Feedback

Teacher's Guide

Video Support

Before viewing the video for this unit, give each student a copy of Handout H6.1. Page 1 of Handout H6.1 has questions for students to answer based on the video. Page 2 of Handout H6.1 provides a short mini-activity that can be used as a follow-up to the video discussion, or as fun practice after Activity 1. After students have viewed the video, use the questions below as the basis for a follow-up discussion.

1. What gives a "feedback loop" its name?

   Sample: The fact that information or causal events "feed back" to the start of the system.

2. Describe a feedback loop involved in driving a car.

   Sample: In steering, you see errors in where you are going, adjust the steering wheel to compensate (causing the car to go a bit too far in the other direction), and repeat the process. Good drivers make small adjustments; beginning drivers tend to weave back and forth. Anything that delays the feedback process, such as alcohol, causes these adjustments to be less smooth.

3. Suggest one other feedback system that affects you.

   Sample: students might link study time, TV time, sleep, athletics, grades, pride, etc.

LESSON ONE
What Lies Ahead

PREPARATION READING

Modeling

See Annotated Teacher's Edition.

ACTIVITY 1

Catching a CLD

See Annotated Teacher's Edition.

INDIVIDUAL WORK 1

Behind the Scenes

See Annotated Teacher's Edition.

ACTIVITY 2

Decisions, Decisions . . . .

See Annotated Teacher's Edition.

INDIVIDUAL WORK 2

The Cast of Characters

See Annotated Teacher's Edition.

SUPPLEMENTAL ACTIVITY S6.1

Name that Model

CD-ROM: LINEAR.XLS, QUADRATI.XLS, and EXPMIXED.XLS

This activity is a slightly more directed version of Activity 2, supplying specific suggestions for graphical tests for particular forms of growth. Items 4 and 5 of this supplemental activity are identical to Items 3 and 4 of Activity 2 in the student text. These items serve as the "launch point" for closer consideration of contextual assumptions in the search for good models.

Students are asked for long-range predictions based on their models. The CD-ROM Excel files LINEAR.XLS, QUADRATI.XLS, and EXPMIXED.XLS provide sample spreadsheets for such work.

Discuss how predictions from one source of data might vary, due only to when the prediction was made.

SUPPLEMENTAL ACTIVITY S6.2

Webs

Transparency T6.1

This activity develops web diagrams assuming no previous experience with them. Items 1–5 develop one geometric representation of recursion. This development is preliminary work to the discussion of web diagrams. These items may be omitted if students have a working understanding of recursive calculations and recursive graphs. However, if students have not seen web diagrams before, this preliminary work will be well worth the time spent. For students who just need review, you may wish to use only selected items. Item 2 is good for this purpose.

You may wish to use Transparency T6.1 to illustrate the process discussed in Item 1(b) and Figure 3.

Items 3 and 4 invite students to make connections between time-series and recursive graphs. Items 3(c) and 4(c) show a converging sequence.

Item 5 applies the newly developed feedback diagram to a new setting. Students should recognize this as mixed growth.

Items 6–11 are a detailed review of web diagrams, first introduced in Course 2, Unit 6, *Growth*.

Note that Assessment Problems are listed in the Annotated Teacher's Edition with the specific activities and individual works in which the assessed concepts first appear. Although use of an assessment at that point is appropriate, be cautious about assessing too soon. You may prefer to use the assessment after concepts have been revisited.

ASSESSMENT PROBLEM A6.1

Milk for Piglets

This problem may be used at any time, even before Lesson 1, and serves to check student familiarity with properties of quadratic data. The data are given as a scatter plot only, so numerical methods such as regression must be replaced with more fundamental thinking. The skills needed for Item 5 are important to the development of logistic models in Lesson 2.

ASSESSMENT PROBLEM A6.2

Olympic Women Athletes

This problem asks for contextual-based extrapolation and informal curve sketching. The focus is more on comparison with known functions and on critical thinking about the situation being modeled than on computations and formulas. You may use this problem any time during or after Lesson 1.

ASSESSMENT PROBLEM A6.3

Reverse Commuters Sail

This problem is similar to that in A6.2, except that no graphical or numerical data are given. Instead, students must reason directly from information given in news clippings. Again, the emphasis is on building a graphical interpretation of the situation and in explaining the reasons for that interpretation. Stronger students should realize there might be an interaction between the two groups of commuters in which some drivers move from one category to the other. Consider asking for a CLD as well as graphs. This problem may be used at any time during the unit.

LESSON TWO

Another Model

PREPARATION READING

It Can't Do That Forever!

See Annotated Teacher's Edition.

ACTIVITY 3

Improving a First Model

See Annotated Teacher's Edition.

INDIVIDUAL WORK 3

But What Do You Believe?

See Annotated Teacher's Edition.

ACTIVITY 4

How Does This Rate?

See Annotated Teacher's Edition.

INDIVIDUAL WORK 4

But What Does It All Mean?

See Annotated Teacher's Edition.

SUPPLEMENTAL ACTIVITY S6.3

Quadratic Webs

The purpose of this activity is to show something of the range of behavior that a "simple" quadratic system can produce. For example, students may be surprised to see that it can lead to (discrete) oscillation just as regular as that produced by trig functions. The activity also reinforces the roles of the control numbers—the intrinsic rate and the carrying capacity.

