THE HOT DOG STAND LOCATION PROBLEM

DAN TEAGUE AND FLOYD BULLARD

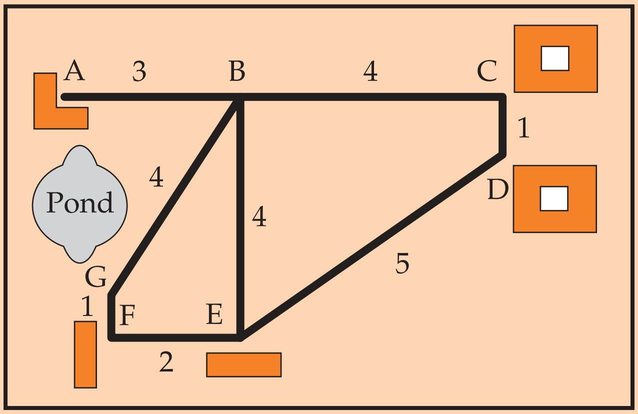

FIGURE 1: MAP OF CAMPUS.

A map of a portion of a college campus is given in **Figure 1**. The map shows the walking paths and dormitories in this section of campus and the approximate distances (in 100 feet) between locations. Your roommate has convinced you to open a hot dog stand on weekends at one of the intersections along the walkways. You would like the stand to be as convenient as possible for the students. Where on campus should you set up your stand?

Questions to get started:

What is your definition of convenient? What do you want to optimize when placing the stand? For example, do you want the shortest average distance for students? Do you want the smallest maximum distance to the stand? What other criterion could be used?

Suppose the dormitories are located at positions A, C, D, E, and F and the numbers of students in each dorm are 200 in A, 300 in each of C and D, and 100 each in E and F. Does the desired position of the stand change?

How do other variations in the scenario effect your model? For instance, consider placing the stand along a path, adding a path, or having two hot dog stands.

Hot Dog Stand Location

When given this problem, many students will want additional information about the campus. Where are the classroom buildings located relative to the dorms? Can students walk across the grass lawns? Are there benches around the pond? Are any of the dorms popular hangout spots?

Such student questions should not be discouraged. In a real-life situation the answers to these questions may be not merely relevant, but decisive in determining the best place for a hot dog stand. However, the additional information also makes the problem more complex. Modeling problems are often best answered by first solving a simplified version of the problem, then refining the solution by gradually introducing more realism and complexity to the model. What is presented here is a simplified problem, and the different solutions (all proposed by students) are based on no more than the information given.

We can make a model of the campus by setting up a network that displays the paths across campus. Associated with each path is the length of the path from intersection to intersection. **Figure 2** illustrates one such model.

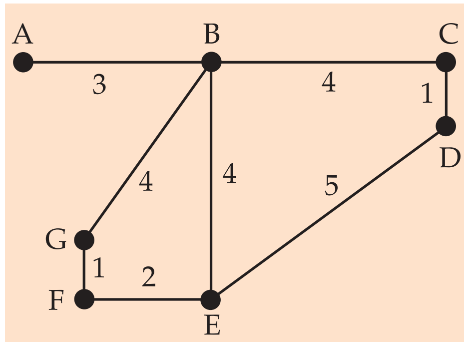

FIGURE 2: MAP OF CAMPUS.

Since students come to the stand primarily from the dorms, one approach is to find the distance from each of the dorms to a stand placed at every possible location. **Table 1** contains the distances from the dorms to the possible locations of the stand. The columns represent the possible locations of the hot dog stand and the rows provide the distance from the dorm sites to that position. Also included in the table are the maximum distance from a dorm to the location of the hot dog stand and the average distance from the stand to the dorms.

Based on this table, if we place the hot dog stand at Location B, then none of the students have to walk more than 500 feet to get from their dorm to the stand. Placing the stand at Location A would mean that some students would have to walk 800 feet to reach the stand. If we want to minimize the farthest anyone has to walk from the dorms we would place the hot dog stand at Location B. However, by placing the stand at location B, the hot dog stand will be an average of 420 feet from each dorm. By placing the hot dog stand at location E, the average distance from the dorms to the stand will be only 400 feet. If we want to minimize the average distance from the dorms, we should place the stand at Location E.

