2010 HiMCM, Judges Commentary

Problem A: Bicycle Club

Several cities in the US are starting bike share programs. Riders can pick up and drop off a bicycle at any rental station. These bicycles are typically used for short trips within the city center, either one-way or roundtrip. The idea is to help people get around town on a bike instead of a car. Those making longer trips (such as commuting to work) are likely to use their own bikes.

Some of the challenges are how to determine where to locate the rental stations, how many bikes to have at each station, how/where to add new locations as the program grows, how many bikes to move to another location and when (time of day, day of week).

The downtown city maps, the bike rental locations and the number of bikes at each location for Chicago, Denver and Des Moines are available from the following websites:

http://chicago.bcycle.com/

http://denver.bcycle.com/

http://desmoines.bcycle.com/

You have been asked to develop an efficient bike rental program for these cities.

- List the traffic/bike usage and other information that you would need to collect in order to plan the bike rental program for these cities.
- Develop a mathematical model that the city could use to plan the program, including the location of new rental stations for the next 5 years.
- Assume that the bike usage in the program will grow by 30% per year.

In your analysis consider the existing bike paths in the city center, attractions such as museums, theaters, etc in the city center, and the other transportation hubs in the city center. When your analysis is complete, prepare a short letter to the mayor explaining the benefits and recommendations of your analysis.

Judge's Comments:

Author: Veena Menderatta

William P. Fox, HiMCM Contest Director

This problem is of interest to the author. Originally proposed as a problem for the college level MCM, the problem was selected for the HiMCM contest by the contest directors because it can be addressed with high school mathematics.

This raises the question: What are reasonable expectations for a high school team, in 36 hours, to model this situation? As a regional head judge and one of the national judges, and as the problem author, I offer the following insights.

First, and this comment is made each year, if the problem asks for a letter, news article, or position paper, the student group must provide one if they hope to be recognized. Additionally, the letter should be written after the problem is solved, as the letter must contain facts and results that will excite the reader to look at the entire analysis. Most of the letters did not report their facts and thus, would not motivate a mayor or anyone else to examine the analysis more closely.

Second, the problem statement explicitly called out three components to be addressed (see problem statement above). Addressing all three greatly increases the chance of recognition.

Although not explicitly asked for it would be hard to address these three issues without considering costs.

The better papers this year attempted to present frameworks for choosing solutions. The mathematics required to do this are very accessible at the high school level. The approaches for traffic density were mostly good. Few teams, if any, considered the impact of sharing the road or riding in darkness as most teams had the bike kiosks open to 8-11PM.

This year's papers have many strong points. Almost all the papers did a reasonable job of estimating the demand bikes but few discussed the issue of weather and how that might impact the use of bikes. Several of the papers did excellent jobs modeling the bike usage and comparing to other modes of traffic.

There were a wide variety of approaches used from simple algebra or statistics through simulation models. We found many simulation models were not ever well explained nor were flow charts presented and used. It was as if these techniques were a black box. As models, they should be explained as to what they do and why they could be used in the scenario.

A few of the papers did outstanding jobs of representing their strategies graphically.

There were some notable patterns of weakness. Many papers never considered foul weather, like snow and ice in cities such as Chicago. Many did not look at all three facets that were required. Others forced the use of calculus in their solution, although it was not really appropriate. On the other hand, some papers offered no mathematical treatment of the problem at all.

Student groups should remember that the problems posed in these contests are not going to have a unique solution--they are designed **not** to have one, in fact. Students should remember that general high school mathematics are adequate to the task at hand--what we are looking for is evidence of good modeling of the problem with these tools, and then discussion of the implications of the model and its solution(s). We are looking for the quality of creative modeling and a thorough job of implementing the modeling process.

Problem B: Curbing City Violence

A regional city has had lots of problems with gangs and violence over the years. The mayor, chief of police, and city council need your help. Data are available for the following: Incidents of violence, Homicides, Assaults, Regional Population (Census data), Unemployment, Unemployment rate, High School enrollment, High school drop outs, Graduation rate, Drop out rate, Prison population, Released on parole, Parole violations, Percent of parole violations, and Juvenile Inmates.

Analyze and model these data to give the city a plan to reduce violence. After you complete your analysis and model, prepare a news release for the mayor briefly outlining your proposals that recommend a campaign strategy to curb the violence.

Some real data was provided to the students.

Judge and Author's Comments

William P. Fox, HiMCM Contest Director

This problem's statement is concise but clearly has elements for the students to consider. Students should have clearly defined what they considered "violence" and how they were going to measure it.

Most students completed most of the required tasks. Many did not pick the variables that impacted violence the most and discuss how to control those variables. Nearly all teams did the letter, but few did what we would call an excellent job of concisely telling their story and relating the facts in the letter. Thus, teams should ensure that that they complete and include all the required tasks in their submission.

The executive summaries for the most part were either absent or poorly written. This has been ongoing since the beginning of the contest. Faculty advisors should spend some time with teams on how to write a good summary. Many summaries read like technical reports or were too vague to be helpful. Summaries need to contain the results of the model as well as brief explanation of the problem. A summary should entice the reader, in our case the judge, to read the paper.

