6.23\%$. (d) For a benchmark, use a portfolio of 2/3 a stock index and 1/3 a bond index. Its $\beta = 1$ and $\alpha = 0$. (e) By Formula .5, $ear_p = .287 + (6.23 - .287)(1.02) = 6.349\%$. Since $ear_p < ar_p$, the fund did better than its expected rate of return based on its beta. $\alpha = 6.39 - 6.349 = .04$. (f) Most of the time for Fund X, its annual return would be between $6.39\% + 14.53\%$ and $6.39\% - 14.53\%$.

5. (b) There is just a shift in notation. The variable still represent averages.

7. (a) The expected return for Fund C is $ear_p = -10(.9) + (.1)ar_f$.

$ear_p = -10\%(.9) = -9\%$ if $ar_f$ is small. (b) Sharpe ratio $= \frac{2.54}{14} = .18$ (c) For Fund X, Sharpe ratio $= .42 = \frac{6.39 - ar_f}{14.53}$. $ar_f = .287\%$. The risk adjusted rate of return is $ar_p - ar_f = 6.39 - .287 = 6.1\%$. Standard deviation = 14.53. Fund C did not perform as well as Fund X.

8. (d) $V(x+b) = \frac{\sum(x^2 + 2xb + b^2)}{N} - (\bar{x}^2 + 2\bar{x}b + b^2) = \frac{\sum x^2}{N} - \bar{x}^2 + 2b\bar{x} + b^2 - 2b\bar{x} - b^2 = V(x)$ so $S(x+b) = S(x)$. 
9. (a) Mean of \(L_1 = 2.57 = \text{mean of } r_M - r_f\). Standard deviation of \(L_1 = 7.56\) per quarter. Mean of \(L_2 = 1.703 = \text{mean of } (r_p - r_f)\). Standard deviation of \(L_2 = 9.225\).
(b) \(\text{Mean}(r_M - r_f) = \text{mean}(r_M) - \text{mean}(r_f) = 4.7 - 2.13 = 2.57\).
(b) \(2.57 = \text{mean}(r_M - r_f) - \text{mean}(r_M) = \text{mean}(r_f)\). Let \(\text{mean}(r_f) = 2.13\).
\(2.57 = \text{mean}(r_M) - 2.13. \text{mean}(r_M) = ar_M = 2.57 + 2.13 = 4.7\).
For \(ar_p, 1.703 = ar_p - 2.13. ar_p = 3.83\). (c) To estimate standard deviation of \(r_M\), use a constant \(c\), and standard deviation of \((x+c) = \text{standard deviation of } x\). Assume \(c = r_f\), a constant. So \(7.56 = \text{standard deviation of } r_M\). Similarly standard deviation of \(r_p = 9.225\).
(d) Code and commentary: On the home screen, 
\[
\text{2nd L1 } ÷ \text{100 } + \text{1 } \rightarrow \text{L5 2nd Quit List } < \text{6 You see prod( Type L5 ) Enter } \text{You see 1.4403. Then calculate } i = \sqrt[16]{1.4403} - 1 = .023 = 2.3\%.
\] This is sometimes called the geometric mean.

11. For Fund D, \(\alpha = .08\%\). For Fund D, Sharpe ratio = .05 per quarter. For the stock index, Sharpe ratio = .0625 per quarter. Fund D did a good job of managing risk and outperformed the index on quarterly excess returns. The stock index had a better Sharpe ratio.

**Side Bar Notes**

**Note about averages.** Alexander and Sharpe, from which we are adapting this derivation, used Arithmetic Averages for \(ar_M, ar_f\), and \(ar_p\). See pages 392-394. Wikipedia says that the market rate of return is usually estimated by the Geometric Average of historical returns. The risk free rate of return is usually the Arithmetic Average of historical returns. (http://en.wikipedia.org/wiki/Capital_asset_pricing_model, page 2). The literature applies beta and alpha to annual geometric rates of return.

**Code for calculating on the TI84 the mean and standard deviation of L1 and L2.**

To calculate the mean of L1, code and commentary: 
\[
\text{2nd Quit 2nd List } < \text{Math 3 for mean You see on the home screen mean( Type 2nd L1 ) Enter You see the mean of L1: } 2.567. \text{For standard deviation: 2nd List } < \text{to Math 7 for stdDev( 2nd L1 ) Enter You see the standard deviation 7.56 (for quarterly returns).}
\]

**Linear Regression** is a method of fitting a line \(y = a + bx\) to the data points by choosing values for \(a\) and \(b\) which minimize the sum of the squares of the distances from the points to the line. See a calculus book.

**Correlation coefficient** always lies from \(-1\) to \(+1\). A value of \(-1\) represents perfect negative correlation, for example a scatter plot on \(y = -x\). A value of \(+1\) represents perfect positive correlation, for example a scatter plot in \(y = x\). This high positive correlation means that when one of the two securities have a relatively high return, then so does the other. If the scatter plot is represented by a downward sloping
line, with negative correlation, then the two securities move in opposite directions. If the scatter plot cannot be represented by an upward or downward sloping line, then the security returns are not linearly related, the correlation coefficient is close to zero. (See a book on statistics.)

