

2011 MCM Problems

Problem B: Repeater Coordination

Users of the VHF radio spectrum, including police, fire, public safety, amateur radio, etc., attempt to overcome the line-of-sight limitation of radio signals at these frequencies by means of a “repeater.” A repeater is a fixed radio station containing a receiver and a transmitter. The receiver picks up weaker signals from users in the geographical area in which it can hear users on its receive frequency and retransmits them, using its higher-power transmitter, at a frequency separated enough from the receive frequency so that undesirable conditions (such as damage to the receiver) does not occur. In this way, low-power users (such as mobile stations) can communicate with each other using a repeater in situations where direct user-to-user contact would not be possible. However, repeaters may interfere with each other unless they are either geographically sufficiently far apart or operate on sufficiently separated frequencies.

Other schemes are available to help mitigate possible interference. One of the most popular such schemes is the “continuous tone-coded squelch system”, or CTCSS, sometimes also nicknamed “private line” or PL. This functions by associating a fixed, but subaudible, audio frequency tone with each repeater, transmitted by all users who wish to communicate through that repeater. The repeater does not respond to any received signal that does not contain its specific PL tone. In this way, two nearby repeaters could share the same frequency pair (for receive and transmit) and so more repeaters, and therefore more users, can be accommodated in the same geographical area.

For a circular area of 100 miles radius, comprising flat terrain, determine the minimum number of repeaters necessary to accommodate 1,000 simultaneous users. Assume that the spectrum available is 145 to 148 MHz, the spacing between receiver and transmitter in a repeater is 600 kHz (transmitting either above or below the receiver frequency), and there are 54 different PL tones available. How does your solution change if there are 10,000 users? Discuss the case where there might be defects in the line-of-sight propagation caused by mountainous areas.