The *letter to the mayor (news release)* should have been a concise explanation of the modeling results to include (1) defining violence and why controlling it is important (2) listing the variables that most influence violence, and (3) a brief description or statement of the potential impacts and changes to reduce violence. Again, many teams failed to do this in their submission.

The judges felt the first critical task was to define violence and define measures that could be used to measure such the violence.

The modeling seen was not based on first principles of the modeling process. Students obtained scatter plot looking for correlations and build linear regression models. Some build multiple regression models. Almost all teams used the variables as presented without forming maybe ratios or creating new variables. The data were integer data, yet models often had so many decimals points it was absurd. Some teams built regression models of higher order (8 data pair and a 7^{th} order polynomial). Teams did not graph their polynomials (lower or higher order) to check to see if the trends were always captured. Few teams looked at residuals, percent errors, or anything other than r^2 to determine the adequacy of the model.

Students should consider this example:

Consider the following 4 sets of data:

I		II		III		IV	
x	*y*	*x*	*y*	*x*	*y*	*x*	*y*
10.0	8.04	10.0	9.14	10.0	7.46	8.0	6.58
8.0	6.95	8.0	8.14	8.0	6.77	8.0	5.76
13.0	7.58	13.0	8.74	13.0	12.74	8.0	7.71
9.0	8.81	9.0	8.77	9.0	7.11	8.0	8.84
11.0	8.33	11.0	9.26	11.0	7.81	8.0	8.47
14.0	9.96	14.0	8.10	14.0	8.84	8.0	7.04
6.0	7.24	6.0	6.13	6.0	6.08	8.0	5.25
4.0	4.26	4.0	3.10	4.0	5.39	19.0	12.50
12.0	10.84	12.0	9.13	12.0	8.15	8.0	5.56
7.0	4.82	7.0	7.26	7.0	6.42	8.0	7.91
5.0	5.68	5.0	4.74	5.0	5.73	8.0	6.89

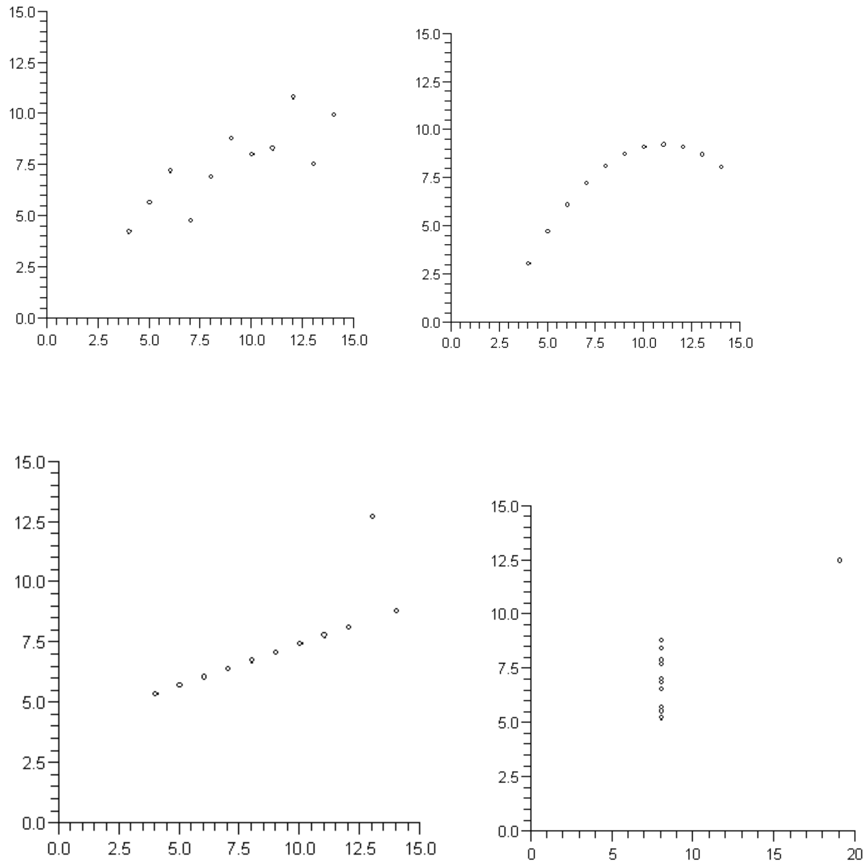
Suppose we fit the model $y = ax + b$ to each data set using the least-squares criterion. In each case the following model results:

$$y = 3 + 0.5x$$

The correlation coefficient in each case is 0.82, and $r^2 = 0.67$. The sum of the squared deviations between observed and predicted values is also the same. In particular,

$$\sum_{i=1}^{11}[y_i - (3+5x)]^2 = 13.75$$

These two numerical measures imply that for each case $y = 3 + 0.5x$ does about the same job explaining the data, and that it is a reasonable fit ($r^2 = 0.67$). However, the following scatter plots convey a different story:



A point to consider is how well the model $y = 3 + 0.5x$ captures the trend of the data. (This example is adapted from F. J. Anscombe, "Graphs in Statistical Analysis," Amer. Stat., 27, 1973, 17-21.)

