The Remarkable History of Returns of Small Cap Stocks (Preliminary Version)
Floyd Vest, Nov. 2014

In March 1981, Ibbotson and Singfield (See the References) reported a study of the affect of market capitalization of stocks (price of stock × number of shares outstanding) on stock returns. They reported the remarkable earnings of small cap stocks in comparison to the S&P 500 Index of Stocks. From the beginning of 1926 through 1983, for 58 years, the average annual return for small cap was 17.05% and for the S&P 500 was 11.26%, a 5.7% per year difference. See Table 1.

<table>
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<tr>
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<th>Small Stocks</th>
<th>S&amp;P 500</th>
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<tbody>
<tr>
<td>Average annual return</td>
<td>17.05%</td>
<td>11.26%</td>
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<tr>
<td>Annual standard deviation</td>
<td>32.35%</td>
<td>20.62%</td>
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<tr>
<td>Percent of months with positive earnings</td>
<td>58.3%</td>
<td>58.6%</td>
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For their study, all stocks in the New York Stock Exchange were ranked by market capitalization for Dec. 31, 1925. A small cap portfolio was selected as the bottom quintile (20%). For the small cap portfolio, stocks were selected in proportion to their market capitalization. Every five years, the portfolio was revised. On average, small cap outperformed the S&P 500 by 5.79% per year or 51.7% over time. (See the Exercises.)

For current stock indexes, we use the Center for Research in Security Prices (CRSP), University of Chicago. (See CRSP in the Side Bar Notes.) For Jan. 1926, they divided the U. S. stock market into deciles based on market capitalization. We give approximate comparisons in Table 2 which are only approximations from a graph. See CRSP for more accurate returns. You might have to pay money or copy from someone who did.

<table>
<thead>
<tr>
<th></th>
<th>CRSP small cap (9-100)</th>
<th>CRSP small cap (6-10)</th>
<th>CRSP (1-5)</th>
<th>CRSP large cap (1-2)</th>
</tr>
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<tbody>
<tr>
<td>Annual Rate of Return</td>
<td>12.4%</td>
<td>11.8%</td>
<td>9.5%</td>
<td>9.4%</td>
</tr>
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McDermott and D’Auria, AAIJ Journal, July 2014 (See the References.)
IFA.com gives for small cap stocks for the 86 years, 1928 to 2013, an average return of 11.20%.

For the largest cap stocks, the top two deciles, CRSP (1-2), the average annual return was approximately 9.4%. For the smallest two deciles, CRSP small cap (9-10) it was 12.4%, and the smallest five deciles CRSP small cap (6-10) it was 11.8%. What would be the growth of $1 for each, over the 88 years? See the Exercises. You can see that there are different definitions for small cap stocks. The indexes are periodically revised.

According to McDermott and D’Auria, one remarkable feature of CRSP (6-10) small cap is that value small cap stocks significantly outperform other small cap stocks. From Jan. 1926 to Dec. 2013, value small cap stocks earned approximately 17% annualized while CRSP (6-10) earned approximately 11.8%. Thus we have an interest in not only small cap stocks but also small cap value stocks. (See Vanguard Small-Cap Value Index Fund (VISVX) in the Side Bar Notes.)

In order to rank small cap stocks by value, CRSP divided small cap stocks (CRSP 6-10) into five asset classes based on Book to Market Ratio, BtM = \( \frac{book \text{ value}}{market \text{ cap}} \). BtM is the inverse of the common price-to-book ratio. Stocks with highest BtM were considered value stocks. Note for this, the value of the company was greater in proportion to the market cap. For example, if you look at the holdings of the Vanguard Small-Cap Value Index Fund which follows the CRSP U. S. Small Cap Value Index (9/30/2014), you will see the largest holding is Snap-on Inc. (SNA). From finance.yahoo.com, Key Statistics, we get

\[
\text{Price/Book} = 3.39 \quad \text{and BtM for SNA} = \frac{1}{3.39} = .295
\]

The larger the BtM the greater the value rating of the stock. You can look at the Constituent companies in the CRSP U. S. Small Cap Value Index and see 806 stocks (Nov. 2014). This CRSP index holds stocks in proportion with market cap weightings meaning the portfolio holds each stock in the same proportion as its market cap represented in the asset class (small value). Snap-on (SNA), with a market cap of $7.810B, the largest holding, is held in the proportion \( .005119 = .5119\% \) and HGG (the smallest, market cap $146.80M) is held in the proportion \( .0000196 = .00196\% \). Notice that the market cap of small value ranges from $7810M to $146.80M. With 806 stocks in small value, an “average weight” would be \( \frac{1}{806} = .0012 \).

A person might wonder if this outperformance by small value is still current. To investigate, we compare Vanguard Small-Cap Value Index Fund with Vanguard 500 Index Fund Admiral Shares (9/30/14) in Table 3.

