Judges Commentary, 14th Annual Contest
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Problem A: Space Shuttle Problem: No More Space Shuttles

On July 21, 2011, the 135th and final US Space Shuttle landed in Florida after its 13-day mission into orbit complete with a docking at the International Space Station (ISS). NASA will now have to rely on other nations or commercial endeavors to travel into space until a replacement vehicle is developed and constructed. Develop a comprehensive ten-year plan complete with costs, payloads, and flight schedules to maintain the ISS.

Some interesting facts possibly worthy of your consideration:

- The ISS is at full capacity with 6 astronauts, but can surge during shuttle docks to as high as 13.
- The ISS is scheduled to remain in service until at least the year 2020.
- Historically, it has cost between $5000-10,000 per pound to transport to the ISS using the US Shuttles. Shuttle missions have lasted approximately 10-14 days on orbit. Missions onboard the ISS are typically around six months.
- Recently, progress has been made within the private industry to launch unmanned rockets into space.
- Russia is willing to launch US astronauts into space for about $60 million each.

Judge's Comments: Professor William P. Fox, Naval Postgraduate School
Author: Jack Picciuto, United States Military Academy

This problem was of interest to the author who was an army aviator. First, the problem statement explicitly called for a plan that presented costs, payloads, flight schedules. Many teams failed to provide this schedule. Addressing all three greatly increases the chance of recognition.

Although not explicitly asked for it would be hard to address these three issues without considering schedule slippage. Have you ever seen or heard of a space flight taking off and landing on time. Very few teams considered or mentioned schedule slippage nor did they make an assumption that there would not being schedule slippage due to any factors.

Many papers started with an equation for costs and then tried to explain them. Modeling says we start with variables and assumptions that led to a model. We might assume linear or even nonlinear relationships but only after we examine the variables and assumptions.

The better papers this year attempted to present frameworks for choosing solutions. The mathematics required to do this are very accessible at the high school level.

There were many strengths in this year's papers. Almost all the papers did a reasonable job of estimating the costs but few, if any, discussed the issue of bad weather and how weather caused delays might impact the costs of the mission.

There were a wide variety of approaches used from simple algebra, statistics, and regression techniques. For those using regression techniques very few ever examined the residuals or errors of the model to insure the regression was useful. The R^2 is not always a good indicator.

We provide an example of regression that shows why examining residuals is important. Consider the following 4 sets of data:

I		II		III		IV	
x	y	x	y	x	y	x	y
10.0	8.04	10.0	9.14	10.0	7.46	8.0	6.58
8.0	6.95	8.0	8.14	8.0	6.77	8.0	5.76
13.0	7.58	13.0	8.74	13.0	12.74	8.0	7.71
9.0	8.81	9.0	8.77	9.0	7.11	8.0	8.84
11.0	8.33	11.0	9.26	11.0	7.81	8.0	8.47
14.0	9.96	14.0	8.10	14.0	8.84	8.0	7.04
6.0	7.24	6.0	6.13	6.0	6.08	8.0	5.25
4.0	4.26	4.0	3.10	4.0	5.39	19.0	12.50
12.0	10.84	12.0	9.13	12.0	8.15	8.0	5.56
7.0	4.82	7.0	7.26	7.0	6.42	8.0	7.91
5.0	5.68	5.0	4.74	5.0	5.73	8.0	6.89

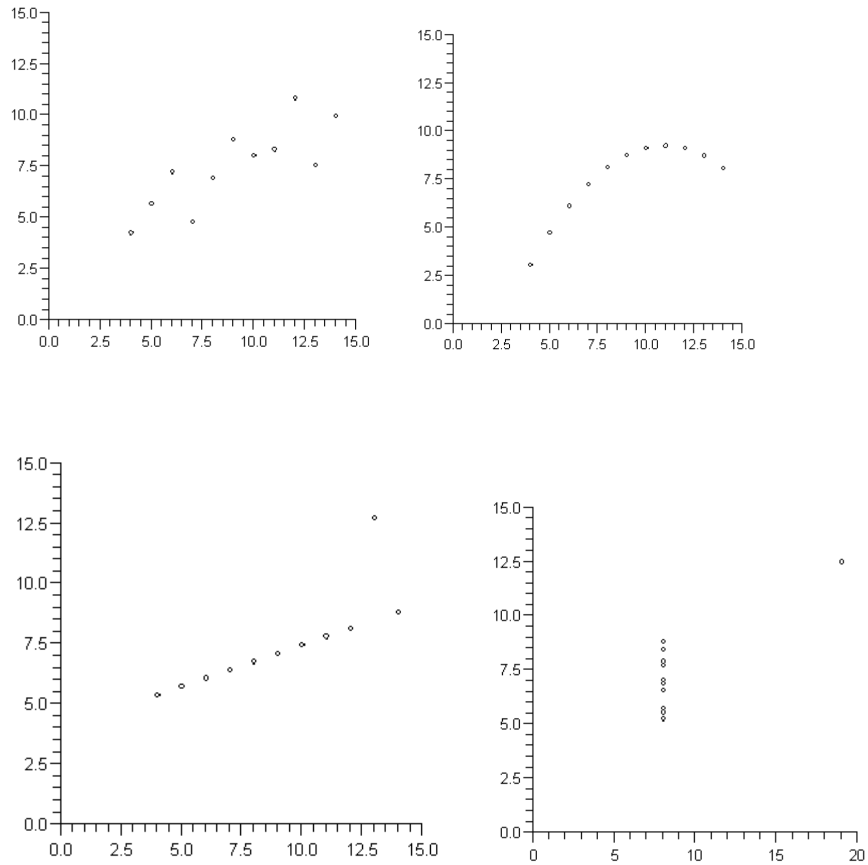
Suppose we fit the model $y = ax + b$ to each data set using the least-squares criterion. In each case the following model results:

$$y=3+0.5x$$

The correlation coefficient in each case is 0.82, and $r^2=0.67$. The sum of the squared deviations between observed and predicted values is also the same. In particular,

$$\sum_{i=1}^{11}[y_i-(3+5x)]^2=13.75$$

These two numerical measures imply that for each case the model y=3+0.5x does about the same job explaining the data, and that it is a reasonable fit ($r^2=0.67$). However, the following graphs of the data sets convey a different story:



A point to consider is how well the model y=3+0.5x captures the trend of the data. (This example is adapted from F. J. Anscombe, "Graphs in Statistical Analysis," Amer. Stat., 27, 1973, 17-21.)

There were some notable patterns of weakness. Many papers never considered foul weather, like storms in Russia or other countries. Few teams thought about returning to earth or delays in getting back. Schedule slippage was not addressed.

Some teams did consider training and costs to get space travelers to the country of departure.

Student groups should remember that the problems posed in these contests are not going to have a unique solution – they are designed **not** to have one, in fact. Students should remember that general high school mathematics are adequate to the task at hand--what we are looking for is evidence of good modeling of the problem with these tools, and then discussion of the implications of the model and its solution(s). We are looking for the quality of creative modeling and a thorough job of implementing the modeling process.

Problem B: Search and Find

Finding lost objects in not always an easy task, even when you have knowledge of a general location. Consider the following scenario: you have lost a small object, such as a class ring, in a small park, see map 1. It is getting dark and you have your pin light flashlight available. If your light shines on the ring, you assume that you see it. You cannot possibly search 100% of the region. Determine how you will search the park in minimum time. An average person walks approximately 4 mph. You have about 2 hours to search. Determine the chance you find the lost object.

Then assume using map 2, a jogger is lost who was going on a 5 mile run. Determine how you search the region to have a good chance of finding the lost jogger (who might be not conscious). Assume it is night and you still only have your pin light.

Two maps were provided to the student.

Judge and Author's Comments: William P. Fox, HiMCM Contest Director

The judges were amazed at the mathematics applied for this problem. They were amazed because the use of sophisticated mathematical concepts was unexpected. A majority of teams used graph theory to search for the objects with many teams trying to define nodes and then minimize the circuit length in which to traverse. Most judges felt that graph theory was not appropriate and further teams using it should have maximized the coverage over time not minimized a circuit. With that said, we allowed the approach and read with the students.

Students also were expected to randomly place the lost objects and then try to find the object through some mathematics plan or model. Very few teams considered the randomness of the lost object.

Students that compared areas to obtain the chance of finding the object were on a good track. The object is in an area of size X and the search area is size Y. The ratio of Y/X provides a good first calculation for the chance of finding the object. Some teams held the light vertically to obtain a circular search pattern on the ground; other teams tilted the light to maximize the coverage with a more elliptical shape.

Some teams only solved for one or the other problem and the problem asked for both to be solved.

The executive summaries for the most part were still poorly written although getting a little better. This has been ongoing since the beginning of the contest. Faculty advisors should spend some time with their teams and advise to write a good summary. Many summaries tend to be written before the teams start and only state how they will solve the problem. Summaries need to be written last and should contain the **results** of the model as well as brief explanation of the problem. The executive summary should entice the reader, in our case the judge, to read the paper.

Few teams, if any, did any sensitivity analysis or error analysis on their model. With the randomness of a lost item this is a critical element.