Which of the two criteria given above, maximum distance or average distance, do you think is better for this problem? What position best combines these two characteristics? If you read the Winter 1995 *Consortium* Number 56, you may recognize the question found in *Everybody's Problems*, "The Computer Problem." In that problem we considered several different ways to combine two desired criteria. The most commonly used methods involved the sum or product of the two measurements. These two combined measures are added to the table in **Table 2.**

From Table 2, we see that Location B does the best job of combining the criteria of maximum distance and average distance.

"Center of mass" Solution

Students, particularly those who have studied physics, will often try to find the "center of mass" of the campus, and select a location as close to this center as possible. To find the center of mass, they will generally use a model of the campus that is coordinatized as shown in **Figure 3**.

To find the center of mass of the campus, we can use the coordinates of the dorms, and find the average of the x-coordinates and the y-coordinates. This is $\bar{x} = \frac{-3+-2+0+4+4}{5} = \frac{3}{5}$ and $\bar{y} = \frac{0+0+3+4+4}{5} = \frac{11}{5}$. The location (0.6, 2.2) is the center of the campus and is marked with a * on the graph in Figure 3. The closest location to this center on a walkway is indicated by Location H. If the hot dog stand is set up at Location H, the maximum distance to any dorm is 6.8 (from D) and the average is 4.76. The sum is

Distance from Dorm to Stand

Location of Stand	A	C	D	E	F	Max.	Avg.
A	0	7	8	7	8	8	6.0
B	3	4	5	4	5	5	4.2
C	7	0	1	6	8	8	4.4
D	8	1	0	5	7	8	4.6
E	7	6	5	0	2	7	**4.0**
F	8	8	7	2	0	8	5.0
G	7	8	8	3	1	8	5.4

TABLE 1: DISTANCES TO STAND.

Distance from Dorm to Stand

Location of Stand	A	C	D	E	F	Max.	Avg.	Sum	Prod.
A	0	7	8	7	8	8	6.0	15.2	55.8
B	3	4	5	4	5	**5**	4.2	**10.4**	**26.4**
C	7	0	1	6	8	8	4.4	14.0	45.0
D	8	1	0	5	7	8	4.6	13.6	41.4
E	7	6	5	0	2	7	**4.0**	11.2	29.4
F	8	8	7	2	0	8	5.0	14.2	46.8
G	7	8	8	3	1	8	5.4	13.4	43.2

TABLE 2: COMBINING THE TWO CRITERIA.

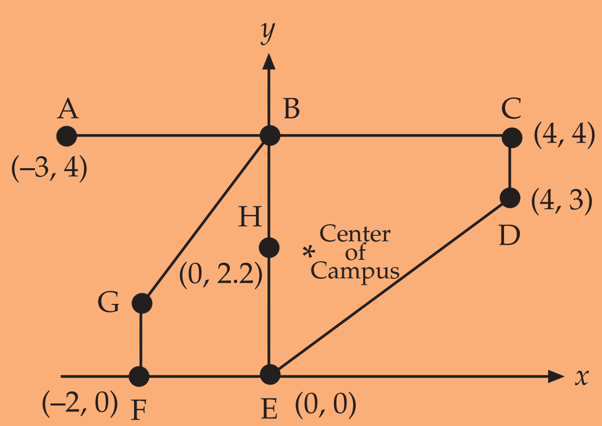

FIGURE 3: COORDINATIZED GRAPH OF CAMPUS.

11.56 and the product is 32.368. Location B is better on all measures. This solution would be particularly attractive if it was common for students to walk across the grass lawns.

Least Squares Solution

Students who have studied some curve fitting often try a least squares method. This will progressively "penalize" a location for being too far away. For example, a distance of 3 is more than twice as bad as a distance of 2, since $2^2 = 4$ and $3^2 = 9$.

By this measure, Location B is the best and Location A the worst. (See **Table 3.**) Notice that while Location E had the smallest average distance, it does not have the smallest sum of squares.