<table>
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<tr>
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<th>1 year</th>
<th>5 years</th>
<th>10 years</th>
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<tbody>
<tr>
<td>Vanguard small value</td>
<td>12.47</td>
<td>15.50</td>
<td>8.83</td>
</tr>
<tr>
<td>Vanguard 500</td>
<td>19.68</td>
<td>15.67</td>
<td>8.10</td>
</tr>
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(Expense ratio .24%)

(Expense ratio .05%)

Adding back in the above expense ratios.
According to McDermot and D’Auria, Fama and French created the well-known small-cap-premium factor called SmB (Small minus Big) to measure the outperformance of U. S. small-cap stocks relative to U. S. large-cap stocks. The annualized return of SmB from Jan. 1926 through Dec. 2013 is positive 2.22%. Thus we have evidence of the small-cap-premium. But, SmB is positive in only a little more than 50% of calendar years. So for a specific year, we have a 50% guess. For SmB, the longest period of consecutive months for a negative SmB was 220 months, 18.3 years. For CRSP (6-10) small cap, the longest draw down in consecutive months was 73 months and for CRSP (1-5) large cap, the longest was 34 months, about half. (See the Exercises for investments for a retiree or near retiree.) In the 32 year period (1982-2013), SmB’s annualized return was 0.96% and not statistically significant from zero. What about small cap value stocks?

There are several strategies which coordinate with value investing which enhance the returns of a portfolio. Novy-Marx (2013) finds that gross profitability (revenue minus cost of goods sold, scaled by assets) along with traditional metrics does this. A simple value strategy does not distinguish between companies that are profitable and those that are not. Including value and profitability in selecting stocks is a better value strategy. This applies particularly to small cap stocks. (Search the internet for Robert Novy-Marx, “The Quality Dimension of Value Investing,” University of Illinois at Chicago.) There are funds which incorporate this and other strategies, such as momentum, to generate larger abnormal returns. Momentum involves stocks which performed well recently. Historical studies have shown that such stocks are likely to perform well for a period of time.

An index such as the S&P 500 has rules for selection of stocks so that a stock can be added by purchase, or a stock can be removed by sale, all in one day. The Russell 2000 is probably the most widely followed U.S. small cap index. Historically, studies indicate the index lost from 0.38% to 1.84% annualized in market these impact costs. Traders make money off such an event. One way to avoid this market impact cost is to own the Vanguard Total Stock Market Fund. Study this fund to see how it manages about 5000 stocks. What kind of new-offerings- stocks does it buy, and what kind of stocks does it sell? (The Morningstar Guide to ETF Investing, 2014). For the S&P 500, a stock worth 1% of the index getting removed means $16B worth of the stock is sold in one day.

Side Bar Notes:

Small cap stocks. Investopedia.com says that small cap stocks vary in market capitalization from $300M to $2B. They constitute 17.6% of company stocks. Definitions of small cap stocks have changed over time. What was consider large cap in 1980 is small cap today.

CRSP. The Center for Research in Security Prices, Chicago Booth Business School, University of Chicago. Go to crsp.com and click your way around and see what you can learn. You can read about the CRSP U. S. Small Cap Value Index. They feature the work of highly
regarded investment researchers Fama and French. CRSP maintains the most complete data base for stocks, bonds, and mutual funds.

Masters degrees in financial mathematics. Major universities offer masters degrees with emphasis in financial mathematics. Some degrees are referred to as Masters in Financial Engineering, but at Stanford a Master of Science in Financial Mathematics (MSFM) is offered, Chicago offers a MSFM, Boston a MSFM, Minnesota a MSFM, and Rutgers a Master of Science in Mathematical Finance (MSMF). Other titles are MSFE, MMF. See the internet for programs and rankings, and job offerings.

Beta and Sharpe ratio for small caps. McDermott and D’Auria report beta for CRSP 6-10 (small caps) of 1.31 and a Sharpe ratio of 0.413. For the market (CRSP 1-10), they report a Sharpe ratio of 0.419. Interpret. See the articles in the references.

Rebalancing a portfolio of 60% stocks and 40% bonds. If the market rises, the equity exposure will exceed the 60%. This is an opportunity to sell high. If the market drops, the exposure will be less than 60%. This is an opportunity to buy low. The rebalanced portfolio will produce greater returns than its constituents. If handled well, volatility can outperform other returns.

Ifa.com a good read. Eugene Fama and Kenneth French developed for stock investment asset allocation and portfolio analysis a Three factor model consisting of the Value factor premium, the Small factor premium, and the Market factor premium (ifa.com). You can learn a lot more by reading ifa.com. The Value factor premium is Value minus growth companies, the Small factor premium is US small minus US large, and the Market factor premium is Total US market minus T-Bills. Ifa.com is a fee only advisor, and supplies index funds based on CRSP indexes which have 86 years of historical studies.