We found that most team did not do any research on the problem to see if there were any search and find methods to get started. The references were generally weak.

We expected to see a wide variety of approaches used from simple algebra through simulation models but certainly not graph theory. We found very few simulation models and they were not ever well explained nor were flow charts used. It was as if these techniques were a black box. As models, they should be explained as to what they do and why they could be used in the scenario.

Issues with graph theory included teams using Dykstra's algorithm for a minimal spanning tree. This does not insure you find the lost item. You would need maximum coverage in minimal time.

We expect to draft a possible solution to Part I of this problem using typical high school mathematics and publish it in the following Consortium for students and advisors to review.

General Comments from Judges:

Variables and Units:

Teams must define their variables and provide units for each variable.

Computer generated solutions:

Many papers used extensive computer code especially the A* code. Computer code used to implement mathematical expressions can be a good modeling tool. However, the judges expect to see an algorithm or flow chart from which the code was developed. Successful teams provided some explanation or guide to their algorithm(s)--a step-by-step procedure for the judges to follow. Code may only be read for the papers that reach the final rounds, but not unless the code is accompanied by a good algorithm in words. The results of any simulation need to be well explained and sensitivity analysis preformed. For example, consider a flip of a fair coin. Here is a general algorithm:

INPUT: Random number, number of trails
OUTPUT: Heads or tails
Step 1. Initialize all counters

Step 2. Generate a random number between 0 and 1.
Step 3. Choose an interval for heads, like [0.0.5]. If the random number falls in this interval, the flip is a heads. Otherwise the flip is a tails.
Step 4. Record the result as a heads or a tails.
Step 5. Count the number of trials and increment: Count = Count + 1

An algorithm such as this is expected in the body of the paper with the code in the appendix.

Graphs:

Judges found many graphs that were not labeled nor explained. Many graphs did not appear to convey information used by the teams. All graphs need a verbal explanation of what the team expects the reader (judge) to gain (or see) from the graph. **Legends, labels, and points of interest** need to be clearly visible and understandable, even if hand written. Graphs taken from other sources ***should be referenced and annotated***.

Summaries:

These are still, for the most part, the weakest parts of papers. These should be written after the solution is found. They should contain results and not details. They should include the "bottom line" and the key ideas used in obtaining the solution. They should include the particular questions addressed and their answers. Teams should consider a brief three paragraph approach: a *restatement of the problem* in their own words, a short description of *their method and solution* to the problem (without giving any mathematical expressions), and the *conclusions* providing the numerical answers in context.

Restatement of the Problem:

Problem restatements are important for teams to move from the general case to the specific case. They allow teams to refine their thinking to give their model uniqueness and a creative touch.

Assumptions/Justifications:

Teams should list only those assumptions that are vital to the building and simplifying of their mathematical model. Assumptions should not be a reiteration of facts given in the problem statement. Assumptions are variables (issues) acting or not acting on the problem. Every assumption should have a justification. We do not want to see "smoke screens" in the hopes that some items listed are what judges want to see. Variables chosen need to be listed with notation and be well defined.

Model:

Teams need to show a **clear link** between the assumptions they listed and the building of their model or models. Too often models and/or equations appeared without any model building effort. Equations taken from other sources should be referenced. It is required of the team to show how the model was built and why it is the model chosen. Teams should not throw out several model forms hoping to WOW the judges, as this does not work. We prefer to see sound modeling based on good reasoning.

Model Testing:

Model testing is not the same as testing arithmetic. Teams need to compare results or attempt to verify (even with common sense) their results. Teams that use a computer simulation must provide a clear step-by-step algorithm. Lots of runs and related analysis are required when using a simulation. Sensitivity analysis should be done in order to see how sensitive the simulation is to the model's key parameters. Teams that relate their models to real data are to be complimented.

Conclusions: This section deals with more than just results. Conclusions might also include speculations, extensions, and generalizations. This is where all scenario specific questions should be answered. Teams should ask themselves what other questions would be interesting if they had more time and then tell the judges about their ideas.

Strengths and Weaknesses:

Teams should be open and honest here. What could the team have done better?

References:

Teams may use references to assist in their modeling. However, they must also ***reference the source*** of their assistance. Teams are reminded that only ***inanimate resources*** may be used. Teams cannot call upon real estate agents, bankers, hotel managers, or any other real person to obtain information related to the problem. References should be cited where used and not just listed in the back of the paper. Teams should also have a reference list or bibliography in the back of the paper.

Adherence to Rules:

Teams are reminded that detailed rules and regulations are posted on the COMAP website. Teams are reminded that they may use only ***inanimate sources*** to obtain information and that the ***36-hour time limit is a consecutive 36 hours.***