Few teams, if any, did any sensitivity analysis on the data or the model.

We found that many of the assumptions and research were very good. Teams did researched history of violence and data on violence but none used their modeling efforts to see if the new data followed the same trends. We encourage teams who take data from other sources or graphics to ***include the reference at that point*** as well as a reference page at the end. We saw the use of data from blogs and Wikipedia---such information can be suspect, and we encourage teams to obtain data and information from reliable sources.

There were a wide variety of approaches used from simple algebra through simulation models. We found that many simulation models were not ever well explained, nor were flow charts used. It was as if these techniques were a black box. As models, they should be explained as to what they do and why they could be used in the scenario.

One issue was with significant digits. The models built by the teams were in number of violent acts of some magnitude. Yet numerical values were presented to (at times) many decimal places (as many 20 decimal places). Clearly, this was not necessary.

General Comments from Judges:

Variables and Units:

Teams must define their variables and provide units for each variable.

Computer generated solutions:

Many papers used computer code. Computer code used to implement mathematical expressions can be a good modeling tool. However, the judges expect to see an algorithm or flow chart from which the code was developed. Successful teams provided some explanation or guide to their algorithm(s)--a step-by-step procedure for the judges to follow. Code may only be read for the papers that reach the final rounds, but not unless the code is accompanied by a good algorithm in words. The results of any simulation need to be well explained and sensitivity analysis preformed. For example, consider a flip of a fair coin. Here is a general algorithm:

INPUT: Random number, number of trails
OUTPUT: Heads or tails
Step 1. Initialize all counters
Step 2. Generate a random number between 0 and 1.
Step 3. Choose an interval for heads, like [0.0.5]. If the random number falls in this interval, the flip is a heads. Otherwise the flip is a tails.
Step 4. Record the result as a heads or a tails.
Step 5. Count the number of trials and increment: Count = Count + 1

An algorithm such as this is expected in the body of the paper with the code in the appendix.

Graphs:

Judges found many graphs that were not labeled or explained. Many graphs did not appear to convey information used by the teams. All graphs need a verbal explanation of what the team expects the reader (judge) to gain (or see) from the graph. **Legends, labels, and points of interest** need to be clearly visible and understandable, even if hand written. Graphs taken from other sources ***should be referenced and annotated***.

Summaries:

These are still, for the most part, the weakest parts of papers. They should be written after the solution is found. They should contain results and not details. They should include the "bottom line" and the key ideas used in obtaining the solution. They should include the particular questions addressed and their answers. Teams should consider a brief three paragraph approach: a *restatement of the problem* in their own words, a short description of *their method and solution* to the problem (without giving any mathematical expressions), and the *conclusions* providing the numerical answers in context.

Restatement of the Problem:

Problem restatements are important for teams to move from the general case to the specific case. They allow teams to refine their thinking to give their model uniqueness and a creative touch.

Assumptions/Justifications:

Teams should list only those assumptions vital to the building and simplifying of their mathematical model. Assumptions should not be a reiteration of facts given in the problem statement. Every assumption should have a justification. We do not want to see "smoke screens" in the hopes that some items listed are what judges want to see. Variables chosen need to be listed with notation and be well defined.

Model:

Teams need to show a **clear link** between the assumptions they listed and the building of their model or models. Too often models and/or equations appear without any model building effort. Equations taken from other sources should be referenced. It is required of the team to show how the model was built and why it is the model chosen. Teams should not throw out several model forms hoping to impress the judges, as this does not work. We prefer to see sound modeling based on good reasoning.

Model Testing:

Model testing is not the same as testing arithmetic. Teams need to compare results or attempt to verify (even with common sense) their results. Teams that use a computer simulation must provide a clear step-by-step algorithm. Lots of runs and related analysis are required when using a simulation. Sensitivity analysis should be done in order to see how sensitive the simulation is to the model's key parameters. Teams that relate their models to real data are to be complimented.

Conclusions:

This section deals with more than just results. Conclusions might also include speculations, extensions, and generalizations. This is where all scenario specific questions should be answered. Teams should ask themselves what other questions would be interesting if they had more time and then tell the judges about their ideas.

Strengths and Weaknesses:

Teams should be open and honest here. What could the team have done better?

References:

Teams may use references to assist in their modeling. However, they must also ***reference the source*** of their assistance. Teams are reminded that only ***inanimate resources*** may be used. Teams cannot call upon real estate agents, bankers, hotel managers, or any other real person to obtain information related to the problem. References should be cited where used and not just listed in the back of the paper. Teams should also have a reference list or bibliography in the back of the paper.

Adherence to Rules:

Teams are reminded that detailed rules and regulations are posted at the COMAP website. Teams are reminded that they may use only ***inanimate sources*** to obtain information and that the ***36-hour time limit is a consecutive 36 hours.***