INCLUDING THE DORMS

How does the size of the dorms affect our placement? One way to include the size of the dorm is to weight the distance by the number of students. In this situation the location may need to be closer to the large dorms and farther from the smaller dorms. Rather than measure in yards, we will measure in student-yards. Perhaps we would rather have 50 students walk 100 feet, than 100 students walk 80 feet. We can accomplish this weighting by multiplying each distance by the number of students having to walk that distance. Equivalently, we could weight all distances from A by 2, from C and D by 3, and E and F by 1. This will keep the numbers more managably small. (See **Table 4.**)

Distance from Dorm to Stand

Location of Stand	A	C	D	E	F	SS.
A	0	7	8	7	8	243
B	3	4	5	4	5	**102**
C	7	0	1	6	8	183
D	8	1	0	5	7	159
E	7	6	5	0	2	127
F	8	8	7	2	0	198
G	7	8	8	3	1	187

TABLE 3: SUM OF SQUARES SOLUTION.

Distance from Dorm to Stand

Location of Stand	A	C	D	E	F	Max.	Avg.	SS.
A	0	21	24	7	8	24	12.0	1130
B	6	12	15	4	5	15	8.4	446
C	14	0	3	6	9	**14**	**6.2**	**322**
D	16	3	0	5	7	16	**6.2**	339
E	14	18	15	0	2	18	9.8	749
F	16	24	21	2	0	24	16.6	1277
G	14	24	24	3	1	24	13.2	1350

TABLE 4: DISTANCES TO STAND WEIGHTED BY DORM POPULATION.

When the sizes of the dorms are considered, we have new locations that satisfy our various criteria. Now Location C minimizes the maximum of the weighted distances. The smallest average weighted distance is found at either Location C or Location D. Because dorms C and D are the largest, our weighting system shifts the location of the hot dog stand toward these two dorms. Based on these two criteria, Location C seems to be a good choice.

TWO STANDS

How will your location change if you get to set up two stands rather than just one. With two stands there are 21 different combinations to consider. We don't need to consider them all, since some will clearly be poor choices. First, look at using Locations A, B, C, and D, since those are the locations nearest the largest dorms. (See **Table 5.**)

With two stands, we see that, for these pairs, Locations B and D give the smallest maximum weighted distance whereas Locations A and D give the smallest average distance.

OTHER MODIFICATIONS

How would your choice for the location change if you knew that dorms A and C were female dorms and D, E, and F male dorms, and that only 30% of the females would likely eat at your stand while 80% of the males would likely eat at your stand?

Suppose the path between B and C and the path between E and D went uphill and that it is twice as hard to walk uphill as downhill. How would your choice of location change?

OTHER SCENARIOS

Emergency Phones

Suppose instead of a hot dog stand, you are interested in locating an emergency phone on campus. Which is the better criteria: maximum distance or average distance? Would it be good to weight the distances by the number of students in each dorm? Do you want the phone beside a dorm or close to where students gather? Where on campus would you locate one emergency phone? Two emergency phones?

Newspapers

You want to deliver newspapers to the dorms, with each student getting a copy of the newspaper. The papers will be delivered to a central depository from which you must pick up the papers and walk them to the dorms. You can carry only 100 at a time. Where should you ask to have the 1000 papers delivered to minimize the total distance you have to walk? ❏

Pairs	A	C	D	E	F	Max.	Avg.	LS
AB	0	12	15	4	5	15	7.2	410
AC	0	0	3	6	8	8	3.4	109
AD	0	3	0	5	7	7	**3.0**	**83**
BC	6	0	3	4	5	6	3.6	86
BD	6	3	0	4	5	**6**	3.6	86
CD	14	0	0	5	7	14	5.2	306

TABLE 5.

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Dan Teague is an instructor of Mathematics at the North Carolina School of Science and Mathematics. He is a Presidential Awardee for North Carolina. You may email him at teague@ncssm.edu or write to: Dan Teague North Carolina School of Science and Mathematics 1219 Broad Street, Durham, North Carolina 27705

Floyd Bullard teaches math at the North Carolina School of Science and Mathematics. He attended the Johns Hopkins University (B S. 1991) and the University of North Carolina ath Chapel Hill (M.S. in statistics, 1997) and has taught math in Benin, West Africa as a Peace Corps volunteer. You may email him at bullard@ncssm.edu

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