Many graphing calculators include a "web" setting. For example, this is one option under the graphing "format" menu on a TI-83 when "Seq" mode is selected. To use it, enter the recursive equation under Y=, set an appropriate window (remember, both horizontal and vertical axes show the same quantities), and press "Trace." Then each press of the "right arrow" cursor control produces one additional segment of the web diagram.

ASSESSMENT PROBLEM A6.4

Decay of Animal Bones

This problem may be used as assessment after Activity 3, or as review before starting the lesson. It involves computations based on a recursively defined process, a topic that should be familiar to students already. It also asks for creation of a variety of graphs, each of which is used throughout the remainder of the unit.

ASSESSMENT PROBLEM A6.5

The Tongue of a Snake, the Tentacles of a Squid

This problem may be assigned after Individual Work 3 and the definition of joint proportionality. Its main focus is on applying joint proportionality to a geometric setting. The context provides an opportunity for students to have an interesting "Aha!" experience related to biology, in addition to practicing some mathematics.

ASSESSMENT PROBLEM A6.6

The Restraining Influence

This problem is best used after the definition of logistic growth in Individual Work 4. However, you may be able to use it as early as after Individual Work 3 for exceptional classes. While this problem does not deal directly with that type of growth, it does incorporate the notion of a "negative feedback" factor in a descriptive formula. The formula used here is given in closed form rather than recursively, but students should note its similarity with the corresponding expression in the equations defining logistic growth. Item 2(c) may cause some students difficulty since it asks for geometric thinking.

ASSESSMENT PROBLEM A6.7

Harvesting Models

This problem is actually an extended investigation, including suggestions for independent exploration of additional ideas. It is probably best used as a project, with additional time allowed for its completion. These investigations are best suited to working in pairs.

This problem tests students' abilities to read and interpret technical writing, a skill that will be needed again in Unit 7, *Modeling Your World*. It asks students to look at familiar ideas in new ways and is another example of the modeling cycle.

Since it is related to logistic growth, you may assign this problem any time after Lesson 2. Implicit in its development is the fact that renewable resources can only be "harvested" at rates less than or equal to their regeneration, or the renewability is lost.

LESSON THREE

It's Going Around

PREPARATION READING

Give and Take

See Annotated Teacher's Edition.

ACTIVITY 5

The Joke's On You

See Annotated Teacher's Edition.

INDIVIDUAL WORK 5

That's Catchy!

See Annotated Teacher's Edition.

ACTIVITY 6

Dear S-I-R

See Annotated Teacher's Edition.

INDIVIDUAL WORK 6

What Goes Around Comes Around

See Annotated Teacher's Edition.

SUPPLEMENTAL ACTIVITY S6.4

The Joke's On You

Handout H6.2

This activity is provided as a somewhat more guided version of Activity 5 in the student text. Although both activities leave the simulation design up to the students, Handout H6.2 is provided as a sample design. If possible, though, help students come up with a design of their own.

In Item 6, be sure that students graph the relative rate from one time to the next versus the earlier of the two times; that is, $(Q(n + 1) - Q(n))/Q(n)$ versus $Q(n)$.

Items 6 and 7 provide the mathematical "punch line." Each of these two graphs is characteristic of logistic growth, and each provides easy estimates of the key control numbers in the model.

ASSESSMENT PROBLEM A6.8

A Young Student's Experience with Math

This problem connects several units from the *Mathematics: Modeling Our World* program. Key ideas are recursion and slope, but in a contextual setting quite different from the unit itself. This problem is appropriate for use in this lesson, but may be used later.

ASSESSMENT PROBLEM A6.9

Political Mobilization

This problem asks students to interpret a sequence of models describing political support. Other contexts may be used instead. In many ways, the structure is similar to that of disease transmission. For additional background information, see COMAP's UMAP, Unit 304, *The Growth of Partisan Support I* by Carol Weitzel Kohfeld.

LESSON FOUR

An Ecological PushMe-PullYou

PREPARATION READING

Systems

See Annotated Teacher's Edition.

ACTIVITY 7

The Old Runaround

Some graphing calculators, for example the TI-83, have a mode in which relations may be defined recursively and interdependently. This permits modeling *S-I-R-S* and predator-prey systems, among others. In addition, the TI-83 permits time-series, web, and phase-plane graphs.

For the TI-83, select Seq mode. Then define quantities recursively. (Note that it requires defining $Q(n)$ in terms of $Q(n-1)$ instead of $Q(n+1)$ in terms of $Q(n)$.) Graph type is set under the Format menu; each of *uv, vw,* and *uw* is a phase-space graph.

If you have students who are interested in pursuing systems modeling beyond the level of this unit, you may wish to investigate purchasing specialized modeling software. There are several programs that are well suited to this task. Among them are Stella II, ithink, and PowerSim.

INDIVIDUAL WORK 7

Balancing Act

See Annotated Teacher's Edition.

ASSESSMENT PROBLEM A6.10

A Complicated Feedback System

This problem extends the predator-prey system to include four species. Students are asked to read graphs of population time-series and interpret their contextual meaning. While causal reasoning is necessary, the problem does not require computation or formula construction. This problem is appropriate for use after Activity 7.