Morningstar’s Star Ratings. (www.morningstar.com, investing classroom) It is common for mutual funds to report their Star Ratings by Morningstar. The ratings are calculated mathematically based on a fund’s risk adjusted rate of return relative to similar funds with a weighting against downward volatility and favoring upward volatility. The 10% of funds in each category with the highest risk adjusted returns receive 5 stars, the next 22.5% receive 4 stars, and the middle 35%, 3 stars, and so on. Funds are rated for three, five, and ten years. The ratings are based on historical results and don’t guarantee future results.

“5 Top Fund-Picking Tips,” Kiplinger Personal Finance, Aug. 2011, p.38. The editor writes, “measures of a fund’s risk-adjusted returns. … You can check a fund’s alpha, its Sharpe ratio. … domestic stock funds that earned five stars in 2005 beat their index 55% of the time over the next five years. … Five-star domestic stock funds with expense ratios in the bottom 25% of their category beat their index 66% of the time.” (See kiplinger.com/tools/fund finder.)

Standard deviation, the most popular risk measure. According to Morningstar, for standard deviation, they calculate with the most recent three years of monthly returns. Then monthly returns are annualized. (www.morningstar.com) It measures the average dispersion about the mean return for the fund’s returns. For example, if Fund A has a 10% average rate of return and a standard deviation of 9%, most of the time, we would expect the return to range from 10% - 9% to 10% + 9%. A person can compare standard deviations of funds with similar or dissimilar returns and objectives.

If a fund loses money and does so consistently, it can have a low standard deviation. A fund that gains 10%, then 15%, could have a high standard deviation. So volatility as a form of risk, it is not itself a bad thing (www.morningstar.com, investing classroom).

Question: How are alphas based on quarterly returns annualized? In a discussion, Sharpe (Sharpe and Alexander, p. 657) says “Figure 20.8(c) shows quarterly returns for 100 mutual funds. The average alpha was -.50% per quarter, or approximately 4(-.50) = 2% per year. “ In another discussion, monthly returns are annualized and annual alphas are reported. “In Figure 20-8(a), alphas are calculated based on annualized monthly returns. The average alpha was .09% per year ….”

Sortino ratio. The Sortino ratio measures risk-adjusted rate of return. It is a modification of the Sharpe ratio, but penalizes only those returns falling below a user-specified target (www.wikipedia.org, Sortino ratio).

Low Volatility funds – low beta and low price to earnings ratio. The number of such funds has doubled to 35 since the start of 2012. Only one is three years old. They
can still lose money. In 2008, the S&P 500 lost 37% and a leading low volatility fund lost 27%. Some have a beta as low as .66. Interpret what this means. (Money, Oct. 2013, p. 44)

An 8% retirement withdrawal program and 12% return on investments are recommended by a famous financial planner of radio, TV, and print. The retirement program includes withdrawals increasing each year at the rate of inflation. (a) Based on the articles in this course, what do you think of this? Is this good advice? (Money, Oct. 2013, p. 72-78) . (b) Money said that if you simply take out 8% of your portfolio balance every year, your income could drop 40% after a year like 2008. Prove that income drops 40% if and only if the portfolio drops 40%. (c) Money said that if you take the advice to save 15% of your income a year, you’ll be doing okay in the end whether you earn 7.6% or 12%. Do an example to check this out. (If you need help, see the article in this course, “America’s Retirement Challenge.”) (d) According to Money, over the last 20 years after expenses, the return on selected stock mutual funds was 7.6%. If withdrawals begin at 8% and increase at 3.2% per year, how long would the retirement money last? (e) Money said that the front end load on deposits in a certain mutual fund is 5.75%. If beginning earnings is $40,000 and you save 15% increasing each year at 3.2%, for 40 years, and money earns 10%, what is the future value of the front end loads on yearly deposits? (f) For 40 years, what is the rate of return, to which the 10% is reduced, by the front end loads on yearly deposits? What is the reduced rate of return for one year, for 10 years? Could you do a graph of this reduced return from one year to 40 years?

Four year sequences of rates of returns which have an arithmetic average of 12%. Invent three examples from the simplest to the exotic which have an arithmetic average of 12%. Note that for sequences with 12% per year arithmetic average, the average compounded total return can vary? Prove that for returns a and b, if the arithmetic mean equals the average annual total return, then a = b. What is the compounded total return for four years of 12% each year, arithmetic average? If the 12% per year is not compounded, what is the annual total return?

Reducing investing expenses. At Vanguard and Fidelity, it takes only $10,000 invested in a fund to qualify for reduced expense ratio. Example, .17% vs .05% on Vangurad 500 Index Fund, Admiral Shares (Money, Oct. 2013, p. 60) . Do the calculations on how much this makes you. The average expense ratio for mutual funds is about 1.20%, if I remember right.

Teachers’ Notes

For a copy of the TI 84 manual, see http://www.ti.com/calc.


For a free course in financial mathematics with emphasis on personal finance, see COMAP.com. Click on the box for the free financial mathematics course, register, and COMAP will e-mail you a password. Simply click on an article in the annotated bibliography, download it, and teach it.

References

Vest, Floyd, “Standard Deviation as a Measure of Risk for a Mutual Fund,” COMAP.com, in this course.