Modern portfolio theory. Harry Markowitz, Ph. D., in his 1952 Nobel Prize-winning article, “Portfolio Selection” asserted that the best portfolios include non-correlated stocks thus reducing risk for a good return (ifa.com). (See articles in this course.)

Index funds are tax efficient since they do less trading. Tax managed index funds offset realized gains with realized losses, deferring net capital gains and minimizing dividend income. The benefit is that unrealized capital gains remain a growing part of the net asset value. Taxes on gains are not paid until withdrawals, and gains become realized (ifa.com).

Bogle on index funds. After a 15 year study, Bogle says that investors keep 47% of the accumulated returns of an average actively managed mutual fund, investors keep 87% in a low cost market index fund. The actively managed return to the investor is depleted by expense ratio, commission, transaction costs, taxes, cash drag (ifa.com). They report that $10,000 invested in an actively managed fund grew to $49,000 versus $90,000 in an index fund. Explain these numbers.


The first index fund available to individual investors. On Dec. 31, 1975, John Bogle created what is now known as the Vanguard 500 Index Fund. The fund was met with criticism.
and called “Bogle’s folly.” Today, it one of the largest mutual funds. Since then Bogle has become one of the most respected in finance (ifa.com).

**Longevity.** For the average population, chances for one or both of a couple to lie to age 95 is 19%, for the upper middle class it is 43%, and it is calculated that by 2029, for the upper middle class it is 50%. To what age should you plan for retirement and when should you retire? See the articles on retirement calculations in this course and do your own calculations. How much is required to fund retirement? Money reports that for the highest income quartile, about 12% will not have enough retirement resources to live to age 100. Of the third quartile, it is 30%. Of the second quartile, it is 48%. Do you want to be an average retiree or an average student? (Money, Nov. 2014, page 55)

How much is needed to draw $40,000 the first year in retirement? One million dollars is required for a 25 year retirement, $1.18M for 30 years, $1.35M for 35 years. The safe amount to withdraw the first year from $1M is $40,000 for a 25 year retirement, $34,000 for a 30 year retirement, and $30,000 for a 35 year retirement. Note: Assumes a 40% bonds, 60% stocks portfolio, 80% probability of success, David Blanchett, Morningstar. (Money, Nov. 2014, page 58) You can have a lot of fun working with these numbers. What do they assume is withdrawn the second year?

**Part-time work or full retirement.** Assume a $100,000 salary for full-time work, $50,000 for part-time work, $500,000 in savings at age 60, a 40% stock/60% bond investment, average earnings of 6% per year, 10% contributed each year to a 401(k) when working full-time, delay Social Security until full retirement. How much yearly income can you expect? Full retirement at age 62, $50,800; work part-time 62 to 66, $66,700; full retirement at Age 66, $69,500; work part-time age 62 to 70, $71,600; work par-time 66 to 70, $87,900; full retirement at age 70, $93,400. (Money, Nov. 2014, page 62, Source T. Rowe Price.) What did they assume about inflation of salary and withdrawals, longevity, and social security, etc? Have fun with these numbers. Did you know that if you used a real rate of return, this takes care of increasing salary and increasing withdrawals? See an article in this course.

**Taking the pension or investing the cash value?** Assuming a $2500 a month pension with a cash value of $440,000 which could be invested in a 50/50 stock/bond portfolio, 9% annual return on stocks, 3% on bonds. What is the chance that the investment lasting 30 years if your withdrawals equal the pension? Answer: 40%. (Source: Pension Benefits, David Blanchett, Money, Nov. 2014, p. 82) Does the pension have a COLA?

**Variable annuity versus mutual fund.** Assume $100,000 is invested in a 50/50 stock/bond mutual fund, or a variable annuity, with 9% return for stocks and 3% for bonds, mutual fund expense ratio 0.7%, variable annuity expense ratio 1.3%, and after two years start withdrawing $5000 per year for 30 years. The mutual fund grows to about $160,000, the variable annuity to about $110,000, and if the variable annuity is replace by another at 2 years, the variable annuity declines to about $83,000. Explain. (Source: David Blanchett, Morningstar, Money, Nov. 2014, page 83)

**Titles that suggest financial expertise.** The Consumer Financial Protection Bureau has counted more than 50 titles that suggest financial expertise, with some requiring little more than a check and a multiple choice exam. (Money, Nov. 2014, page 85)
Did they go to the free financial dinner? When FINRA surveyed investors age 40 and over, 32% reported having gone to as lunch that turned out to be a sales pitch. (Money, Nov. 2014, page 85)

Small stocks slip into a correction. In the four weeks ending Oct. 1, 2014, the Russell 2000 small cap index dropped more than 10%. As of Oct. 1, for one month, the S&P was down 2.7%. (Money, Nov. 2014, page 102)

What is most important for students, football or financial education? Rampant academic fraud occurs in the nation’s colleges. At the University of North Carolina, the state’s flagship university, over two decades, more than 3100 students received credit for taking nonexistent phantom classes, nearly 50 percent were university athletes. (Denton Record Chronicle, Nov. 8, 2014) How many colleges provide a general financial education?


