

1995 MCM

Problem A **The Single Helix**

Background:

The problem proposed here consists of assisting a small biotechnological company in designing, proving, programming, and testing a mathematical algorithm to locate "in real time" all the intersections of a helix and a plane in general positions in space.

Similar programs for Computer Aided Geometric Design (CAGD) enable engineers to view a plane section of the object that they design, for example, an aircraft jet engine, an automobile suspension, or a medical device. Moreover, engineers may also display on the plane section such quantities as air flow, stress, or temperature, coded by colors or level curves. Furthermore, engineers may rapidly sweep such plane sections through the entire object, to gain a three-dimensional visualization of the object and its reactions to motion, forces, or heat. To achieve such results, the computer programs must locate all the intersections of the viewed plane and every part of the designed object with sufficient speed and accuracy. General "equation solvers" may in principle compute such intersections, but for specific problems specific methods may prove faster and more accurate than general methods. In particular, general software for Computer Aided Geometric Design may prove too slow to complete computations in real time, or too large to fit in the finished medical devices being developed by the company. The considerations just explained have led the company to the following problem.

Problem:

Design, justify, program, and test a method to compute all the intersections of a plane and a helix, both in general positions (at any locations and with any orientations) in space.

A segment of the helix may represent, for example, a helicoidal suspension spring or a piece of tubing in a chemical or medical apparatus.

The need for some theoretical justification of the proposed algorithm arises from the necessity of verifying the solution from several points of view. For instance, through mathematical proofs of parts of the algorithm, and through tests of the final program with known examples. Such documentation and tests will be required by government agencies for medical use.