Back testing for better stock returns has resulted in over 200 discoveries in the last 15 years. They identify factors such as company size that are associated with out performance. Many mutual funds are sold based on claims of such factors and out performance. “But a study by economists Campbell Harvey, Yan Liu, and Heqing Zhu argue that many of these discoveries are probably illusory. The sheer number of findings, the study argues, suggests researchers are simply picking up a lot of random noise,” (Money, Dec. 2014, page 81) Discuss: What are they saying?

ChicagoBoth.edu/impact. In her hedge fund world, Roxanne Martino and the Aurora Fund have consistently outperformed benchmarks. She used the Chicago approach (CRSP). (Bloomberg Business Week, Nov. 10, 2014)

Exercises: Show your work. Label answers, numbers, and variables. Discuss in complete sentences. Label graphs.

#1. On fool.com, Motley Fool said that Fama and French at CRSP developed methods for determining and updating different portfolios and that for the 78 years from 1927 to 2005, Large Cap Value grew $100 to $898,967; Small Cap Value grew $100 to $7,367,903; Large Growth grew $100 to $130,165; and Small Growth grew $100 to $103,165. (a) How does the end result of Small Cap Value compare to that of Small Cap Growth? (b) Calculate the average compound growth rate for each portfolio. (c) On a grapher, graph and report a growth curve for Small Cap Value for the 78 years from 0, and build a table for the results for 10 year intervals. Graph by hand with a linear vertical axis in intervals of $1M, and a horizontal axis in intervals of 10 years. Discuss the graph. What do you know about the increase of funds at various 10 year intervals and the rate of increase of the slope? Interpret this in terms of practical investing. What do you know about the slope of the curve at each point and the rate of increase of the slope? See the $\frac{dy}{dx}$ function on page 3-28 of the TI83 manual.
#2. For Table 1, calculate the compounded sum of $1 invested for 58 years for Small cap stocks and the S&P 500. The final amount for Small cap is what multiple of that for the S&P 500?

#3. For a normal distribution and Table 1, for Small Stocks and the S&P 500, what is $\mu - \sigma$ and the percentage of returns from $\mu$ down to $\mu - \sigma$, what is $\mu - 2\sigma$ and the percentage of returns down to $\mu - 2\sigma$? What is a $3\sigma$ event for each and what is the probability? Is there evidence that such a distribution is not normal but skewed to the left? See articles in this course.

#4. Calculate BtM for SNA by a method different from the one used in the above discussion of BtM of SNA.

#5. The comparison between small value stocks and the S&P 500 in Table 3 does not display a significant advantage of small value stocks, but the comparison of small value cap at 17% and CRSP small cap (6-10) at 11.8% is impressive. (a) Trace the logic for the comparison of the three types of stocks in these figures and Table 1. (b) Try to find some better historical evidence of the outperformance of small value stocks over the S&P 500.

#6. For a retiree or near retiree who feels they can’t tolerate an extensive draw down, what do you think about investments in small cap stocks? Give calculations and historical evidence.

#7. McDermott and D’Auria report that for SmB, a mean annualized return of 0.96% for 32 years is not statistically significant from zero. If 100 years is required for 95% confidence that the long term mean is greater than zero, estimate the standard deviation of SmB.

#8. From #7, if 100 years is required to indicate that a mean for SmB of 0.96% is statistically significant at the 95% confidence level from the long term mean of 2.22%, estimate the standard deviation of SmB.

#9. What would be a simple way for CRSP to separate out growth stocks?

#10. Study finance.yahoo.com and other sources for Snap-on Inc. (SNA), which is a small value stock (See CRSP.) and check to see if it is a momentum stock, and a profitable company. Give your sources, discuss.

#11. Felix Salmon said that in Jan. 2001, he purchased a TiBook for $2600. He says that if he had bought $2600 of Apple stock instead, it would be worth $200,000 today. (Money, Nov. 2014, page 108) (a) What was the rate of return on the Apple stock price? (b) What about dividends and splits? (c) Look up Apple stock on finance.yahoo.com and learn about Apple stock. Report.

#12. A publication claimed that since 1991, by Fall 2014, a Vanguard fund was up 1,324%. What is the compounded annual rate of return? Figure it with two interpretations. What is the annual arithmetic rate of return.
References:

In this course:
“Annual Total Return Table for the S&P 500 Index of Stocks,” July 2013.
“Standard Deviation as a Measure of Risk for a Mutual Fund,” Summer 2011.
“Covariance and Correlation in Finance,” June 2012.

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